PAPR Reduction Based on Parallel Tabu Search for Tone Reservation in OFDM Systems

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***Abstract*—In this letter, we focus on the high peak to average power ratio (PAPR) reduction problem in tone reservation-based orthogonal frequency division multiplexing systems. We first propose a parallel Tabu search (PTS)-based scheme to find a sub-optimal peak reduction tone (PRT) set. After finding the sub-optimal PRT set, we apply it in an adaptive iterative clipping and filtering (AICF) method for PAPR reduction. Furthermore, PAPR reduction and bit error rate (BER) performances are compared among the AICF method, the adaptive scaling, the adaptive amplitude clipping and the fast iterative shrinkage-thresholding algorithm schemes. Simulation results verify that the PTS-based PRT scheme can obtain better secondary peaks with lower computational complexity, and the AICF scheme effectively reduces PAPR with a faster convergence speed while its BER performance is only slightly worse than for the existing methods.**

***Index Terms*—OFDM, PAPR, tone reservation, parallel tabu search.**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has numerous advantages as its channels are orthogonal to each other. This contributes to avoiding narrow-band interference and multi-path fading [1]. But OFDM has several drawbacks such as high peak to average power ratio (PAPR). High PAPR results in bad effects on the orthogonality of transmitted signals in conjunction with a non-linear power amplifier. To reduce high PAPR, numerous conventional methods [2] have been proposed. Among the ways, the tone reservation (TR) method proposed by Tellado firstly [3] is a simple and efficient one without requiring transmission of side information.

The PAPR reduction performance of TR-clipping-based schemes relies on how to select the peak reduction tone (PRT) set and the optimal clipping threshold. However, it's a nondeterministic polynomial-time (NP)-hard issue [4] to find the optimal PRT set. Therefore, sub-optimal solutions are preferable, like the genetic algorithm (GA)-PRT [4], cross entropy (CE)-PRT [5] and invasive weed optimization and particle swarm optimization (IWOPSO)-PRT [6]. These methods converge slowly and have high computation complexity (CC).

To obtain a low PAPR signal, an adaptive scaling (AS)-TR algorithm [7] was proposed to reduce PAPR using a pre-determined clipping threshold. However, for the AS-TR it is hard to select an optimal clipping threshold. An adaptive amplitude clipping (AAC)-TR [4] method was proposed to improve AS-TR performance. A fast iterative shrinkage-thresholding algorithm (FISTA) scheme [8] was presented to address PAPR with power control on magnitude of reserved tones. Although the AAC-TR and FISTA schemes acquire better PAPR performance, the CCs of the AAC-TR and FISTA ones are also higher. The equation-based approach considers only in-band distortion at Nyquist sampling rate and does not take the effect of out-of-band noise into account [9].

In this letter, we focus on the efficiency of finding the PRT set and the CC decrease of methods for PAPR reduction. Specifically, we first propose a novel method based on parallel tabu search (PTS) to search a sub-optimal PRT set. Then, we propose an AICF algorithm to reduce PAPR after finding the sub-optimal PRT set. We compare performances of the AICF algorithm and traditional methods from two aspects of PAPR reduction and bit error ratio (BER). Simulation results validate the effectiveness of our proposed scheme in PAPR reduction and BER performance.

II. OFDM SYSTEMS AND TONE RESERVATION

A. OFDM Systems and PAPR

In OFDM systems, N independent data symbols X_k are modulated through phase shift keying (PSK) or quadrature amplitude modulation (QAM) on a set of N orthogonal subcarriers with the oversampling factor J. The OFDM block is expressed as $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$, where $(\cdot)^T$ means the transpose of a vector. After inverse fast Fourier transform (IFFT), the discrete time domain OFDM signal is generated as

$$x_m = \frac{1}{\sqrt{JN}} \sum_{k=0}^{JN-1} X_k \cdot e^{\frac{j2\pi mk}{JN}}, \quad m = 0, 1, \ldots, JN-1. \tag{1}$$

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The PAPR of $\mathbf{x}$ is defined as the ratio of maximum instantaneous power to the average power,

$$PAPR(\mathbf{x}) = \frac{\max\limits_{0 \le m < JN} |x_m|^2}{E[|x_m|^2]}, \tag{2}$$

where $\mathbf{x} = [x_0, x_1, \ldots, x_{JN-1}]^T$. The complementary cumulative distribution function (CCDF) is used to measure the capability of PAPR reduction. The CCDF is the probability that the PAPR of an OFDM symbol exceeds the predetermined threshold $PAPR_0$,

$$CCDF = Pr(PAPR > PAPR_0). \tag{3}$$

B. Tone-Reservation

For the TR-based technology, M tones are selected to be a PRT set for PAPR reduction. The peak-cancelling signal $\mathbf{c} = [c_0, c_1, \ldots, c_{JN-1}]^T$ is produced by the M reserved tones. Then the peak reduced signal $\mathbf{a}$ is generated by the peak-cancelling signal $\mathbf{c}$ plus the original signal $\mathbf{x}$ in the time domain,

$$\mathbf{a} = \mathbf{x} + \mathbf{c} = \mathbf{Q}(\mathbf{X} + \mathbf{C}), \tag{4}$$

where $\mathbf{Q}$ is the IFFT matrix. To prevent signal distortion, $\mathbf{C}$ and $\mathbf{X}$ are orthogonal in the frequency domain.

With TR [3], PAPR is redefined as

$$PAPR(\mathbf{a}) = \frac{\max\limits_{0 \le n < JN} |x_n + c_n|^2}{E[|x_n|^2]}, \tag{5}$$

where $\mathbf{c}$ should be selected to minimize the peak amplitude of the signal $\mathbf{a}$. According to [3], $\mathbf{c}$ is updated as follows:

$$\mathbf{c}^{(k+1)} = \mathbf{c}^{(k)} - \lambda_k \mathbf{p}[((j - j_k))_{JN}], \tag{6}$$

where λ_k is a scaling factor. $\mathbf{p}[((j - j_k))_{JN}]$ is a circular shift of $\mathbf{p}$ to the right by a value j_k, and $\mathbf{p} = \mathbf{QP}$ is a time domain kernel. The frequency domain kernel $\mathbf{P} = [P_0, P_1, \ldots, P_{N-1}]^T$ closely related to the PRT set is defined by

$$P_n = \begin{cases} 0, & n \in \mathcal{R}^C, \\ 1, & n \in \mathcal{R}, \end{cases} \tag{7}$$

where $\mathcal{R}$ means the index set of the PRT and $\mathcal{R}^C$ is the complementary set of $\mathcal{R}$ in $\mathcal{N} = \{0, 1, \ldots, N-1\}$. To obtain the optimal PRT set or the optimal $\mathbf{p}$, we need to address the optimization problem:

$$\mathcal{R}^* = \arg\min_{\mathcal{R}} ||[p_1, \ldots, p_{JN-1}]^T||_\infty. \tag{8}$$

The secondary peak (SP) of the time domain kernel $\mathbf{p}$ is used as a metric to evaluate the performance of the PRT set whose definition is

$$\text{SP} = \| \, [p_1, \ldots, p_{JN-1}]^T \, \|_\infty \, . \tag{9}$$

According to (5)-(9), the performance for PAPR reduction relies on the choice of the time domain kernel $\mathbf{p}$ or frequency domain kernel $\mathbf{P}$. We will present a parallel tabu search algorithm to solve (8) in the next section.

III. PARALLEL TABU SEARCH ALGORITHM AND ADAPTIVE ITERATIVE CLIPPING AND FILTERING

A. Parallel Tabu Search Algorithm

Tabu search (TS) was first proposed by Glover [10], which is evolved from classical local search methods to deal with combinatorial optimization problems. Like previous local search methods, the TS algorithm explores the neighbourhood to find a better solution. The neighbourhood is generated according to one Hamming distance between the initial solution and neighbour solutions. Differently, TS has a tabu list to avoid local search cycling. The tabu list contains all best solutions of recent searches. These solutions included in the tabu list will not be searched at following iterations.

Algorithm 1 PTS Algorithm for PRT Set

```
Input $N$, $M$, $U$, $K_1$ , $p_c$, $p_m$ and $K$.
Randomly generate four initial sequences. Do the TS for every sequence.
Update initial sequences and tabu list.
Repeat $K_1$ times and obtain four better sequences.
Do crossover and mutation to the four better sequences above. Select the best sequence of smallest SP.
Repeat $K$ cycles. Output the final PRT set with the smallest SP.
```

The parallel tabu search (PTS) algorithm is that several TS algorithms run simultaneously, which avoids the TS algorithm falling into a local optimal solution and can find a better solution. Compared with the TS, the PTS can shorten the search time and improve the search efficiency. In this letter, we combine the crossover and mutation operations from the GA with the PTS to further produce better solutions and accelerate the rate of search. To the best of our knowledge, this is the first application of the PTS to solve the PRT set problem. In order to better understand the PTS and GA algorithms, readers can refer to [4], [10], and [11].

B. PTS-Based PRT Position Search

In this subsection, we use the PTS to find the sub-optimal PRT set. In the PTS, four TSs work simultaneously to find better PRT sets.

Firstly, four initial binary sequences are randomly generated. Each sequence consists of M ones and $N - M$ zeros. The positions of the PRT set are marked by ones. We then compute SPs of every initial binary sequence. Secondly, each initial sequence takes TS independently. The process of TS is to find the sequence of smallest SP in the neighbourhood solution space. Thirdly, these sequences replace the initial four sequences, respectively. Fourthly, the tabu list is updated by adding the sequence of smallest SP in each iteration. Therefore, the solutions in the tabu list are not searched in the following iterations even if these solutions are in the neighbourhood solutions space. The length of the tabu list is U. After K_1 iterations, four parallel tabu searches end and obtain four better sequences.

Then, crossover and mutation operations are used to generate new solutions from the better sequences above at the probability of crossover p_c and mutation p_m. After crossover and mutation operations, the best sequence with the smallest SP is selected.

The whole procedure above is called a cycle. The PTS algorithm finds the best sequence after K cycles. Eventually, the PRT set is provided as output. In conclusion, the PRT search algorithm based on the PTS is summarized in Algorithm 1.

Algorithm 2 AICF-Based Algorithm for PAPR Reduction

```
Input OFDM symbols, initial $\gamma_i$, $i$, maximal iteration number $i_{max}$.
Calculate $P_{av}$ and $T_i$.
Calculate $v_i$, $V_i$, $H_i$ and $h_i$.
Get PAPR reduced signal $x_{i+1}$ by (12).
Update the CR $\gamma_i$ by (13).
$i = i + 1$. The algorithm ends until $i = i_{max}$.
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C. AICF Method to Reduce PAPR

After finding the sub-optimal PRT set by the PTS algorithm, we apply the AICF algorithm to reduce PAPR. The AICF algorithm is similar to the AS-TR [7] and AAC-TR methods [4], which also consist of iterative clipping and filtered operations. Firstly, the initial clipping ratio (CR) γ_i is set and the initial clipping threshold T_i is computed

$$T_i = \gamma_i * P_{av}, \tag{10}$$

where P_{av} is the average power of signal x_i and *i* is the iteration number. Then the AICF clips the time signal x_i to get a clipping noise v_i when the amplitude of x_i exceeds T_i

$$v_i = (|x_i| - T_i)e^{j\arg x_i}, \tag{11}$$

where $|x_i|$ denotes the amplitude of x_i and argx_i represents the phase of x_i.

Secondly, because clipping noise v_i must meet tone reservation constraints, so v_i is first converted to frequency domain V_i by a fast Fourier transform (FFT), i.e., $V_i = \mathrm{FFT}(v_i)$. Thirdly, V_i is projected to the reserved PRT set, and removes the out-of band components of V_i, that can be achieved by a filtering operation. Specifically, we construct a length *JN* filter *F* whose components are 1 in the positions of the PRT set and 0 in other locations. As the V_i passes the filter *F*, we achieve a filtered clipping noise $H_i = V_i * F$ in the frequency domain. Fourthly, by carrying out an IFFT for H_i, we obtain a filtered clipping noise h_i in the time domain, i.e., $h_i = \mathrm{IFFT}(H_i)$. Then, the PAPR reduced signal is given by

$$x_{i+1} = x_i - h_i \tag{12}$$

The CR γ_i is updated [12] by

$$\gamma_i = \frac{A_{max}}{A_{ave}}, \tag{13}$$

where A_{max} and A_{ave} denote the maximum and average amplitude of x_{i+1} respectively. The proposed AICF-based PAPR reduction algorithm is summarized in Algorithm 2.

D. Complexity Analysis of AICF Algorithm

In the AICF algorithm, main computational costs depend on the computations of frequency domain noises V_i and clipping noises h_i after filtering. The computations require a pair of FFT-IFFT, so the complexity of the AICF algorithm is $\mathcal{O}(JN\log(JN))$, which is the same as for the AAC-TR and AS-TR algorithms. But the CR γ of the AAC-TR algorithm is invariable. The CR γ of the AICF algorithm is related to the maximum amplitude and average amplitude of the PAPR reduced signal. This leads to the faster convergence of the AICF algorithm, which will be verified in the next section.

TABLE I
SECONDARY PEAK (SP) AND COMPUTATION COMPLEXITY (CC) COMPARISON

methods	CC	CC gap	SP
GA-PRT	$30 * 170 = 5100$	4080	0.4375
CE-PRT	$120 * 170 = 20400$	19480	0.3908
IWOPSO-PRT	$30 * 170 = 5100$	4080	0.4159
PTS-PRT	$(4 * 10 + 6) * 20 = 920$	-	0.3951

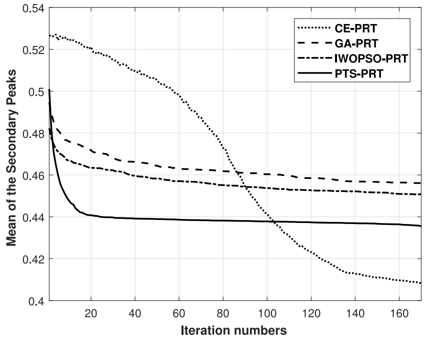

Fig. 1. Comparison of average secondary peak for different PRT sets.

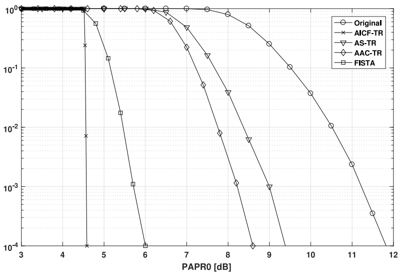

Fig. 2. CCDF comparison with PTS PRT.

IV. SIMULATION RESULTS

To compare the performance of the PTS and other algorithms, i.e., GA, CE and IWOPSO, 10^5 OFDM symbols with $N = 512$, $M = 32$ and $J = 4$ are randomly generated. For the parameters of GA, CE and IWOPSO in the simulation, we refer to [4]–[6], respectively. The PTS takes 4 independent tabu searches, $K_1 = 10$ iterations for each tabu search, the length of the tabu list is $U = 10$, crossover probability $p_c = 0.9$, mutation probability $p_m = 0.05$, 6 crossover and mutation operations are carried out within each cycle, and the number of total cycles is $K = 20$.

The PRT set obtained by the PTS is **PTS-PRT** = {9, 20, 21, 26, 56, 88, 93, 136, 151, 153, 176, 177, 245, 273, 285, 294, 295, 299, 313, 314, 324, 330, 339, 370, 391, 404, 406, 428, 472, 478, 485, 495}.

From Table I, the SP achieved by the CE algorithm is the smallest one in these four algorithms. However, the CC gap between the CE and PTS algorithms is $20400 - 920 = 19480$, while the SPs of the two algorithms only have a gap of 0.0043, which can be ignored. Therefore, the proposed method based on the PTS is better than the other three algorithms in finding the PRT set because it has the lowest CC.

In Fig. 1, the average SPs of the four algorithms are compared. Clearly, the PTS algorithm converges fastest and the

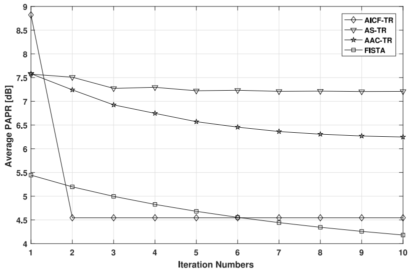

Fig. 3. Average PAPR comparison with different iteration numbers.

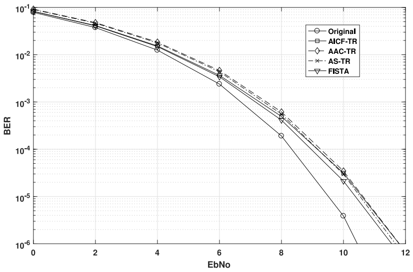

Fig. 4. BER performance comparison among various methods.

CC of the PTS one is the lowest within 100 iterations among the four algorithms. Above 100 iterations, The gap of the average SPs for the four algorithms is small. Our aim is to fast obtain a better PRT set with a lower computational complexity. So the PTS algorithm is the best choice among the four schemes.

In Fig. 2, the PAPR reduction performances of various schemes are compared with the identical PTS-PRT. The maximum number of iterations of the AICF-TR and FISTA schemes is 3, while that for the AAC-TR and AS-TR is 10. Other parameters of the AAC-TR, AS-TR and FISTA are referred to [4], [7], and [8], respectively. When the CCDF equals 10^{-4}, the PAPR of the original OFDM signal, the AS-TR, AAC-TR, FISTA and AICF-TR are 11.9 dB, 9.3 dB, 8.6 dB, 6 dB and 4.5 dB, respectively. Clearly, the PAPR reduction performance of the AICF-TR scheme is superior to others.

In Fig. 3, the average PAPRs of the four schemes are compared as the iteration number increases. The initial CR $\gamma = 4.5\,\text{dB}$ is the same for the AAC-TR, AS-TR and AICF-TR. When the iteration number is 2, the average PAPR of the AICF-TR converges to 4.5 dB while the AAC-TR, AS-TR and FISTA are still at the state of decrease. When the iteration number is 10, the average PAPRs are 4.5 dB, 6.2 dB, 7.2 dB and 4.2 dB for the AICF-TR, AAC-TR, AS-TR and FISTA, respectively. Fig. 2 and Fig. 3 show that the AICF-TR algorithm can acquire better PAPR decrease, lower CC and a faster convergence rate than the AAC-TR, AS-TR and FISTA schemes within 6 iterations.

To evaluate the BER performance of the whole OFDM system, the processed signal passes through a solid-state power amplifier (SSPA) model [1],

$$S_o = \frac{|S_i|}{[1 + (\frac{|S_i|}{A})^{2p}]^{\frac{1}{2p}}} e^{j\Theta}, \tag{14}$$

where S_o and S_i mean the output and input signals, respectively. p is 3 and A is 0.6 in the simulation.

Fig. 4 shows the comparison of the BER performance among the optimized signals over an additive white Gaussian noise (AWGN) channel. With the ideal SSPA (A taking $\max|S_i|$ and $p \rightarrow \infty$), the original signal gets the BER of 10^{-6} when the signal-to-noise ratio Eb/No is 10.3 dB. However, including the above specified SSPA model, Eb/No reaches 11.7 dB, 11.7 dB, 11.6 dB and 11.5 dB for the AICF-TR, AAC-TR, AS-TR and FISTA algorithms, respectively. At the BER of 10^{-6}, the BER performance of the AICF-TR is same with the AAC-TR, while that is inferior by 0.1 dB and 0.2 dB compared with the AS-TR and FISTA schemes.

V. CONCLUSION

In this letter, we propose a novel method based on parallel tabu search (PTS) to find the sub-optimal PRT set for PAPR reduction in OFDM systems. Compared with existing algorithms, the PTS scheme can find better PRT sets with lower computational complexity. Then, we adopt the AICF algorithm to decrease PAPR.Simulation results reveal that the proposed AICF algorithm achieves better PAPR reduction, faster convergence rate and a comparable BER as the AS-TR and FISTA algorithms.

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