Achieving Maximum Reliability in Deadline-Constrained Random Access With Multiple-Packet Reception

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***Abstract*—This paper considers random access in a communication channel, which is shared by N active users with saturated traffic. Following a slotted ALOHA-type protocol, each active user attempts to transmit in every slot with a common probability. It is assumed that the channel has the multiple-packet reception capability to enable the correct reception of up to M $(1 \leq M < N)$ time-overlapping transmissions. To support mission- and time-critical applications that require reliable delivery within a strict delivery deadline D (in units of slot), the aim of this paper is to achieve the maximum deadline-constrained reliability. First, we prove the uniqueness of the optimal transmission probability for any $1 \leq M < N$ and any $D \geq 1$. Second, we show it can be computed by a fixed-point iteration for all the cases. Third, for real-life scenarios where N may be unknown and changing, we develop a distributed algorithm for $M > 1$, which allows each active user to dynamically tune its transmission probability based on a method for estimating N. Simulation results verify our analysis and show that the proposed tuning algorithm is effective with near-optimal performance. In addition, as a special case (i.e., $D = 1$) of our study, the issue of saturation throughput maximization is completely addressed for the first time.**

***Index Terms*—ALOHA, multiple-packet reception, optimal transmission probability, reliability, throughput.**

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I. Introduction

A. Motivation

SINCE Abramson's seminal work [1] in 1970, ALOHA-type random access has been widely used for uncoordinated users to establish initial connection or transmit short packets in a variety of wireless networks. Traditional studies on ALOHA-type protocols are based on the *single-packet reception* (SPR) model: a transmission is successful if it does not overlap with any other transmission on the channel. However, this model has become more and more restrictive due to the advances in *multiple-packet reception* (MPR) techniques that enable the correct reception of time-overlapping transmissions. Hence, there is a natural interest in gaining a clear insight into the impact of MPR on the behavior of ALOHA-type protocols.

With this goal, considerable research efforts [2]–[10] have been made to the stability, throughput or delay issues under MPR. However, to our best knowledge, little has been reported for the deadline-constrained reliability, which is of primary importance in mission- and time-critical applications [14], [15], such as environmental sensing, health care monitoring and military target tracking. For these uses, the collected information needs to be delivered in a timely and reliable fashion to one or more central controllers, where the information is analyzed and the decisions are made.

To fill the gap in this field, this paper concentrates on achieving maximum reliability under MPR subject to a deadline constraint, that is, a head-of-queue packet will be dropped from the queue if it *cannot* be delivered within a strict delivery deadline $D \geq 1$ (in units of slot).

B. Related Work

For slotted networks, a generalized MPR channel can be fully characterized by the complete set of the conditional reception probabilities $q_{\mathcal{R},\mathcal{T}}$ with $\mathcal{R} \subseteq \mathcal{T}$. Here $q_{\mathcal{R},\mathcal{T}}$ is the probability that only and all the packets sent from users in the set $\mathcal{R}$ are received correctly provided that only and all users in the set $\mathcal{T}$ transmit in a particular slot. For many different physical-layer parameters, such as the channel conditions and the signal processing technologies, the complete set of $q_{\mathcal{R},\mathcal{T}}$ can take many different forms.

The first attempt to study ALOHA-type protocols under MPR was made by Ghez *et al.* [2], [3]. They analyzed stability properties for the infinite-user case under the symmetric MPR channel, which assumes that $q_{\mathcal{R},\mathcal{T}}$ depends only on $|\mathcal{R}|$ and $|\mathcal{T}|$. Naware *et al.* [4] extended the stability study to the finite-user case, and investigated the average delay for capture channels. Luo *et al.* [5] derived the throughput and stability regions for a standard MPR channel, which assumes that simultaneous transmissions are unhelpful for the reception of any particular group of transmissions.

Recently, much attention has been devoted to the throughput performance under M-user MPR, which assumes that $q_{\mathcal{T},\mathcal{T}} = 1$ if $|\mathcal{T}| \leq M$, and $q_{\mathcal{R},\mathcal{T}} = 0$ for all $\mathcal{R}$ if $|\mathcal{T}| > M$. Gau [6], [7] derived the saturation and non-saturation throughput for the finite-user case. Zhang *et al.* [8], [9] shown that the maximum throughput increases superlinearly with M for both finite-user and infinite-user cases, and further shown that this superlinear scaling holds under bounded delay-moment requirements. Bae *et al.* [10] investigated the optimal transmission probability that maximizes the saturation throughput in the finite-user case under some unproved technical conditions.

When the number of active users N is unknown and time-varying, to achieve excellent access performance of ALOHA-type protocols, it is essential to enable each active user to estimate N at runtime. Many approaches have been proposed under SPR. The method in [17] estimates N based on the channel state in the previous slot, while the schemes in [18], [19] estimate N with statistics of consecutive idle and collision slots. All of them require N to follow a Poisson distribution. The extension of the estimation algorithm to the MPR case is rarely reported. We are aware of only one previously proposed algorithm in [10], which estimates N according to the collected information on the number of users involved in each successful transmission.

C. Contributions

Following [6]–[13], we restrict our attention to the M-user MPR channel. The contributions of this paper are as follows.

i) We prove the uniqueness of optimal transmission probability for the maximization of deadline-constrained reliability when a finite number, $N > M$, of active users contend to transmit with a common transmission probability, and further show it can be obtained by a fixed-point iteration. This work can be seen as a generalization of the work in [16] that only focused on the SPR channel.

ii) For real-life scenarios where N is unknown and changing over time, we propose a tuning algorithm that allows each active user to dynamically tune its transmission probability based on a method for estimating N. Simulation results show that the tuning algorithm enables each active user to obtain a deadline-constrained reliability close to the theoretical limit under a variety of dynamic scenarios.

The closest work to ours is [10]. It will be shown in Section III that the maximization of saturation throughput considered in [10] can be studied as a special case (i.e., $D = 1$) of the maximization of deadline-constrained reliability. To clarify and highlight new technical challenges, the differences between [10] and our work are discussed as follows.

i) To prove the uniqueness of optimal transmission probability for $D = 1$, the corresponding proof in [10] is based on a key hypothesis $\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\mathbb{E}[m^2|\tau]}{\mathbb{E}[m|\tau]} < N$ (m is the number of users involved in each successful transmission when the transmission probability τ is used). However, it was claimed in [10] that this hypothesis has been proved only when $M \leq 3$ due to extremely complex algebraic manipulations. On the other hand, as this hypothesis is also necessary in our work, we prove $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N - 1$ for an arbitrary M in Lemma 2. Here, $H_1(\tau) = \frac{\mathbb{E}[m^2|\tau]}{\mathbb{E}[m|\tau]} - 1$. To significantly simplify the algebraic manipulations, a recursive technique is used in the proof.

ii) As shown in the proof of Theorem 5, it is straightforward that the unique optimal transmission probability for $D = 1$ can be obtained by a fixed-point iteration under the condition $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N$. But when $D > 1$, the corresponding proof involves not only more complex algebraic manipulations but also more complex logics.

iii) We observe that the method for estimating N in [10] would lead to biased estimates of N when two groups of potential users become active at different time instants, due to that each active user uses the adopted transmission probability to update N. Whereas, the estimation method in our tuning algorithm does not depend on the adopted transmission probability, and thus would not lead to biased estimates. A performance comparison between these two algorithms will be provided in Section V-B.

In sum, compared with [10], the saturation throughput maximization under M-user MPR is completely addressed for the first time, a series of new proofs are provided due to the generalization to introduce the new parameter D, and a tuning algorithm without biased estimates is proposed.

The remainder of this paper is organized as follows. In Section II, we describe the considered system model. In Section III, we prove the uniqueness of optimal transmission probability that maximizes the deadline-constrained reliability, and show it can be computed by a fixed-point iteration. In Section IV, we propose a tuning algorithm, by which each active user can achieve a reliability level close to the theoretical limit at runtime. In Section V, simulation results verify the accuracy of our analytical results and the effectiveness of the proposed tuning algorithm. Section VI concludes this paper.

II. System Model

We develop our analytical model based on the following assumptions as adopted in [8], [10], [16].

A. Network Model

There are a finite number, $N \geq 2$, of active users, each of which has an infinite backlog of packets to send to one or more common receivers. All the active users are within the interference range of each other, and are within the reception range of each receiver. We will first consider that N is known in Section III and then consider that N is unknown and time-varying in Section IV.

The channel time is divided into time slots of an equal length, and every packet exactly fits in a slot. Following a slotted ALOHA-type protocol, each active user is required to transmit a packet with a common transmission probability τ in every time slot, $0 \leq \tau \leq 1$.

Based on the following three considerations, it is assumed that every packet is neither acknowledged nor retransmitted. First, when a packet is multicast to multiple receivers, it is difficult to implement acknowledgments. Second, when the distance between a sender and a receiver is long, the sender is required to wait for an acknowledgment during a long time duration, which is undesirable for a timely delivery. Third, for group-based detection that applies redundancy to avoid false alarms [20], a receiver detects an event if it receives a certain number of reports from different active users within a time interval since the event occurrence. Under this scenario, the mechanism to ensure reliability is usually the built-in redundancy in sensor coverage rather than acknowledgments and retransmissions.

Every packet should be delivered within a strict delivery deadline $D\,(D \geq 1)$ (in units of slot), which is defined as the duration from the moment of its arrival at the head of the queue to the completion of its transmission. Packets that are not transmitted within the deadline will be dropped from the queue. In other words, it is required that the worst-case channel access delay *cannot* be larger than D slots. Some recent work that considered such a deadline constraint can be found in [16], [21]–[23] for an SPR channel, and [24]–[28] for a multichannel system.

B. Reception Model

This paper considers the M-user MPR channel, on which a packet can be correctly received if at most $M-1$ other packet transmissions overlap with it and is unrecoverable otherwise. To avoid some trivial cases, we assume $1 \leq M < N$.

Various packet separation algorithms built on various forms of diversity can be used to enable such MPR capability [29]. An example with spatial diversity is introduced as follows. Assume that a receiver is equipped with an array of M antennas and the channel is constant over a slot. Consider that only users $i_1, i_2, \ldots, i_K$ are transmitting simultaneously at a particular slot. Let $\mathbf{H}$ denote the channel matrix of size $M \times K$, where the entry in the l-th row and j-th column represents the channel coefficient from user i_j to the l-th receiver antenna. Let $\mathbf{x}_{i_j}$ denote the signal vector transmitted by user i_j, and let $\mathbf{y}_l$ denote the signal vector received by the l-th receiver antenna. Then the mixed signal at the receiver, denoted by $\mathbf{Y}$, can be written as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{W},$$

where $\mathbf{X} = [\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \ldots, \mathbf{x}_{i_K}]^T$, $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_M]^T$, and $\mathbf{W}$ denotes the additive noise. Due to the random access nature, the problem to be addressed is to obtain $\mathbf{X}$ from $\mathbf{Y}$ without priori knowledge of K and $\mathbf{H}$, when $K \leq M$. As pointed out in [8], [13], [29], it can be solved by blind packet separation algorithms [30]–[32], which utilize characteristics of the input sources, such as finite alphabet, amplitude and phase variation. Taking the algorithm in [30] as an instance, the basic rationale is to estimate K from the singular values of $\mathbf{Y}$, estimate $\mathbf{H}$ by $\mathbf{YX}^+$ where $\mathbf{X}^+$ is the pseudoinverse of $\mathbf{X}$, and obtain $\mathbf{X}$ as the global minimum of the following minimization problem:

$$\min_{\mathbf{X} \in \Omega} \|\mathbf{Y} - \mathbf{HX}\|_F^2,$$

where Ω is the adopted finite alphabet and $\|\cdot\|_F^2$ is the Frobenius norm.

In practice, packet errors may occur due to wireless channel fading. Denote by $P_{err}(K)$ the packet error rate when K packets are simultaneously transmitted under M-user MPR. Typically, $P_{err}(K) \geq P_{err}(K')$ if $K \geq K'$. In this paper, we assume that packets are well protected by error correction codes and linear multiuser detection is deployed at the receiver. These mechanisms allow us to assume that $P_{err}(K)$ is close to 0 if $K \leq M$ and is close to 1 otherwise [8].

C. Channel Estimation Capability

When considering N is unknown and time-varying in Section IV, we assume that each untransmitting active user can detect if the number of ongoing transmissions is $0, 1, \ldots, \omega$ or $> \omega$, and the time required to do so is shorter than the slot length. This detection can be implemented by equipping each user with an array of ω antennas [8].

The following considerations make such implementation feasible in distributed wireless networks. First, it will be shown in Sections IV–V that we only require $w \leq M$, which indicates that a growth of population would not lead to an increase in hardware complexity. Second, the signal processing algorithms for detection [33] are usually based on the eigenvalue analysis of the correlation matrix, and thus the computational burden is acceptable. Third, the current technology in antenna miniaturization [34] is able to design arrays of 2.4G/5.8 Ghz antennas with acceptable sizes, which can be mounted on sensors or machine-type devices.

On the other hand, the impact of detection errors due to the hardware constraints or hidden terminals will be discussed in Section V.

III. Optimal Transmission Probability

Given any real number $\tau \in [0, 1]$ and integer $D \geq 1$, let $P_D(\tau)$, called *successful delivery probability* (SDP), be the probability that a packet will be successfully received within the delivery deadline D under the common transmission probability τ. Consider a tagged active user. Let λ denote the number of packets transmitted by the other $N-1$ active users in a time slot. It is easy to see that λ follows a binomial distribution with parameters $N-1$ and τ. Then, for $i = 0, 1, \ldots, N-1$, we have

$$\mathbb{P}(\lambda = i) = \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}. \tag{1}$$

Furthermore, the value $P_D(\tau)$ can be obtained as:

$$\begin{aligned} P_D(\tau) &= \sum_{k=1}^{D} \tau (1-\tau)^{k-1} \mathbb{P}(\lambda \leq M-1) \\ &= \left(1 - (1-\tau)^D\right) \sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}. \end{aligned} \tag{2}$$

In this section, we aim to obtain the optimal transmission probability for maximizing $P_D(\tau)$, i.e.,

$$\tau_D^{opt} \triangleq \arg\max_{\tau\in[0,1]} P_D(\tau).$$

Note that τ_D^{opt} may not be unique by definition.

Remark 1: One can observe from (2) that $P_1(\tau)$ and individual saturation throughput have the same expression. Here, individual saturation throughput is defined as the time average of the number of packets successfully transmitted by an active user. Hence, τ_1^{opt} is indeed the optimal transmission probability for maximizing the saturation throughput under M-user MPR, which is the focus of [10].

Remark 2: When $M = 1$, τ_D^{opt} is the optimal transmission probability for maximizing the SDP within the delivery deadline D under SPR, which has been derived in [16].

We see from (2) that, when D is fixed, $P_D(\tau)$ is a continuous function of τ on the closed interval [0, 1]. Hence, by The Extreme Value Theorem, τ_D^{opt} exists. In the remainder of this section, we shall show the uniqueness of τ_D^{opt}, and present how to obtain it by a fixed-point iteration.

A. Uniqueness of τ_D^{opt}

By adopting the notation

$$f_1(\tau) \triangleq \sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}, \tag{3}$$

$$f_2(\tau) \triangleq \sum_{i=0}^{M-1} i\binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}, \tag{4}$$

the derivative of $P_D(\tau)$ with respect to τ can be written as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) &= D(1-\tau)^{D-1} f_1(\tau) \\ &\quad + \left(\frac{f_2(\tau)}{\tau(1-\tau)} - \frac{(N-1)f_1(\tau)}{1-\tau}\right)\left(1-(1-\tau)^D\right) \\ &= \left((N+D-1)(1-\tau)^{D-1} - \frac{N-1}{1-\tau}\right) f_1(\tau) \\ &\quad + \left(\frac{1-(1-\tau)^D}{\tau(1-\tau)}\right) f_2(\tau) \qquad (5) \\ &= \frac{1}{\tau(1-\tau)}\left(\left(1-(1-\tau)^D\right) f_2(\tau)\right. \\ &\quad \left. -\left(N-1-(N+D-1)(1-\tau)^D\right)\tau f_1(\tau)\right). \qquad (6)\end{aligned}$$

Obviously, $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau)$ is continuous on the interval (0, 1).

We first provide some properties of τ_D^{opt} by observing $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau)$ in (5) and (6).

Lemma 1: For any $D \geq 1$ and $1 \leq M < N$,

i) τ_D^{opt} is a solution of $\frac{d}{d\tau}P_D(\tau) = 0$, and

ii) τ_D^{opt} must lie on the interval $\left[1-(\frac{N-1}{N-1+D})^{\frac{1}{D}}, 1\right)$.

The proof of this lemma is provided in Appendix A.

Following (6) and Lemma 1 (i), we see that τ^* is a solution of $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) = 0$ if and only if it is a solution of

$$H_1(\tau) - H_2(\tau) = 0, \tag{7}$$

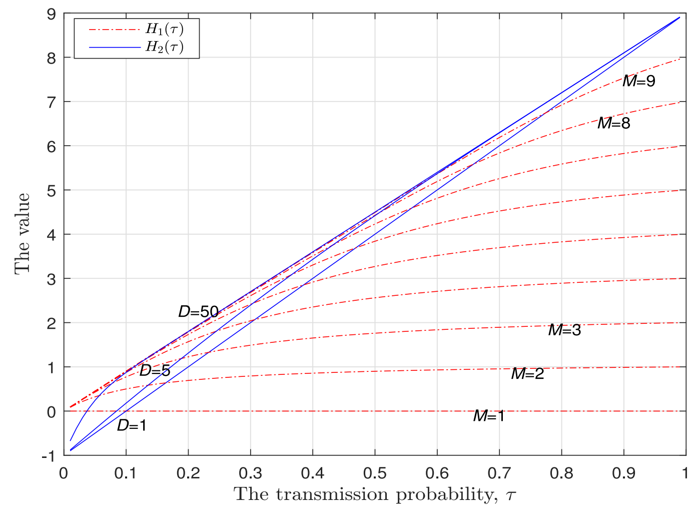


Fig. 1. $H_1(\tau)$ for the varying M and τ, $H_2(\tau)$ for the varying D and τ when $N = 10$.

where

$$H_1(\tau) \triangleq \frac{f_2(\tau)}{f_1(\tau)}, \quad H_2(\tau) \triangleq \tau\left(N+D-1-\frac{D}{1-(1-\tau)^D}\right).$$

In what follows, we will show that the equation (7) has a unique solution on the interval $\tau \in (0,1)$ by investigating the monotonicity of $H_1(\tau)$ and $H_2(\tau)$ separately.

Lemma 2: For any $1 \leq M < N$ and $\tau \in (0,1)$, we have $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) = 0$ if $M = 1$, and otherwise

$$0 < \frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N-1.$$

The proof of Lemma 2 is provided in Appendix B.

Remark 3: In the context of saturation throughput maximization, Bae *et al.* in [10] derived the uniqueness of τ_1^{opt} based on a key hypothesis $\frac{\mathrm{d}}{\mathrm{d}\tau}\big(H_1(\tau)+1\big) < N$. Here, we formally prove it for an arbitrary M, and thus completely address the issue of saturation throughput maximization.

Lemma 3: For any $D \geq 1$, $N \geq 2$ and $\tau \in (0,1)$, we have

$$\frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau) > N-1.$$

The proof of Lemma 3 is provided in Appendix C.

To illustrate that $0 \leq \frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N-1 < \frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau)$ on the interval $\tau \in (0,1)$ as obtained by Lemmas 2–3, a numerical example is presented in Fig. 1, which plots $H_1(\tau)$ and $H_2(\tau)$ for the varying τ, M and D when $N = 10$.

Now, we are ready to derive the uniqueness of τ_D^{opt}.

Theorem 4: For any $D \geq 1$ and $1 \leq M < N$, the equation (7) has a unique solution on τ in the interval (0, 1), denoted by τ^*, and $\tau_D^{opt} = \tau^*$.

Proof: Suppose there are two distinct solutions to the equation (7) in (0, 1). By The Mean Value Theorem, there exists a solution of $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau)$. However, by Lemmas 2 and 3, we have

$$\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N-1 < \frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau), \quad \forall\tau \in (0,1).$$

This implies a contradiction to $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau)$. Hence we conclude that the equation (7) has a unique solution on τ in the interval $0 < \tau < 1$, which by Lemma 1 promises the uniqueness of τ_D^{opt}, and yields $\tau_D^{opt} = \tau^*$. ∎

B. A Fixed-Point Iteration to obtain τ_D^{opt}

For the case $M = 1$, which implies $H_1(\tau) = 0$, we from (7) obtain that

$$\tau_D^{opt} = 1 - \left(\frac{N-1}{N-1+D}\right)^{\frac{1}{D}}.$$

However, for $2 \le M < N$, τ_D^{opt} *cannot* be obtained in a simple closed form like the case $M = 1$. As such, we propose the following theorem to show that τ_D^{opt} can be obtained by a fixed-point iteration for an arbitrary M.

Theorem 5: Define a fixed-point iteration:

$$x_{k+1} = x_k \cdot \frac{H_1(x_k)+1}{H_2(x_k)+1} \qquad \text{for } k = 0, 1, 2, \ldots. \tag{8}$$

Given any initial guess $x_0 \in (0, 1)$, the sequence $x_0, x_1, x_2, \ldots$ converges to the fixed point τ_D^{opt} for any $D \ge 1$ and $1 \le M < N$.

Proof: As $0 \le \frac{\mathrm{d}}{\mathrm{d}x} H_1(x) < N - 1$ for $x \in (0, 1)$ by Lemma 2 and $H_2(x) = N - 1$ when $D = 1$, Theorem 4 implies that τ_1^{opt} can be obtained using the fixed-point iteration (8) with any initial guess $x_0 \in (0, 1)$. We next consider the cases with $D > 1$.

Define

$$g(x) \triangleq \frac{x(H_1(x)+1)}{H_2(x)+1}$$

on the domain (0, 1). By Theorem 4, the equation $g(x) = x$ has a unique solution at $x = \tau_D^{opt}$ in (0, 1). Therefore, the case that $x_0 = \tau_D^{opt}$ obtains the fixed point τ_D^{opt} since $g(\tau_D^{opt}) = \tau_D^{opt}$. So we consider $x_0 \ne \tau_D^{opt}$ in what follows.

Since $g(x) = x$ has a unique solution at $\tau_D^{opt} \in (0, 1)$, to prove the sequence $\{x_n\}_0^\infty$ converges to τ_D^{opt}, it suffices to show that when $D > 1$,

$$\begin{cases} x < g(x) < \tau_D^{opt}, & \text{for } x \in (0, \tau_D^{opt}), \\ \tau_D^{opt} < g(x) < x, & \text{for } x \in (\tau_D^{opt}, 1). \end{cases} \tag{9}$$

When $D > 1$, as proved in Appendix D that

$$\frac{\mathrm{d}}{\mathrm{d}x} g(x) > 0 \quad \text{for } x \in (0, 1), \tag{10}$$

we have

$$\begin{cases} g(x) < \tau_D^{opt}, & \text{for } x \in (0, \tau_D^{opt}), \\ \tau_D^{opt} < g(x), & \text{for } x \in (\tau_D^{opt}, 1). \end{cases} \tag{11}$$

Since that, in the interval (0, 1), $H_1(x) = H_2(x)$ only when $x = \tau_D^{opt}$ and $\frac{\mathrm{d}}{\mathrm{d}x} H_1(x) < \frac{\mathrm{d}}{\mathrm{d}x} H_2(x)$ by Lemmas 2–3, we have $0 \le H_1(x) < H_2(x)$ for $x \in (\tau_D^{opt}, 1)$. Then,

$$g(x) = x\frac{H_1(x)+1}{H_2(x)+1} < x, \quad \text{for } x \in (\tau_D^{opt}, 1). \tag{12}$$

Following the same argument, we have $H_1(x) > H_2(x)$ for $x \in (0, \tau_D^{opt})$. By the result in Lemma 3 that $\frac{\mathrm{d}}{\mathrm{d}x} H_2(x) > 0$ and the L'Hospital's Rule that $\lim_{x\to 0^+} H_2(x) = -1$, we further have $H_1(x) > H_2(x) > -1$ and thus

$$g(x) = x\frac{H_1(x)+1}{H_2(x)+1} > x, \quad \text{for } x \in (0, \tau_D^{opt}). \tag{13}$$

Therefore, (9) can be derived by combining (11)–(13), and then the result follows. ■

Note that Lemma 1 (ii) can be used to set a more reasonable initial guess in Theorem 5.

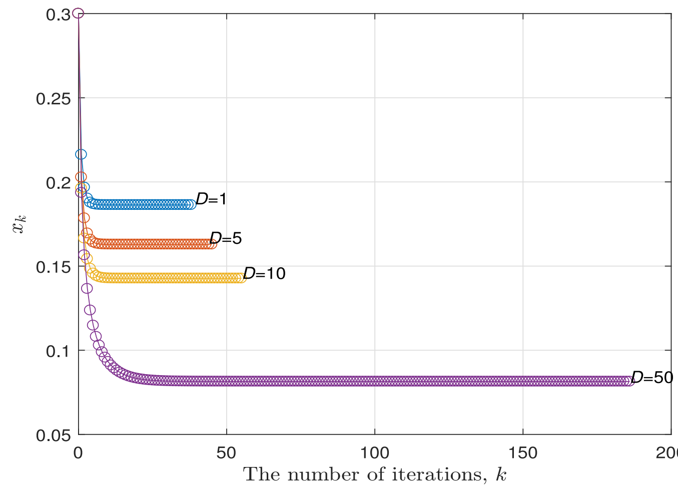


Fig. 2. The iteration procedure of x_k for the case of $N = 20$ and $M = 5$, when x_0 is set to 0.3.

A numerical experiment of applying Theorem 5 to find τ_D^{opt} is illustrated in Fig. 2 for the case of $N = 20$ and $M = 5$.

IV. Runtime Optimization

In the previous section, it has been shown that the optimal transmission probability can be obtained by a fixed-point iteration. However, this method requires a priori knowledge of the number of active users, N, which may be unavailable under some practical scenarios. In this section, we develop a tuning algorithm for $M > 1$, which allows each active user to dynamically tune the transmission probability based on a method for estimating N. It will be shown in Section V that this tuning algorithm is effective when N is unknown and time-varying, with near-optimal performance.

Consider a tagged active user. Recall the variable λ that denotes the number of packets transmitted by the other $N - 1$ active users in a time slot. By the distribution of λ in (1), for $1 \le i_1 < i_2 \le M$, we obtain the following equation:

$$\begin{aligned} &\frac{\mathbb{P}(\lambda = i_1)\mathbb{P}(\lambda = i_2 - 1)}{\mathbb{P}(\lambda = i_2)\mathbb{P}(\lambda = i_1 - 1)} \\ &= \frac{\binom{N-1}{i_1}\binom{N-1}{i_2-1}\tau^{i_1+i_2-1}(1-\tau)^{2N-1-i_1-i_2}}{\binom{N-1}{i_2}\binom{N-1}{i_1-1}\tau^{i_1+i_2-1}(1-\tau)^{2N-1-i_1-i_2}} \\ &= \frac{\binom{N-1}{i_1}\binom{N-1}{i_2-1}}{\binom{N-1}{i_2}\binom{N-1}{i_1-1}} = \frac{i_2(N-i_1)}{i_1(N-i_2)}. \end{aligned} \tag{14}$$

It then directly follows that

$$N = \frac{i_2(i_2 - i_1)}{i_1 \frac{\mathbb{P}(\lambda=i_1)\mathbb{P}(\lambda=i_2-1)}{\mathbb{P}(\lambda=i_2)\mathbb{P}(\lambda=i_1-1)} - i_2} + i_2. \tag{15}$$

It is assumed that each untransmitting user can detect if the number of ongoing transmissions is $0, 1, \ldots, i_2$ or $> i_2$. Hence, $\frac{\mathbb{P}(\lambda=i_1)\mathbb{P}(\lambda=i_2-1)}{\mathbb{P}(\lambda=i_2)\mathbb{P}(\lambda=i_1-1)}$ is locally measurable, which indicates that (15) provides a linear function for the tagged active user to estimate N without priori knowledge of other access parameters.

An update interval is defined as a span of consecutive of L time slots. We require the tagged active user to update the transmission probability at the beginning of the $(n+1)$th update interval, according to the following two estimates of the network status.

i) μ_n: the estimated value of $\frac{\mathbb{P}(\lambda=i_1)\mathbb{P}(\lambda=i_2-1)}{\mathbb{P}(\lambda=i_2)\mathbb{P}(\lambda=i_1-1)}$ at the end of the nth update interval;

ii) N_n: the estimated value of N at the end of the nth update interval.

The tagged active user initially guesses that there are N_0 active users, and sets $\mu_0 = \frac{i_2(N_0-i_1)}{i_1(N_0-i_2)}$. With given N_0, the transmission probability for the first update interval is obtained by Theorem 5. During the nth interval for $n = 1, 2, \ldots$, the proposed algorithm operates as follows.

i) *Update μ_n at the end of the nth update interval:* To evaluate $\frac{\mathbb{P}(\lambda=i_1)\mathbb{P}(\lambda=i_2-1)}{\mathbb{P}(\lambda=i_2)\mathbb{P}(\lambda=i_1-1)}$ at runtime, the tagged active user is required to record $\overline{A}_{i,n}$, the number of the slots during the nth update interval in which i users are simultaneously transmitting and the tagged active user is not transmitting, for $i = i_1 - 1, i_1, i_2 - 1$ and i_2.
Let $\overline{\mu}_n$ be the measure of $\frac{\mathbb{P}(\lambda=i_1)\mathbb{P}(\lambda=i_2-1)}{\mathbb{P}(\lambda=i_2)\mathbb{P}(\lambda=i_1-1)}$ during the nth update interval. Its value is calculated by

$$\overline{\mu}_n = \frac{\overline{A}_{i_1,n} \cdot \overline{A}_{i_2-1,n}}{\overline{A}_{i_2,n} \cdot \overline{A}_{i_1-1,n}}.$$

To avoid sharp changes in the estimated value, the tagged active user further applies an Exponential Moving Average filter as follows:

$$\mu_n = \delta \cdot \mu_{n-1} + (1-\delta) \cdot \overline{\mu}_n$$

where $\delta \in [0, 1]$ is a memory factor.

ii) *Update N_n at the end of the nth update interval:* By (15), the tagged active user obtains

$$N_n = \left\langle \frac{i_2(i_2-i_1)}{i_1\mu_n - i_2} + i_2 \right\rangle$$

where $\langle x \rangle$ is the integer closest to x.

iii) *Update the transmission probability at the beginning of the $(n+1)$th update interval:* With given N_n, the tagged active user updates the transmission probability by Theorem 5.

To avoid harmful measure of $\overline{\mu}_n$ due to occasional fluctuations of the network status, the following special operations are conducted.

i) We assume that the tagged active user knows there are at most $N_{\max}$ active users in the network.[1] If $\overline{\mu}_n$ is found to be smaller than $\frac{i_2(N_{\max}-i_1)}{i_1(N_{\max}-i_2)}$, a contradiction to the fact that (14) must be equal to or larger than $\frac{i_2(N_{\max}-i_1)}{i_1(N_{\max}-i_2)}$ as $M < N \leq N_{\max}$, the tagged active user sets $\overline{\mu}_n = \frac{i_2(N_{\max}-i_1)}{i_1(N_{\max}-i_2)}$.

ii) If $\overline{\mu}_n$ is found larger than $\frac{i_2(M+1-i_1)}{i_1(M+1-i_2)}$, a contradiction to (14) as $N > M$, the tagged active user sets $\overline{\mu}_n = \frac{i_2(M+1-i_1)}{i_1(M+1-i_2)}$.

iii) If $\overline{\mu}_n$ has an invalid value as $A_{i_2,n} \cdot A_{i_1-1,n} = 0$, the tagged active user sets $\overline{\mu}_n = \overline{\mu}_{n-1}$.

[1] All potential users are required to associate with the coordinator during the initialization phase of the network, so that the coordinator can broadcast the information of $N_{\max}$ to each potential user during this phase.

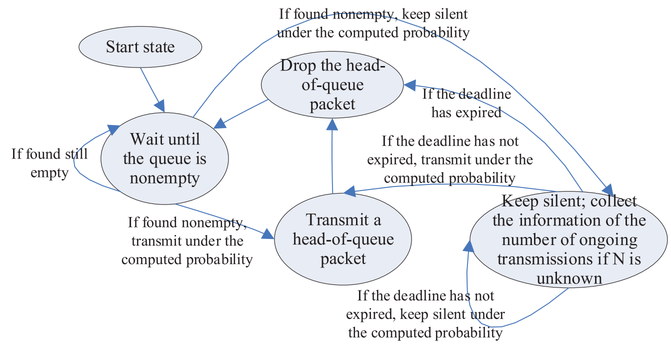


Fig. 3. Finite state machine for an active user.

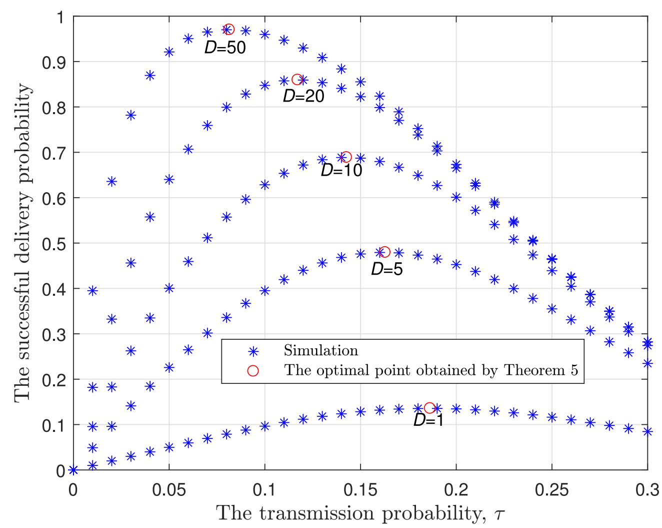


Fig. 4. The SDP as a function of τ for different values of D when $N = 20$ and $M = 5$.

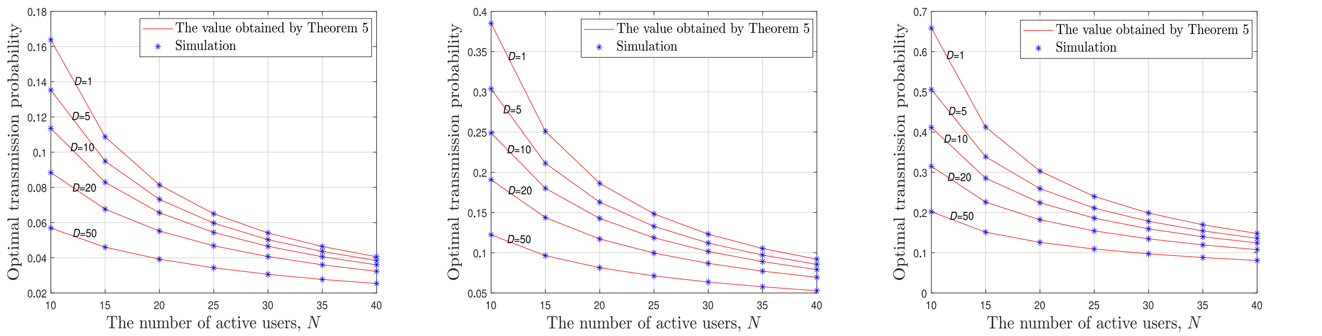
V. Simulation Results

In this section, to demonstrate the accuracy of the analytical results and the effectiveness of the tuning algorithm, we present simulation results with respect to different network configurations. We use the C programming language to develop an event-driven platform, which implements the finite state machines of the active users as shown in Fig. 3. The other simulation settings are in accordance with the descriptions in Section II. Specifically, the transmission rate is set to 54 Mbps, the slot time is set to 8 μs and the packet length is set to 432 bits. Unless otherwise specified, we assume that there is no error for detecting the number of ongoing transmissions, although we also conduct simulations with detection errors to broaden our study.

In the following, we will first consider stationary scenarios when N is known, and then consider dynamic scenarios when N is unknown. For each stationary scenario, the simulation point represents the average value over 10 independent runs, each of which consists of 10^6 slots. For each dynamic scenario, we only present the metrics of interest obtained from a single representative simulation run.

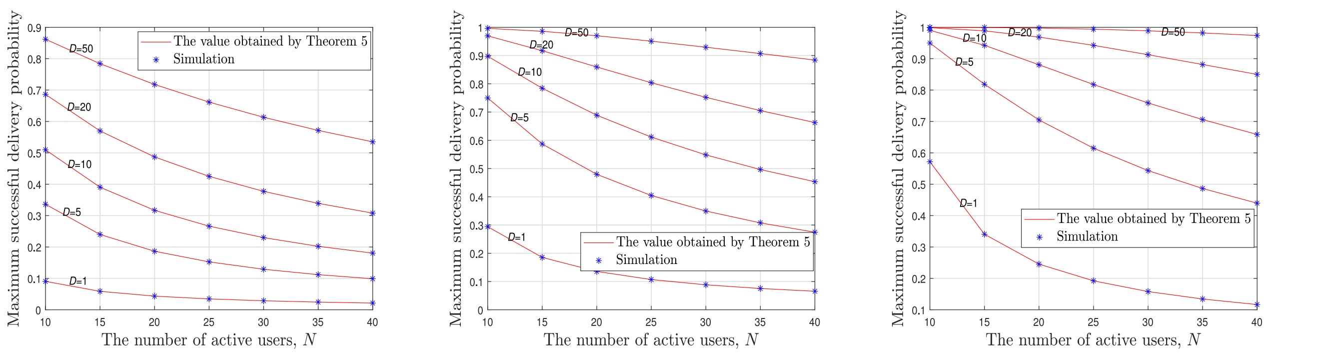
A. Stationary Scenarios When N is Known

Fig. 4 plots the simulative SDP versus the transmission probability τ for different values of D when $N = 20$ and $M = 5$. It is



(a) $M = 2$ (b) $M = 5$ (c) $M = 8$

Fig. 5. The optimal transmission probability as a function of N for different values of M and D.



(a) $M = 2$ (b) $M = 5$ (c) $M = 8$

Fig. 6. The maximum SDP as a function of N for different values of M and D.

verified that there exists a unique τ_D^{opt} for each D. Moreover, we see that τ_D^{opt} is accurately computed by Theorem 5 in each case.

Figs. 5–6 show the τ_D^{opt} and the maximum SDP as a function of N for different values of M and D, respectively. We see a good agreement between the analytical results obtained by Theorem 5 and simulation results in all the scenarios. In Fig. 5, as expected, we see that the τ_D^{opt} becomes smaller when N increases, due to the severer contention level. We also observe that the τ_D^{opt} becomes larger when M increases or D decreases. The reason is that an active user needs to be more aggressive in accessing the channel if more concurrent packets can be successfully received or the active user wants to successfully send out a packet within a shorter delivery deadline. On the other hand, as expected, Fig. 6 shows that any of a decrease of N, an increase of M and an increase of D would lead to an increase of the maximum SDP.

In particular, we note that when N and M are given, the τ_D^{opt} for $D > 1$ is smaller than that for $D = 1$. This phenomenon indicates that the throughput maximization degrades the SDP for $D > 1$, and vice versa.

B. Dynamic Scenarios When N is Unknown

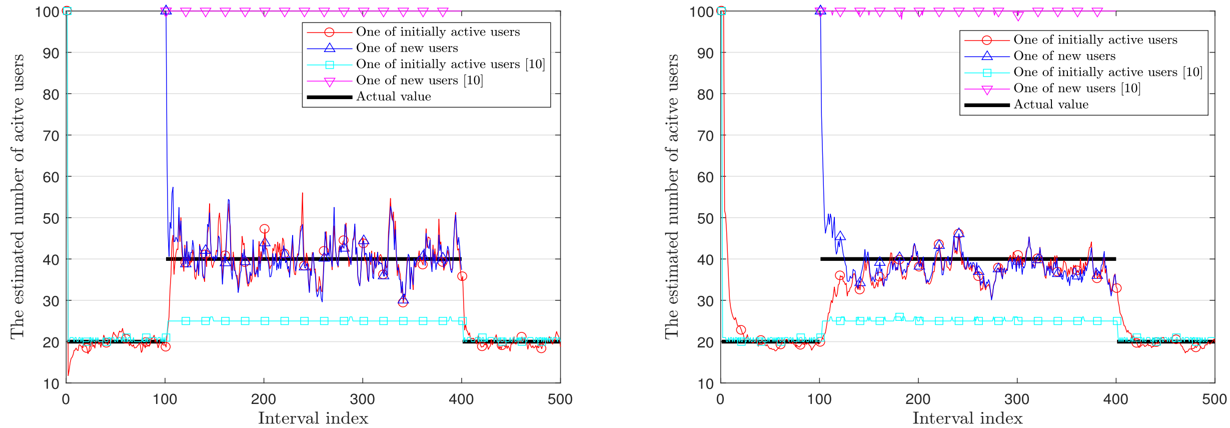
By setting $i_2 = M$ and $i_1 = 2$ in (15), we investigate the performance of the proposed tuning algorithm when N sharply changes. In detail, 20 users are always active from the 1st to the 500th updated interval, and 20 new users are active only from the 101st updated interval to the 400th updated interval. Each active user knows there are at most $N_{max} = 100$ active users, and initially guesses there are $N_0 = 100$ active users.

1) The Cases With $D = 1$: As the SDP for $D = 1$ is identical to the individual saturation throughput, we start by discussing the performance for $D = 1$.

Fig. 7 shows the estimated N under $M = 5$ for different tuning parameters. To characterize individual behavior and improve the readability, we only present the real trajectories of two representative users in the same run. We observe that, when the actual N is changed to 20, the estimate rapidly adapts to the change and keeps a high level of accuracy; when the actual N is changed to 40, the estimate is still able to capture the change within a few update intervals, but the slope increases, i.e, the accuracy of the estimation degrades. This phenomenon can be attributed to the fact that as the actual N increases, the fluctuation in measuring $\overline{\mu}_n$ is amplified.

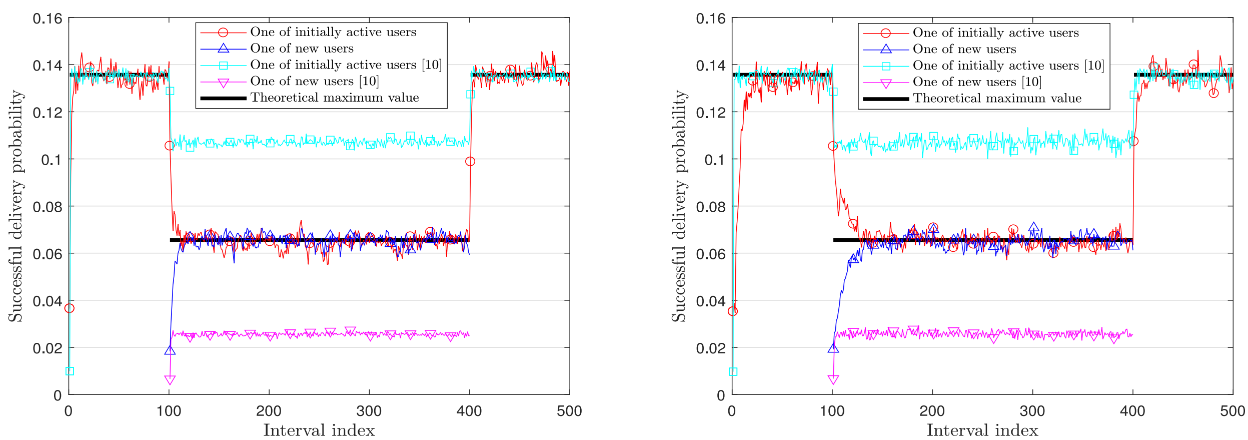
By tuning the transmission probability according to the estimated N, these two representative users achieve SDPs close to the theoretical limit at runtime as shown in Fig. 8. Moreover, we find that even when the oscillation of the estimated N becomes higher as actual N increases, the SDP still keeps relatively stable. This phenomenon is due to the fact that the SDP is less sensitive to the change in transmission probability as actual N increases. Notice that the transient period is a dominant factor to cause SDP degradation.

To evaluate the fairness of the proposed tuning algorithm, we further examine the mean and standard deviation of individual SDPs for every stage. As shown in Table I, we find that the



(a) $L = 50000, \delta = 0.7$ (b) $L = 20000, \delta = 0.9$

Fig. 7. The estimated number of active users for $M = 5$ and $D = 1$ when the number of active users sharply changes.



(a) $L = 50000, \delta = 0.7$ (b) $L = 20000, \delta = 0.9$

Fig. 8. Individual SDP for $M = 5$ and $D = 1$ when the number of active users sharply changes.

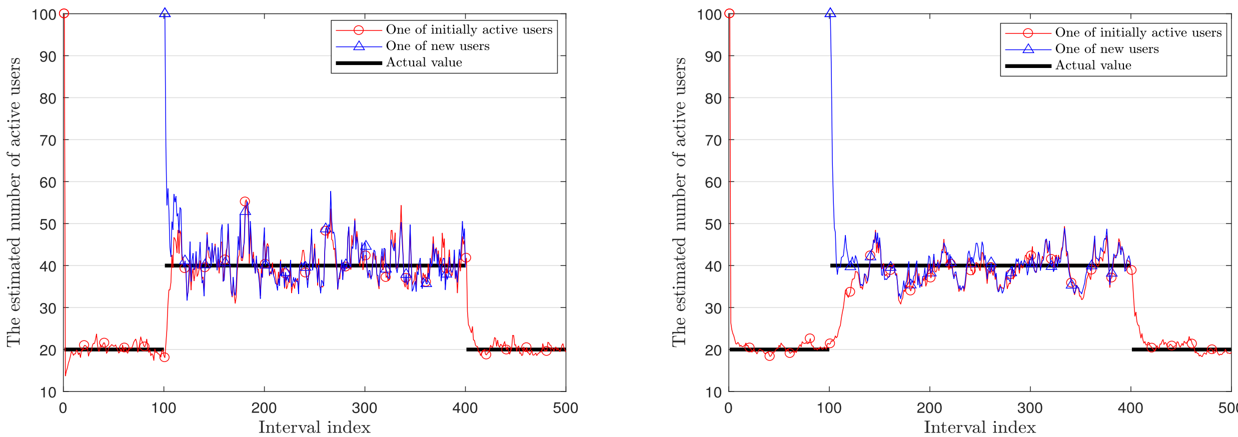
TABLE I
THE MEAN AND STANDARD DEVIATION OF INDIVIDUAL SDPS FOR EACH STAGE WHEN $M = 5$ AND $D = 1$

Tuning parameters	Interval index	Theoretical maximum	Mean in our simulation	Standard deviation in our simulation
$L = 50000$, $\delta = 0.7$	$1 - 100$	0.1357	0.1328	1.240×10^{-3}
	$101 - 400$	0.0656	0.0646	5.539×10^{-4}
	$401 - 500$	0.1357	0.1344	6.796×10^{-4}
$L = 20000$, $\delta = 0.9$	$1 - 100$	0.1357	0.1290	2.744×10^{-3}
	$101 - 400$	0.0656	0.0647	1.650×10^{-3}
	$401 - 500$	0.1357	0.1337	1.045×10^{-3}

mean value achieves at least 95.06% of the theoretical maximum value, while the standard deviation is at most 2.550% of the mean. These results indicate that the proposed tuning algorithm enables every active user to achieve near-optimal performance under this dynamic scenario.

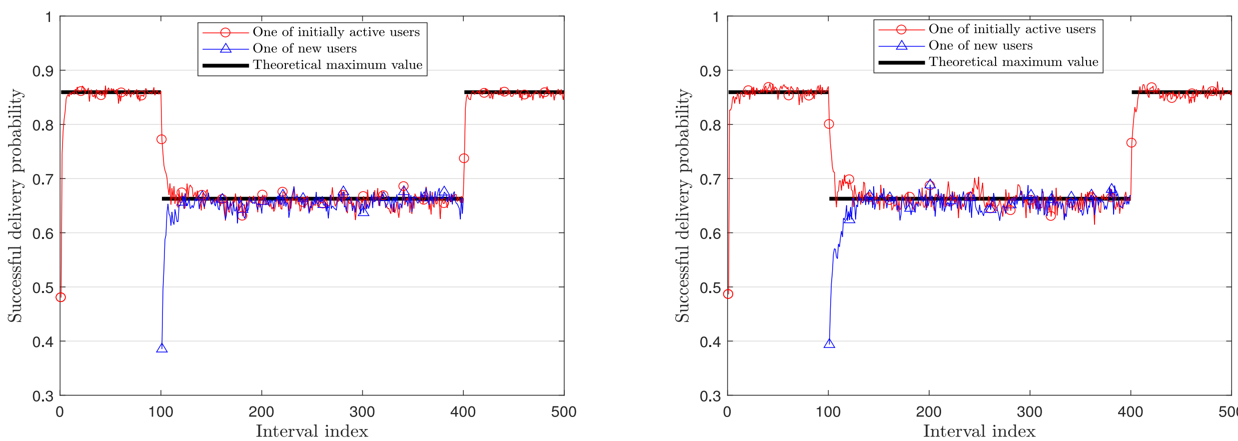
In addition, we consider the runtime throughput optimization in [10] for comparison purposes. The method therein updates the transmission probability with an estimate of N, and then uses the measure of $\frac{\mathbb{E}[m^2|\tau]}{\mathbb{E}[m|\tau]}$ to estimate new N relying on the adopted transmission probability. As shown in Fig. 7, we find that the estimate in [10] is not able to keep track of the change when 20 new users become active. This phenomenon is because when two groups of potential users become active at different time instants, they may adopt different transmission probabilities and then have biased estimates of N. Due to this bias, as shown in Fig. 8, unfair throughput performance is unavoidably caused. Whereas, our algorithm does not depend on the adopted transmission probability, and thus would not lead to biased estimates under this scenario.

2) The Cases With $D > 1$: Fig. 9 shows the estimated N of two representative users for $M = 5$, $D = 20$ and different tuning parameters. We find that the estimate is still able to capture the changes within a few update intervals, although the accuracy degrades when the actual N increases. Fig. 10 further shows that with the estimated N, these two representative users are able to achieve SDPs close to the theoretical limit. We also



(a) $L = 50000, \delta = 0.7$ (b) $L = 20000, \delta = 0.9$

Fig. 9. The estimated number of active users for $M = 5$ and $D = 20$ when the number of active users sharply changes.



(a) $L = 50000, \delta = 0.7$ (b) $L = 20000, \delta = 0.9$

Fig. 10. Individual SDP for $M = 5$ and $D = 20$ when the number of active users sharply changes.

TABLE II
MEAN AND STANDARD DEVIATION OF INDIVIDUAL SDPS FOR EACH STAGE WHEN $M = 5$ AND $D = 20$

Tuning parameters	Interval index	Theoretical maximum	Mean in our simulation	Standard deviation in our simulation
$L = 50000$, $\delta = 0.7$	$1 - 100$	0.8595	0.8525	1.726×10^{-3}
	$101 - 400$	0.6628	0.6578	2.848×10^{-3}
	$401 - 500$	0.8595	0.8558	1.072×10^{-3}
$L = 20000$, $\delta = 0.9$	$1 - 100$	0.8595	0.8547	5.257×10^{-3}
	$101 - 400$	0.6628	0.6579	5.418×10^{-3}
	$401 - 500$	0.8595	0.8545	2.157×10^{-3}

observe from Table II that the mean value of all individual SDPs achieves at least 99.92% of the theoretical maximum value, while the standard deviation of all individual SDPs is at most 0.08235% of the mean. Comparing with Table I for $D = 1$, we note that an increase of D leads to less SDP degradation and better fairness. This phenomenon can be attributed to the fact that the SDP is less sensitive to the change in transmission probability as D increases.

3) The Impact of Detection Errors: Finally, we examine the impact of errors for detecting the number of ongoing transmissions. Under real-life scenarios, such detection errors may occur due to the hardware constraints or hidden terminals. We assume these errors follow the Gaussian distribution with the zero mean and the variance σ^2, which is the most natural error distribution in measuring. As such, different active users may obtain the different numbers of ongoing transmissions at the same slot. Under the setting $M = 5$, $D = 20$, $L = 50000$ and $\delta = 0.7$, Fig. 11 shows the mean and standard deviation of individual SDPs for each stage as a function of σ. As expected, we see that an increase of σ leads to performance loss at every stage. Nevertheless, even when σ is as high as 0.5, which implies only 68.27% correct detection probability, we find that the mean value achieves at least 89.23% of the theoretical maximum and the standard deviation is at most 1.164% of the mean. These results indicate that the tuning algorithm is still able to provide acceptable performance under severe detection errors.

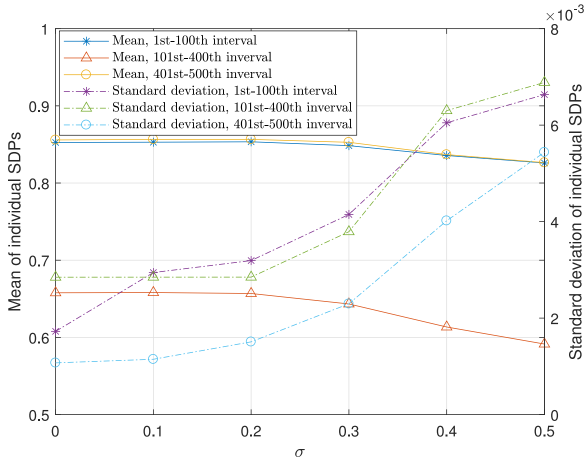


Fig. 11. The mean and standard deviation of individual SDPs for each stage as a function of σ.

VI. Conclusion

In this paper, the joint impact of MPR capability M and delivery deadline D on the optimal transmission probability, which maximizes the reliability of a ALOHA-type protocol when N active users have infinitely backlogged packets to send, have been investigated. We have proved that the optimal transmission probability is unique and can be obtained by a fixed-point iteration for any $1 \le M < N$ and any $D \ge 1$. Then, for real-life scenarios where N is unknown and time-varying, we have developed a tuning algorithm for $M > 1$, which enables each active user to obtain a deadline-constrained reliability close to the theoretical limit under dynamic scenarios. It is worth pointing out that, as a special case of our study, the issue of saturation throughput maximization is completely addressed for the first time. Our future work is to extend the present study by integrating retransmissions or considering a more general MPR channel.

Appendix

A. Proof of Lemma 1

Since that $P_D(\tau) > P_D(0) = P_D(1) = 0$ if $\tau \in (0,1)$, we know the continuous function $P_D(\tau)$ has a local maximum at τ_D^{opt}, which lies in (0, 1). As $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau)$ always exists on the interval $\tau \in (0,1)$, by the Fermat's Theorem, τ_D^{opt} is a solution of $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) = 0$.

Furthermore, we from (5) obtain that $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) > 0$ for $\tau \in (0,1) \setminus [1-(\frac{N-1}{N-1+D})^{\frac{1}{D}}, 1)$, and from (6) obtain that $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) < 0$ as $\tau \to 1^-$. By The Intermediate Value Theorem, the solutions of $\frac{\mathrm{d}}{\mathrm{d}\tau}P_D(\tau) = 0$ must be in $[1-(\frac{N-1}{N-1+D})^{\frac{1}{D}}, 1)$, i.e., τ_D^{opt} must lie in $[1-(\frac{N-1}{N-1+D})^{\frac{1}{D}}, 1)$.

B. Proof of Lemma 2

The case for $M = 1$ obviously holds as $H_1(\tau) = 0$. In the following, we only consider $2 \le M < N$.

We first show that $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) > 0$ by a previously known result in [10]. Let

$$T(\tau) \triangleq \frac{\sum_{i=1}^{M} i^2 \binom{N}{i} \tau^i (1-\tau)^{N-i}}{\sum_{i=1}^{M} i \binom{N}{i} \tau^i (1-\tau)^{N-i}}. \tag{16}$$

By letting $j = i - 1$, after some algebraic manipulations, we have

$$T(\tau) - 1 = \frac{\sum_{j=0}^{M-1} j(j+1)\binom{N}{j+1}\tau^{j+1}(1-\tau)^{N-1-j}}{\sum_{j=0}^{M-1} (j+1)\binom{N}{j+1}\tau^{j+1}(1-\tau)^{N-1-j}} = H_1(\tau). \tag{17}$$

Since it has been proven in [10] that $\frac{\mathrm{d}}{\mathrm{d}\tau}T(\tau) \ge 0$ for $\tau \in (0,1)$ and $M > 1$, we have $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}T(\tau) > 0$ by (17).

Now, we will show that $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau) < N - 1$. By the binomial theorem, $H_1(\tau)$ can be rewritten as

$$\begin{aligned} H_1(\tau) &= \frac{(N-1)\tau - \sum_{i=M}^{N-1} i\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}{\sum_{i=0}^{M-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}} \\ &= (N-1)\tau - \frac{\sum_{i=M}^{N-1}\left(i-(N-1)\tau\right)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}{\sum_{i=0}^{M-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}} \\ &= (N-1)\tau - (N-M)\binom{N-1}{M-1}R(\tau), \end{aligned} \tag{18}$$

where

$$R(\tau) \triangleq \frac{\tau^M(1-\tau)^{N-M}}{\sum_{i=0}^{M-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}.$$

The proof of the equality in (18) is as follows.

For $m = 2, 3, \ldots, N-1$, we have

$$\begin{aligned} &\binom{N-1}{m-1}(N-m)\tau^m(1-\tau)^{N-m} \\ &\quad + \left(m-1-(N-1)\tau\right)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m} \\ &= \binom{N-1}{m-1}(N-m)\tau^m(1-\tau)^{N-m} \\ &\quad - (N-m)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m} \\ &\quad + (N-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\ &= -\binom{N-1}{m-1}(N-m)\tau^{m-1}(1-\tau)^{N-m+1} \\ &\quad + (N-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\ &= (m-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\ &= \binom{N-1}{m-2}(N-m+1)\tau^{m-1}(1-\tau)^{N-m+1}. \end{aligned}$$

Then by recursively using the above equation, we have

$$\begin{aligned} &\sum_{i=M}^{N-1}\left(i-(N-1)\tau\right)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i} \\ &= \binom{N-1}{N-2}\tau^{N-1}(1-\tau) \end{aligned}$$

$$+\sum_{i=M}^{N-2}\big(i-(N-1)\tau\big)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}$$
$$=\binom{N-1}{M-1}(N-M)\tau^M(1-\tau)^{N-M}.$$

To prove $\frac{\mathrm{d}}{\mathrm{d}\tau}H_1(\tau)<N-1$, by (18), it suffices to show that $\frac{\mathrm{d}}{\mathrm{d}\tau}R(\tau)>0$. Let

$$Q(x)\triangleq\frac{x^{M-1}}{\sum_{i=0}^{M-1}\binom{N-1}{i}x^i}.$$

By plugging $x=\frac{\tau}{1-\tau}$, we have

$$R(\tau)=\tau Q(x). \tag{19}$$

Note that as τ increases from 0 to 1, x increases from 0 to ∞, and hence $Q(x)>0$. By taking the derivative of $Q(x)$ with respect to x, we have, for $x>0$,

$$\frac{\mathrm{d}}{\mathrm{d}x}Q(x)=\frac{\sum_{i=0}^{M-1}(M-1-i)\binom{N-1}{i}x^{M+i-2}}{\left(\sum_{i=0}^{M-1}\binom{N-1}{i}x^i\right)^2}>0. \tag{20}$$

Then,

$$\frac{\mathrm{d}}{\mathrm{d}\tau}R(\tau)=\frac{\mathrm{d}}{\mathrm{d}\tau}\Big(\tau Q(x)\Big)=Q(x)+\frac{\tau}{(1-\tau)^2}\cdot\frac{\mathrm{d}}{\mathrm{d}x}Q(x)>0. \tag{21}$$

Hence the result follows.

C. Proof of Lemma 3

Taking the derivative of $H_2(\tau)$ with respect to τ derives that

$$\frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau)=N-1+D\left(1-\frac{1-(1-\tau)^D-D\tau(1-\tau)^{D-1}}{\big(1-(1-\tau)^D\big)^2}\right)$$

So we have

$$\frac{\mathrm{d}}{\mathrm{d}\tau}H_2(\tau)>N-1\Leftrightarrow(1-\tau)^{D+1}+(D+1)\tau-1>0. \tag{22}$$

Let $G(\tau)\triangleq(1-\tau)^{D+1}+(D+1)\tau-1$. We have $\frac{\mathrm{d}}{\mathrm{d}\tau}G(\tau)=(D+1)(1-(1-\tau)^D)$, which is larger than 0 for $\tau\in(0,1)$. It follows that $G(\tau)>\lim_{\tau\to0^+}G(\tau)=0$. Hence we complete the proof by (22).

D. Proof of (10)

We first have

$$\frac{\mathrm{d}}{\mathrm{d}x}g(x)=\frac{1}{(H_2(x)+1)^2}\left(x(H_2(x)+1)\frac{\mathrm{d}H_1(x)}{\mathrm{d}x}\right.$$
$$\left.+(H_1(x)+1)(H_2(x)+1)-x(H_1(x)+1)\frac{\mathrm{d}H_2(x)}{\mathrm{d}x}\right).$$

Define a function $W(x,y)$ on the domain $\mathcal{A}\triangleq(0,1)\times[1,\infty)$ by

$$W(x,y)\triangleq x(H_2(x,y)+1)\frac{\mathrm{d}H_1(x)}{\mathrm{d}x}$$
$$+(H_1(x)+1)(H_2(x,y)+1)-x(H_1(x)+1)\frac{\partial H_2(x,y)}{\partial x},$$

where $H_2(x,y)\triangleq x\big(N+y-1-\frac{y}{1-(1-x)^y}\big)$ is defined on $\mathcal{A}$. Note that $\frac{\mathrm{d}}{\mathrm{d}x}g(x)=\frac{1}{(H_2(x)+1)^2}W(x,y)$ and $\frac{\mathrm{d}}{\mathrm{d}x}H_2(x)=\frac{\partial H_2(x,D)}{\partial x}$. Thus, to prove $\frac{\mathrm{d}}{\mathrm{d}x}g(x)>0$ when $D>1$, it suffices to show $W(x,y)>0$ when $y>1$.

Taking the partial derivative of $W(x,y)$ with respect to y derives that

$$\frac{\partial}{\partial y}W(x,y)=\left(x\frac{\mathrm{d}}{\mathrm{d}x}H_1(x)+H_1(x)+1\right)\frac{\partial}{\partial y}H_2(x,y)$$
$$-x\big(H_1(x)+1\big)\frac{\partial^2}{\partial y\partial x}H_2(x,y).$$

As we have showed in Lemma 2 that $\frac{\mathrm{d}}{\mathrm{d}x}H_1(x)\geq0$, the above implies

$$\frac{\partial}{\partial y}W(x,y)\geq\big(H_1(x)+1\big)\left(\frac{\partial}{\partial y}H_2(x,y)-x\frac{\partial^2}{\partial y\partial x}H_2(x,y)\right)$$
$$=\frac{(H_1(x)+1)x^2y(1-x)^{y-1}}{(1-(1-x)^y)^3}$$
$$\cdot\Big(2(1-x)^y-y\big((1-x)^y+1\big)\ln(1-x)-2\Big).$$

It is not hard to verify that $2(1-x)^y-y\big((1-x)^y+1\big)\ln(1-x)-2>0$ for $(x,y)\in\mathcal{A}$. Then, by the fact that $H_1(x)\geq0$ for $x\in(0,1)$, we have

$$\frac{\partial}{\partial y}W(x,y)>0,$$

which indicates $W(x,y)$ is increasing with respect to y for any fixed $x\in(0,1)$. Therefore, when $y>1$, we have

$$W(x,y)>W(x,1)=Nx^2\frac{\mathrm{d}}{\mathrm{d}x}H_1(x)\geq0,$$

where the last inequality is obtained by Lemma 2 again. Hence we complete the proof.

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References

[1] N. Abramson, "The ALOHA system: Another alternative for computer communications," in *Proc. Fall Joint Comput. Conf.*, Montvale, NJ, USA, 1970, vol. 44, pp. 281–285.

[2] S. Ghez, S. Verdù, and S. Schwartz, "Stability properties of slotted ALOHA with multipacket reception capability," *IEEE Trans. Autom. Control*, vol. 33, no. 7, pp. 640–649, Jul. 1988.

[3] S. Ghez, S. Verdu, and S. Schwartz, "Optimal decentralized control in the random access multipacket channel," *IEEE Trans. Autom. Control*, vol. 34, no. 11, pp. 1153–1163, Nov. 1989.

[4] V. Naware, G. Mergen, and L. Tong, "Stability and delay of finite-user slotted ALOHA with multipacket reception," *IEEE Trans. Inf. Theory*, vol. 51, no. 7, pp. 2636–2656, Jul. 2005.

[5] J. Luo and A. Ephremides, "On the throughput, capacity, and stability regions of random multiple access," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2593–2607, Jun. 2006.

[6] R.-H. Gau, "Performance analysis of slotted ALOHA in interference dominating wireless ad-hoc networks," *IEEE Commun. Lett.*, vol. 10, no. 5, pp. 402–404, May 2006.

[7] R.-H. Gau, "Performance analysis of finite-user slotted ALOHA in wireless networks with multiple packet reception and random traffic," *IEEE Commun. Lett.*, vol. 12, no. 2, pp. 140–142, Feb. 2008.

[8] Y. J. Zhang, P. Zheng, and S. C. Liew, "How does multiple-packet reception capability scale the performance of wireless local area networks?" *IEEE Trans. Mobile Comput.*, vol. 8, no. 7, pp. 923–935, Jul. 2009.

[9] Y. J. Zhang, S. C. Liew, and D. R. Chen, "Sustainable throughput of wireless LANs with multipacket reception capability under bounded delay-moment requirements," *IEEE Trans. Mobile Comput.*, vol. 9, no. 9, pp. 1226–1241, Sep. 2010.
[10] Y. H. Bae, B. D. Choi, and A. S. Alfa, "Achieving maximum throughput in random access protocols with multipacket reception," *IEEE Trans. Mobile Comput.*, vol. 13, no. 3, pp. 497–511, Mar. 2014.
[11] D. S. Chan, T. Berger, and L. Tong, "Carrier sense multiple access communications on multipacket reception channels: Theory and applications to IEEE 802.11 wireless networks," *IEEE Trans. Commun.*, vol. 61, no. 1, pp. 266–278, Jan. 2013.
[12] A. Mukhopadhyay, N. Mehta, and V. Srinivasan,"Design and analysis of an acknowledgment-aware asynchronous MPR MAC protocol for distributed WLANs," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2068–2079, May 2013.
[13] Y. Liu, V. O. K. Li, K-C. Leung, and L. Zhang, "Topology-transparent scheduling in mobile ad hoc networks with multiple packet reception capability," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 5940–5953, Nov. 2014.
[14] P. Suriyachai, U. Roedig, and A. Scott, "A survey of MAC protocols for mission-critical applications in wireless sensor networks," *IEEE Commun. Surv. Tut.*, vol. 14, no. 2, pp. 240–264, Apr.–Jun. 2012.
[15] H. Farag, M. Gidlund, and P. Österberg, "A delay-bounded MAC protocol for mission- and time-critical applications in industrial wireless sensor networks," *IEEE Sensors J.*, vol. 18, no. 6, pp. 2607–2616, Mar. 2018.
[16] Y. H. Bae, "Analysis of optimal random access for broadcasting with deadline in cognitive radio networks," *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 573–575, Mar. 2013.
[17] R. L. Rivest, "Network control by Bayesian broadcast," *IEEE Trans. Inf. Theory*, vol. IT-33, no. 3, pp. 323–328, May 1987.
[18] B. Hajek and T. van Loon, "Decentralized dynamic control of a multiaccess broadcast channel," *IEEE Trans. Autom. Control*, vol. AC-27, no. 3, pp. 559–569, Jun. 1982.
[19] H. Wu, C. Zhu, R. La, X. Liu, and Y. Zhang, "FASA: Accelerated SALOHA using access history for event-driven M2M communications," *IEEE/ACM Trans. Netw.*, vol. 21, no. 6, pp. 1904–1917, Dec. 2013.
[20] Y. Wang, M. C. Vuran, and S. Goddard, "Analysis of event detection delay in wireless sensor networks," in *Proc. IEEE Infocom*, Shanghai, Apr., 2011, pp. 1296–1304.
[21] Y. H. Bae, "Optimal retransmission-based broadcasting under delivery deadline constraint," *IEEE Commun. Lett.*, vol. 19, no. 6, pp. 1041–1044, Jun. 2015.
[22] Y. H. Bae, "Random access scheme to improve broadcast reliability," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1467–1470, Oct. 2013.
[23] Y. Zhang, F. Guan, and Y.-H. Lo, "Coding-based slotted ALOHA for broadcasting multi-slot messages with delivery deadline," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7882–7886, Aug. 2018.
[24] Y. Birk and Y. Keren, "Judicious use of redundant transmissions in multichannel ALOHA networks with deadlines," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 2, pp. 257–269, Feb. 1999.
[25] D. Baron and Y. Birk, "Multiple working points in multichannel ALOHA with deadlines," *Wireless Netw.*, vol. 8, no. 1, pp. 5–11, Jan. 2002.
[26] D. Baron and Y. Birk, "Coding schemes for multislot messages in multichannel ALOHA with deadlines," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 292–301, Apr. 2002.
[27] Y. Birk and U. Tal, "Maximizing delay-constrained throughput in multichannel DS-CDMA ALOHA networks through power diversity and successive decoding," *Wireless Netw.*, vol. 15, no. 8, pp. 1126–1139, Nov. 2009.
[28] Y. Zhang, F. Guan, Y.-H. Lo, F. Shu, and J. Li, "Optimal multichannel slotted ALOHA for deadline-constrained unicast systems," *IEEE Syst. J.*, vol. 13, no. 2, pp. 1308–1311, Jun. 2019.
[29] L. Tong, Q. Zhao, and G. Mergen, "Multipacket reception in random access wireless networks: From signal processing to optimal medium access control," *IEEE Commun. Mag.*, vol. 39, no. 11, pp. 108–112, Nov. 2001.
[30] S. Talwar, M. Viberg, and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array, Part I: Algorithms," *IEEE Trans. Signal Process.*, vol. 44, no. 5, pp. 1184–1197, May 1996.
[31] A. Orozco-Lugo, M. Lara, D. McLernon, and H. Muro-Lemus, "Multiple packet reception in wireless ad hoc networks using polynomial phasemodulating sequences," *IEEE Trans. Signal Process.*, vol. 51, no. 8, pp. 2093–2110, Aug. 2003.
[32] R. Djapic, A. van der Veen, and L. Tong, "Synchronization and packet separation in wireless ad hoc networks by known modulus algorithms," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 51–64, Jan. 2005.
[33] L. C. Godara, "Application of antenna arrays to mobile communications-part II: Beam forming and direction-of-arrival considerations," *Proc. IEEE*, vol. 85, no. 8, pp. 1193–1245, Aug. 1997.
[34] M. Gustafsson, D. Tayli, and M. Cismasu, *Physical Bounds of Antennas.* Berlin, Germany: Springer-Verlag, 2015, pp. 1–32.

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