Joint Constellation-Labeling Optimization for VLC-CSK Systems

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Abstract—In this letter, we have optimized the joint constellation-labeling for a finite number of signals in visible light communication (VLC) systems with color-shift keying (CSK) modulation by maximizing the pragmatic mutual information (PMI). First, an equivalent parallel channel model of the VLC-CSK system is constructed. Then the PMI is derived to be the objective function of the newly formulated joint constellation-labeling optimization problem of the VLC-CSK system. Simulated annealing (SA) algorithm and modified interior point method (MIPM) are adopted to solve the joint constellation-labeling optimization problem. Simulation results show that the proposed methods improve the PMI performance compared to the existing constellation-labeling scheme of signals, especially in the low signal-to-noise ratio (SNR) region. Besides, it can been seen that the MIPM saves a large amount of computational time compared to the SA algorithm while achieving the same performance improvement.

Index Terms—Visible light communication, color-shift keying, pragmatic mutual information, joint signal-labeling optimization.

I. INTRODUCTION

VISIBLE light communication (VLC) using light-emitting diodes (LEDs) has become a promising candidate for indoor wireless communication due to its appealing advantages such as reusability of LED lighting infrastructures, freedom from hazardous electromagnetic radiation and costly frequency licensing [1]. Two kinds of LEDs can be used as the light source (i.e., the transmitter) for VLC system. One is the white phosphorescent LEDs, which utilize the optical spectrum inefficiently while preferred due to cost and complexity. The other one is the multi-color LEDs making use of the optical spectrum with high efficiency, of which, the red/green/blue (RGB) LEDs are the most widely used. If each color of RGB LEDs is modulated independently, it is possible to achieve a threefold increase in data throughput by wavelength division multiplexing. However, since undesirable light source that vary in both intensity and color over time may be created in this

way, color-shift keying (CSK) is outlined in standard IEEE 802.15.7 as an intensity modulation scheme that transmits data imperceptibly through the variation of the light color emitted by RGB LEDs [2].

In [3] and [4], the constellation optimization problem of maximizing the minimum Euclidean distance between constellation points was proposed and solved by billiards algorithm and interior point method, respectively. It was noticed that the labeling of constellations was not considered in [3] and [4], though it indeed had impact on the system performance. In fact, Gray mappings have been proved to be optimal labeling at high signal to noise ratios (SNRs) [5]. Unfortunately, Gray mapping is known for only several kinds of particular modulations such as QAM, QPSK, and PSK, but unsuitable for CSK modulation. To the best of our knowledge, how to label the constellations is not exploited when optimizing constellation positions. In this letter, we focus on the joint optimization of constellation-labeling for a finite number of signals. Our main contributions are summarized as follows:

1) Due to the fact that the encoder is separated from the modulator [2], an equivalent parallel channel model of the VLC-CSK system is established. The PMI expression for the VLC-CSK system is deduced as a function of the constellation positions and labelings based on the parallel channel model.

2) The joint constellation-labeling optimization problem maximizing the PMI is formulated under the constraints of the CSK modulation. As the PMI cannot be expressed in a closed form, we can resort to the simulated annealing (SA) algorithm to solve the jointly optimization problem [6]. Due to the fact that the PMI is differentiable, a modified interior point method (MIPM) is proposed to solve the joint optimization problem. Finally, simulation results show that MIPM can achieve the same PMI performance as the SA algorithm with significantly low-complexity.

The rest of this letter is organized as follows. In Section II, the system model is introduced. In Section III, the expression for the PMI with a finite number of signal constellations is deduced to formalize the joint constellation-labeling optimization problem and SA and MIPM algorithms are used to solve the problem. In Section IV, simulation results obtained by SA algorithm and MIPM are presented and analyzed. Finally, this letter is concluded in Section V.

II. SYSTEM MODEL

Fig. 1 illustrates a typical M-CSK modulated VLC-CSK system with RGB-LEDs, where $M = 2^m$ with m being a positive integer. In Fig. 1, a binary sequence of 0's or 1's with
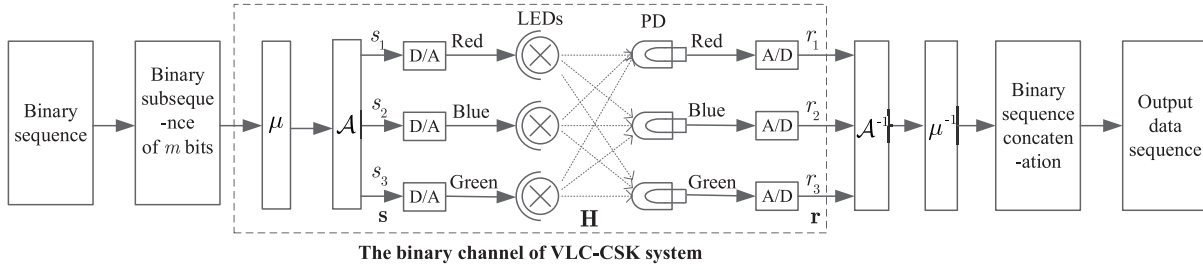
Fig. 1. Block diagram of transmission in VLC-CSK system.

equal probability, representing the data source, is splitted into subsequences of m bits. Each subsequence corresponds to a labeling code $\boldsymbol{b} = (b_1, \ldots, b_m)$ in the binary labeling code set $\mathcal{B} = \{\boldsymbol{b}^{(1)}, \boldsymbol{b}^{(2)}, \ldots, \boldsymbol{b}^{(M)}\} = \{0, 1\}^m$. Here, the probabilities $p(\boldsymbol{b}^{(1)}) = p(\boldsymbol{b}^{(2)}) = \cdots = p(\boldsymbol{b}^{(M)}) = \frac{1}{M}$. Let $\mathcal{A} = \{\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \ldots, \boldsymbol{s}^{(M)}\}$ be the signal set of CSK modulation with each signal vector $\boldsymbol{s}^{(i)} = (s_1^{(i)}, s_2^{(i)}, s_3^{(i)})$ representing a specific constellation point in the signal space. Then, a one-to-one label mapping μ is applied to the subsequences,

$$\mu : \mathcal{B} \ni \boldsymbol{b} = (b_1, \ldots, b_m) \rightarrow \boldsymbol{s}^{(i)} = \left(s_1^{(i)}, s_2^{(i)}, s_3^{(i)}\right) \in \mathcal{A}. \quad (1)$$

Similarly, $p(\boldsymbol{s}^{(1)}) = \cdots = p(\boldsymbol{s}^{(M)}) = 1/M$. Finally, the resulting signal $\boldsymbol{s}^{(i)}$ is transmitted over the VLC channel by emitting light from red, green and blue LEDs with intensities $s_1^{(i)}$, $s_2^{(i)}$ and $s_3^{(i)}$, respectively.

The channel model of VLC-CSK system can be denoted as

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \quad (2)$$

where $\boldsymbol{r} \in \mathbb{R}^3$, $\boldsymbol{H} \in \mathbb{R}^{3 \times 3}$ and $\boldsymbol{n} \in \mathbb{R}^3$ denote the receive vector, channel matrix and the noise vector, respectively. The light intensities emitted from the red/green/blue LEDs are measured by the photo-detectors (PDs) with red/green/blue optical filters, respectively, which constitute the received signal vector $\boldsymbol{r} = (r_1, r_2, r_3) \in \mathbb{R}^3$. The element of the channel matrix, $\boldsymbol{H} = [h_{ji}]_{3 \times 3}$, h_{ji}, represents the channel gain between the ith LED and the jth PD. $\boldsymbol{H}$ is assumed to be deterministic and invertible generally [2]. $\boldsymbol{n} = (n_{\text{r}}, n_{\text{g}}, n_{\text{b}}) \in \mathbb{R}^3$ is the zero-mean Gaussian noise vector with covariance matrix $\sigma^2 \boldsymbol{I}_3$, i.e., $\boldsymbol{n} \sim \mathcal{N}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_3)$, where $\boldsymbol{I}_3$ is the identity matrix.

At the receiver, the received signal $\boldsymbol{r}$ is demodulated and then demapped to get a binary code in $\mathcal{B}$. Finally, the binary codes are concatenated to recover the original binary sequence.

Due to the illumination requirements in CSK systems, the symbol vector $\boldsymbol{s}$ should satisfy the non-negativity and peak intensity constraint, lighting intensity constraint and color constraint [2].

$$\boldsymbol{0} \leq \boldsymbol{s} \leq \boldsymbol{p} \quad (3)$$

$$\boldsymbol{1}^T \boldsymbol{s} = I > 0 \quad (4)$$

$$E[\boldsymbol{s}] = \frac{1}{M} \sum_{i=1}^{M} \boldsymbol{s}_i = \boldsymbol{s}_{avg} \quad (5)$$

where $\boldsymbol{0} = (0, 0, 0)$, $\boldsymbol{1} = (1, 1, 1)$, $\boldsymbol{p} = (P_r, P_g, P_b)$ with P_r, P_g and P_b denoting the peak intensity of the red, green and blue LEDs, respectively. I is a constant representing the
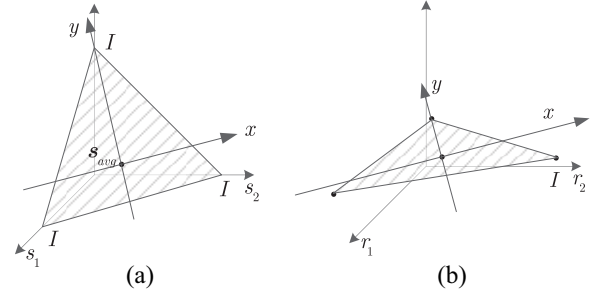

Fig. 2. Signal space of VLC-CSK modulation signals. (a) Ideal channel. (b) Linear distortion channel.

total operating intensity. $\boldsymbol{s}_{avg}$ is the expected perceived color. Constraints (4) and (5) guarantee that the intensity and color of the luminary will not fluctuate, thus limiting potential human health complications related to flickering light [7].

The signal space of $\boldsymbol{s}$ is a convex polygon in $\mathbb{R}^3$ according to the intensity constraints (3) and (4). If the channel is ideal, i.e., $\boldsymbol{H} = \boldsymbol{I}_3$ and $P_{r,g,b} \geq I$, the polygon is an equilateral triangle as shown in Fig. 2 (a) and all the constellations should be distributed inside the shadowed triangle. Specifically, $\boldsymbol{s}_{avg}$, located in the center of the equilateral triangle, represents white perceived color. Hence, we can construct a new 2-dimensional coordinate system $x\boldsymbol{s}_{avg}y$ as shown in Fig. 2 (a) to conveniently present the constellation positions and labelings. In reality, the transmitted signal may suffer from linear channel distortions arising from cross-channel leakage, noise correlation or wavelength-dependent received signal [3], which means, $\boldsymbol{H} \neq \boldsymbol{I}_3$. As a result, the constellations are distributed in a distorted 2-D triangle in another plane in $\mathbb{R}^3$ which could be described according to $\boldsymbol{H}$. Take the channel matrix considering the cross-channel leakage and noise correlation $\tilde{\boldsymbol{H}}$ in [3] as an example:

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} 0.5949 & -0.2805 & -0.0959 \\ -0.6068 & 1.0000 & -0.0311 \\ -0.1506 & -0.0230 & 0.3844 \end{bmatrix}$$

The signal space with $\tilde{\boldsymbol{H}}$ is given in Fig. 2 (b), where a new 2-D coordinate system *xoy* can be constructed as well.

III. OPTIMIZATION PROBLEM FORMULATION

To jointly optimize the signal constellation and labeling, the PMI is taken as the optimization function since it is decided by
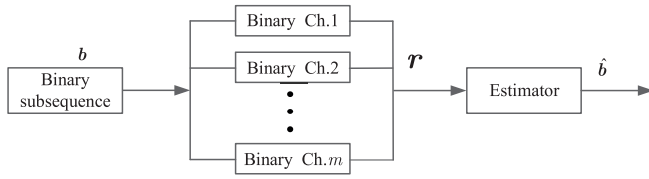
Fig. 3. Equivalent parallel channel model.

not only the signal constellation set $\mathcal{A}$ but also the binary labeling $\mathcal{B}$. To derive the PMI of VLC-CSK system, an equivalent parallel channel model should be firstly constructed.

A. Equivalent Parallel Channel Model

As the encoder is separated from the modulator to reduce the complexity of the receiver [2], an equivalent parallel channel model corresponding to the dotted-squared part in Fig. 1 is derived following the approach as in [8]. To do this, the following notations are adopted. For any given signal $\boldsymbol{s}$, let $\mu^i(\boldsymbol{s})$ denote the ith bit of the binary label associated to it, and $\mathcal{A}_b^i$ the subset of all the signals in $\mathcal{A}$ whose label takes on the value b in the ith bit position (i.e., $\mathcal{A}_b^i = \{\boldsymbol{s} \in \mathcal{A} | \mu^i(\boldsymbol{s}) = b\}$). Further, let $\bar{b}$ denote the complement of b.

Given $\mu^i(\boldsymbol{s}) = b \in \{0, 1\}$, the conditional probability density function (PDF) of $\boldsymbol{r}$ at the receiver is written as

$$p(\boldsymbol{r}|\mu^i(\boldsymbol{s}) = b) = \sum_{\boldsymbol{s} \in \mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s}, \mu^i(\boldsymbol{s}) = b)p(\boldsymbol{s}|\mu^i(\boldsymbol{s}) = b)$$
$$= 2^{-(m-1)} \sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s}). \qquad (6)$$

Suppose we have ideal channel state information (CSI) in the optimum receiver. The maximum likelihood bit metrics of the demodulator per symbol should be $p(\boldsymbol{r}|\boldsymbol{s}, \mu^i(\boldsymbol{s}) = b)$. To simplify calculation, ignoring the constant multiplier and taking a logarithm of it, the maximum log likelihood bit metrics can be expressed as

$$\lambda^i(\boldsymbol{r}, b) = \log \sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s}), \qquad (7)$$

for $b \in \{0, 1\}$ and $i = 1, \ldots, m$. Therefore, the input binary sequence is estimated according to the rule

$$\hat{\boldsymbol{b}} = \arg \max_{\boldsymbol{b} \in \mathcal{B}} \sum_i \lambda^i(\boldsymbol{r}, b). \qquad (8)$$

Given (8), the system model is reconstructed as an equivalent parallel binary channel model of m sub-channels as shown in Fig. 3. The parallel binary sub-channels are independent and memoryless. Each sub-channel corresponds to one bit in the label of the signals in $\mathcal{A}$.

B. Pragmatic Mutual Information

In accordance with the equivalent parallel channel model, let b denote the binary input for the ith subchannel. Since b and $\boldsymbol{r}$ are conditionally jointly distributed as $p(b, \boldsymbol{r}|i) = 2^{-m} \sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})$, the conditional mutual information of b

and $\boldsymbol{r}$ can be calculated by

$$I(b; \boldsymbol{r}|i) = 1 - E_{b,\boldsymbol{r}} \left[\log_2 \frac{\sum_{\boldsymbol{s} \in \mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s})}{\sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})} \right]. \qquad (9)$$

Then, the conditional average mutual information is obtained by averaging over all the subchannels, $I(b; \boldsymbol{r}) = \frac{1}{m} \sum_{i=1}^m I(b; \boldsymbol{r}|i)$. Because the m parallel channels are independent, the PMI can be expressed as m times $I(b; \boldsymbol{r})$, i.e.,

$$I(\mathcal{A}, \mu) = m - \sum_{i=1}^m E_{b,\boldsymbol{r}} \left[\log_2 \frac{\sum_{\boldsymbol{s} \in \mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s})}{\sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})} \right] \qquad (10)$$

In general, expectations in (10) cannot be calculated in a closed form. It is normally obtained by numerical integral via the Monte Carlo method.

C. Joint Constellation-Labeling Optimization Problem

The joint constellation-labeling optimization problem of maximizing the PMI can be formulated as

$$\max \ I(\mathcal{A}, \mu)$$
$$s.t. \ \boldsymbol{0} \le \boldsymbol{s} \le \boldsymbol{P}$$
$$\boldsymbol{1}^T \boldsymbol{s} = I > 0$$
$$E[\boldsymbol{s}] = \frac{1}{M} \sum_{i=1}^M \boldsymbol{s}_i = \boldsymbol{s}_{avg} \qquad (11)$$

Since $I(\mathcal{A}, \mu)$ can not be expressed in a closed form, it is difficult to solve optimization problem (11) directly.

Actually, SA algorithm can be used to solve the joint constellation-labeling optimization problem [6]. Since SA is a well-developed heuristics algorithms, we briefly explain how the SA algorithm works. An initial constellation-labeling scheme and a cooling schedule have to be chosen first. If the logarithm cooling schedule $T_t = A / \log(t + 1)$ is adopted, then the global optimal can be reached, provided A is large enough [9]. Whereas the linear cooling schedule $T_t = T_{t-1} - \alpha_t$ can not guarantee the global optimality, it speeds up the SA algorithm. Then, at any step t, we randomly choose a constellation point and move it to a random new position inside the shadowed triangle in Fig. 2. If the PMI is increased by this movement, we accept the new position. On the other hand, if the PMI decreases, the new position is accepted with probability $e^{T_t \Delta \mathcal{I}}$, where $\Delta \mathcal{I}$ is the difference between the PMI of the constellation and labeling at time t and $t - 1$.

Since the SA algorithm belongs to a global randomized searching algorithm in essence, it needs a large amount of computation which is really time consuming. A MIPM with a tricky assumption is proposed to solve the joint optimization problem (11).

We realize that $I(\mathcal{A}, \mu)$ is differentiable if we apply the tricky assumption of fixing the labeling of the constellations when computing $I(\mathcal{A}, \mu)$. For instance, given a signal set $\mathcal{A} = \{\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \ldots, \boldsymbol{s}^{(M)}\}$, we can assume that the binary labeling for $\boldsymbol{s}^{(i)}$ is the binary expression of $(i - 1)$. As a result, only the constellation position remains to be the optimization variable of the joint optimization problem and it is continuous and differentiable. Consequently, interior point method (IPM)

Algorithm 1 Modified Interior Point Method

Input: maximum iteration time N; SNR γ
Initialize: iteration time $n = 1$; the PMI $\mathcal{I}^* = 0$;
While: $n \leq N$

    Choose an initial constellation points $\mathcal{A}_n^0$ randomly
    Calculate the optimum constellation points $\mathcal{A}_n^*$ by
interior point method and record corresponding PMI $\mathcal{I}_n^*$

      If $\mathcal{I}_n^* > \mathcal{I}^*$
          $\mathcal{A}^* = \mathcal{A}_n^*$; $\mathcal{I}^* = \mathcal{I}_n^*$
      End If
      $n = n + 1$
End While
Return: the optimal constellation $\mathcal{A}^*$, $\mathcal{I}^*$

Fig. 4. PMI comparison between standard and optimal constellation and labeling.

can be used to solve the joint optimization problem under such assumption. However, since $I(\mathcal{A}, \mu)$ is not convex, the optimized result is locally optimal. Therefore, we can run the IPM several times with different start points and then choose the $(\mathcal{A}, \mu)$ that maximizes the PMI as the optimal constellation-labeling scheme. The concrete MIPM algorithm is stated in Algorithm 1.

IV. OPTIMIZATION RESULTS

In the simulation, the SNR is defined as $\gamma = I^2/(3\sigma^2)$ since there are totally three PDs in VLC-CSK systems [10]. Without loss of generality, I is normalized to be 1. Therefore, different SNR implies different noise power σ^2. To observe the optimal constellation and labeling straightforward, the channel is assumed to be ideal, i.e., $\boldsymbol{H} = \boldsymbol{I}_3$ and the color constraint can be ignored temporarily.

The PMI lines calculated in accordance with the optimal constellations and labeling obtained by MIPM, SA algorithm and the standard ones for 4-CSK, 8-CSK and 16-CSK are shown in Fig. 4 for comparison. Note that the signal constellations in the standard are the same as the optimal constellations in [4] when the color constraint was not considered. It can be seen that the PMI performance improvement is significant in the low SNR region. As SNR increases, the PMI performance benefit based on the optimization is on the decrease. This is because the limit of $I(\mathcal{A}, \mu)$ is $\lim_{\gamma \to +\infty} I(\mathcal{A}, \mu) = m$ as $\gamma \to +\infty$. Even though, the joint constellation-labeling optimization remains useful for VLC-CSK systems due to the dimming demand. Given that CSK dimming employs amplitude dimming and controls the brightness by changing the current driving the light source [7], the SNR may fall into a relatively low level. Besides, if adaptive modulation is employed to achieve high information rate by improving the modulation order in high SNR situation, the optimized constellation-labeling scheme could improve the PMI performance before $I(\mathcal{A}, \mu)$ tends to its limit.

Though the SA algorithm and the MIPM achieve the almost same PMI performance in general, the SA algorithm performs a little bit worse occasionally. This is because the linear cooling schedule of SA algorithm is adopted in simulation which may lead to a local optimum. In addition, the computational complexity of the proposed MIPM algorithm is
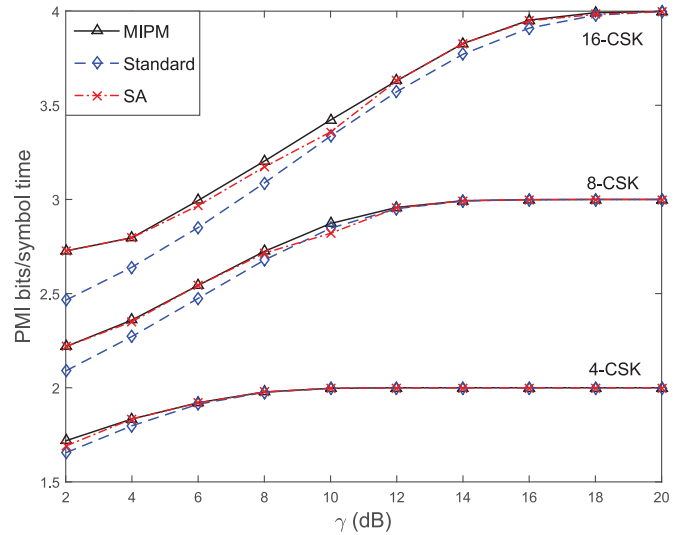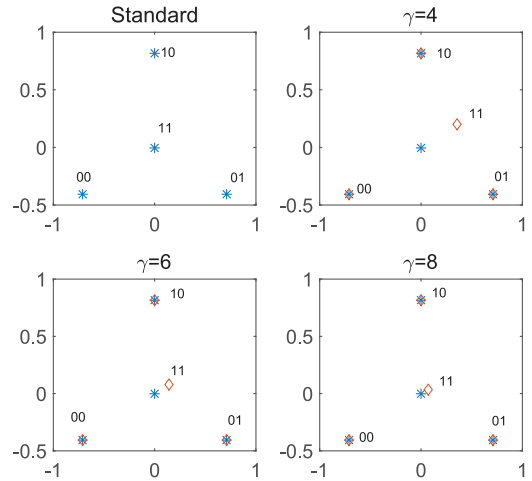

Fig. 5. Optimal constellation positions and labeling for 4-CSK.

$N\mathcal{O}(1)\sqrt{(6 + 2M)}\log(1/\epsilon)$ for a given convergence tolerance of ϵ [11]. Since SA algorithm is essentially a kind of exhaustive search algorithm, the complexity is far more than the MIPM algorithm. As the complexity of SA algorithm is hard to be written explicitly, we will show the complexity comparison in terms of the run time of the MIPM and SA program in the same personal computer configured to Inter Core I5 processor and 8G RAM. For instance, when we run the simulation program to jointly optimize the constellations and labelings for 8-CSK at $\gamma = 6$dB, MIPM consumes only about 5 minutes whereas SA algorithm needs about 6 hours. The time-saving quality of MIPM make sense when adaptive constellation and labeling is applied.

In addition, we present the optimization results of the pair $(\mathcal{A}, \mu)$ which maximize $I(\mathcal{A}, \mu)$ at given SNR for 4-CSK, 8-CSK and 16-CSK in the $x\boldsymbol{s}_{avg}y$ coordinate system in Fig. 5, Fig. 6 and Fig. 7, respectively. The signal constellations and labeling in the standard are shown in the first sub-figures for comparison. The optimal signal constellation
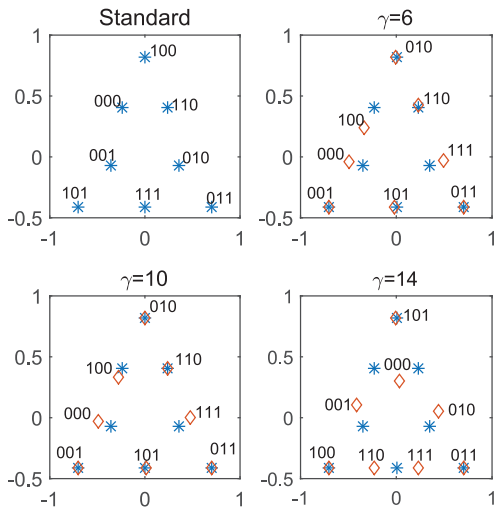
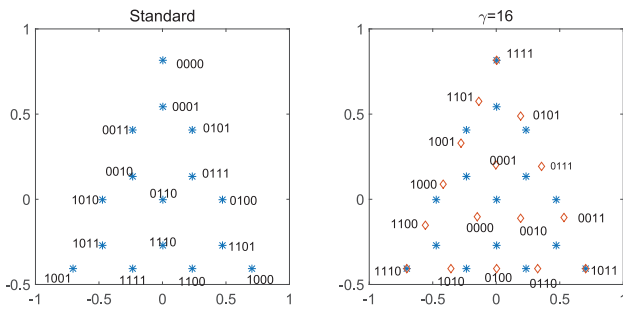Fig. 6.   Optimal constellation positions and labeling for 8-CSK.

Fig. 7.   Optimal constellation positions and labeling for 16-CSK.

Fig. 8.   Optimal constellation positions and labeling for 8-CSK with $\tilde{H}$.

The optimized constellation-labeling with channel $\tilde{H}$ is given in Fig. 8 for 8-CSK as an instance.

V. CONCLUSION

We have made an investigation of the joint constellation-labeling optimization problem with a finite number of signals in VLC-CSK systems in this letter. The PMI was defined and proposed as the objective function of the joint constellation-labeling optimization problem. Due to the fact that PMI has no closed-form, we can resort to SA algorithm to solve the optimization problem. However, SA algorithm's computation complexity is very high and time-consuming. Then, we proposed the MIPM algorithm to solve the joint optimization problem, which can save a large amount of time and achieve an even better performance.

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and labeling for selected γ are represented by the red diamonds and the binary labelings nearby in the rest sub-figures. It can be seen that if the constellation and labeling for 4-CSK in the standard is adopted, two-bits error occurs once the constellation labeled as 11 is demodulated to be the constellation labeled as 00. Thus, it is better to enlarge the distance between the constellation labeled as 11 and the constellation labeled as 00 a little bit. All in all, no matter what the labelings of the constellations are, the key point is that the constellation moves further from the constellations labeled with large-Hamming-distance labeling and gets closer to the constellations labeled with low-Hamming-distance labeling. The move distance and direction are calculated according to the SNR based on the proposed algorithm. Besides, as γ increases, the optimal constellations and labelings approach the ones in the standard for 4-CSK modulation whereas the optimal constellations and labelings are different from the standard ones for 8-CSK and 16-CSK. The proposed joint constellation-labeling optimization may be more effective for large modulation orders.

For non-ideal channels, the joint constellation-labeling optimization problem and the algorithm to solve it are all the same a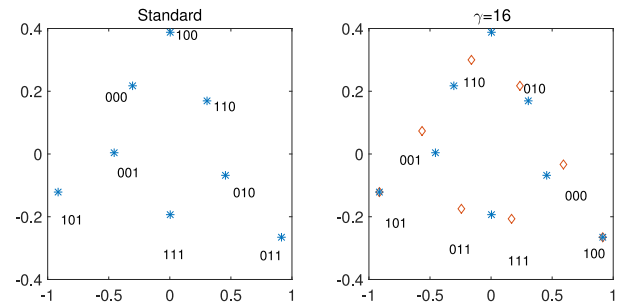s the ideal channel except for the distorted signal space.