Performance Analysis of an Opportunistic Relaying Power Line Communication Systems

Yuwen Qian, Jun Li, Yijin Zhang, and Dushantha Nalin K. Jayakody

Abstract—**Powerline communication (PLC) is an attractive approach to providing information transfer services for future smart grids. However, due to high attenuation and interference, it is a great challenge to achieve reliable transmissions over PLC channels. In this letter, we propose an opportunistic decode-and-forward-based multiple-relaying (ODFR) scheme to enhance the performance of PLC networks, where the optimal relay is selected dynamically for data forwarding. Specifically, we first formulate the cumulative distribution functions (CDFs) of the received signal-to-noise ratio by exploiting statistical properties of the PLC channels. Then, drawing upon this derived CDF, we further develop the closed-form expressions of outage probability and approximate capacity for the proposed ODFR scheme. Simulations validate the consistency of our analytical results with Monte Carlo simulations.**

Index Terms—**Decode-and-forward (DF), in-home network, opportunistic relaying, outage probability, power line communications (PLCs).**

I. Introduction

Usually being distributed in harsh and hostile environments, power line communication (PLC) channels are unavoidably subject to the time-varying nature and frequency-selective attenuation [1]. In addition, different from wireless channels, impulsive noises in the PLC caused by random switching of power appliances may lead to severe performance deterioration. These undesirable features make the design of reliable PLC schemes for long-distance transmissions a very challenging task. Relaying techniques, which have been extensively studied in the context of wireless networks, can be levered to the PLC for coping with this issue [2]–[4].

As a pioneer work, a time-division-multiple-access-based relay-aided PLC scheme for a network management system was studied in [2]. Following [2], there have been a number of investigations for cooperative PLC relaying systems. For example, Lampe *et al.* [3] employed a distributed space-time coding (DSTC) scheme to reduce transmission delay in a relay-aided PLC system. Furthermore, opportunistic relaying scheme in wireless communications, as an efficient relaying approach, has been applied to PLC networks for enhancing the capacity and coverage [4]. Specifically, in the opportunistic relaying scheme, only one of the relays having the best link toward the destination will be selected to retransmit the data packet.

At the same time, a number of frameworks have been proposed [3], [5], [6] to investigate the performance of various PLC relaying systems. Lampe *et al.* [3] derived a closed form of the outage probability for a decode-and-forward (DF) based relaying system with DSTC, and [5] analyzed the potential performance of PLC systems with amply-and-forward relaying schemes. However, neither of these works took into account the frequency-selective characteristic of the PLC channels. More practically, Tonello *et al.* [4] investigate the cooperative DF relay protocol in the PLC in-home network and the performance is studied via the use of a statistical topology model. However, this work neglected the impulsive noises that are rooted in the PLC channels.

In this letter, we model the PLC channel as the combination of PLC subchannels with maximal-ratio combining (MRC) to obtain the received signal-to-noise ratio (SNR). Based on the PLC channel mode, we propose an opportunistic DF-based multiple-relaying (ODFR) scheme, where the best relay is dynamically selected for each time slot according to the channel state to forward information. By adopting the precise approximation for sum log-normal function, we develop the cumulative distribution functions (CDFs) of the received SNR at the destination, i.e., the CDF of the optimal relaying channel. Then, based on this derived CDF, we further develop the closed-form expression of the capacity by converting the integrals to the forms of Hermite polynomial, in which the channel noise is modeled as Bernoulli–Gaussian noise with two states. Next, we work on the formulation of the outage probability. Simulations show that the derived analytical results are consistent with the results by Monte Carlo simulations.

The remainder of this paper is organized as follows. In Section II, we set up the system model for a PLC network with the ODFR scheme. Section III presents the analysis of the CDF, outage probability, and the capacity for the proposed scheme. Section IV compares the performance between the analytical results and the simulation ones, and the conclusions are given in Section V.

II. System Design

As widely adopted, a typical in-home PLC infrastructure is composed of several subcircuits, as shown in Fig. 1. Such a structure can improve system security and energy efficiency, but, at the same time, may conversely result in instable communications due to avoidably lengthened power line between terminal nodes. To cope with this instability, multiple relay nodes are installed to enhance data transmissions. However, according to Fig. 1, these relay nodes are arranged as far as possible among each other to improve the network scope and reduce the number of relay nodes.

In a PLC network system, generally, two PLC nodes are connected by several cables, which are used as separate PLC channels. These

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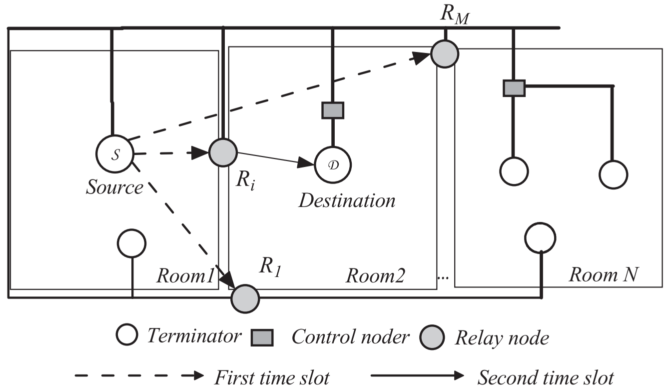

Fig. 1. Structure of a typical in-home PLC relaying system, and the ODFR scheme is adopted in this system. $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_M$ are the relaying nodes, and $\mathcal{R}_i$ is selected optimal relay for the data transmission.

channels have multiple subfrequency bands with sufficient separation to avoid cross-channel interference according to [7]. Thus, we consider the PLC channel between two PLC nodes as an L-branch diversity channel, where the transmitter transmits the same stored data as BPSK modulated symbols over the L PLC subchannels. The L subchannels can be formed using combinations from frequency, time, and space at the receiver. In this scenario, we set up a model of the ODFR scheme.

As shown in Fig. 1, a source terminal $\mathcal{S}$ communicates with the destination terminal $\mathcal{D}$ via several relaying nodes $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_M$. Since the ODFR scheme is applied, there are two stages for the transmissions. In the first stage, $\mathcal{S}$ broadcasts a message to the relay nodes. For a relaying node $\mathcal{R}_m$, $m = 1, \ldots, M$, there are L subchannels of the channel from $\mathcal{S}$ to $\mathcal{R}_m$. The received signal from one subchannel at $\mathcal{R}_m$ in one time slot of the first stage can be written as

$$y_{R_{m,l}} = h_{SR_{m,l}} \sqrt{P} x_S + n_{R_{m,l}}, \quad l = 1, 2, \ldots, L \tag{1}$$

where x_S is the symbol transmitted at a time slot with an unit power, $h_{SR_{m,l}}$ is the channel gain for the lth subchannel between $\mathcal{S}$ and $\mathcal{R}_m$, P is the transmission power, and $n_{R_{m,l}}$ is the noise sample of the subchannel at the time slot.

After receiving signals from L branch, $\mathcal{R}_m$ employs the MRC to combine the received signal. The receiving SNR at $\mathcal{R}_m$ can be given as

$$\gamma_{SR_m} = \sum_{l=1}^{L} \gamma h^2_{SR_{m,l}} \tag{2}$$

where γ is the transmission SNR, defined as $\gamma \triangleq \frac{P}{\sigma_0^2}$, where σ_0^2 is the variance of the noise $n_{R_{m,l}}$. According to [5], the noise in PLC channel is Bernoulli-Gauss distributed, subject to a combination of the additive white Gaussian noise (AWGN) and the impulsive noise. Hence, the noise variance σ_0^2 can be further formulated as

$$\sigma_0^2 = \sigma_G^2 + p\sigma_I^2 \tag{3}$$

where σ_G^2 and σ_I^2 are the variances of the AWGN and the impulsive noise, respectively, and p is the parameter of Bernoulli random sequence.

In the second stage, the optimal relay node, say, $\mathcal{R}_i, i \in \{1, \ldots, M\}$, is selected to forward data to the terminal $\mathcal{D}$. The selection criterion for the optimal relay node will be presented later. Assume that each relay node transmits with the same power P as at the source. Then, the received signal via the lth subchannel from $\mathcal{R}_i$ to $\mathcal{D}$ can be written as

$$y_{R_{i,l}D} = h_{R_{i,l}D} \sqrt{P} \hat{x}_S + n_{R_{i,l}D} \tag{4}$$

where $\hat{x}_S$ is the decoding result of x_S at R_i, $h_{R_{i,l}D}$ is the channel gain from $\mathcal{R}_i$ to $\mathcal{D}$, and $n_{R_{i,l}D}$ is the corresponding noise at the destination with the same variance σ_0^2.

By using the same method with (2), we can obtain the received SNR from $\mathcal{R}_i$ at the destination terminal $\mathcal{D}$ as

$$\gamma_{R_iD} = \sum_{l=1}^{L} \gamma h^2_{R_{i,l}D}. \tag{5}$$

The capacity of the channel $\mathcal{S} \to \mathcal{R}_m \to \mathcal{D} \quad \forall m$, is determined by minimum one of the channels $\mathcal{S} \to \mathcal{R}_m$ and $\mathcal{R}_m \to \mathcal{D}$. Thus, the equivalent SNR of the channel $\mathcal{S} \to \mathcal{R}_m \to \mathcal{D}$, namely γ_{SR_mD} can be written as

$$\gamma_{SR_mD} = \min\left(\gamma_{SR_m}, \gamma_{R_mD}\right). \tag{6}$$

In our ODFR scheme, the optimal relay $\mathcal{R}_i$ is selected based on the max-SNR criterion, i.e.,

$$i = \arg\max_{m=1,\ldots,M} \gamma_{SR_mD}. \tag{7}$$

Hence, the equivalent SNR, denoted by γ_0, for our ODFR scheme can be formulated as [8]

$$\gamma_o = \max_m \left(\min\left(\gamma_{SR_m}, \gamma_{R_mD}\right)\right) = \gamma_{SR_iD}. \tag{8}$$

III. Performance Analysis

In this section, we analyze the performance of a PLC system adopting the ODFR scheme. We characterize the statistics of the received SNR at destination, and then derive the expressions of the instantaneous capacity and outage probability .

A. CDF of Received SNR γ_o

In a PLC channel, the probability density function (PDF) of the amplitude h of the lth subchannel is log-normal [7], given as

$$f_{h_l}(x) = \frac{1}{\sqrt{2\pi\sigma^2}x} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \tag{9}$$

where μ and σ^2 are the mean and the variance of $\ln(x)$, respectively, which is a normal random variable.

According to (2) and [9], we can accurately approximate PDF of the receiving SNR of the PLC channel with the log-normal sum distribution, given by

$$f_{\gamma_{SR_m}}(x) = \frac{a_1 a_2 (x/\gamma)^{-(a_2/\lambda+1)}}{\lambda\sqrt{2\pi}\gamma} \times \exp\left(-\frac{\left(a_0 - a_1 (x/\gamma)^{-a_2/\lambda}\right)^2}{2}\right) \tag{10}$$

where $\lambda = \frac{\ln 10}{10}$, and a_0, a_1, and a_2 are constants.

Since the two-stage transmission is adopted to simplify our analysis, we assume that h_{SR_m} and h_{R_mD} are independent variables, which identically follow the same distribution. Then, the CDF of γ_{SR_m} and γ_{R_mD} can be written as

$$F_{\gamma_{SR_m}}(x) = F_{\gamma_{R_mD}}(x) = \Phi\left(a_0 - a_1 (x/\gamma)^{-a_2/\lambda}\right). \tag{11}$$

Follow (6) and (11) and define the variable $z \triangleq \min(u, v)$, the CDF of $\gamma_{SR_m D}$ can be given as

$$F_{\gamma_{SR_m D}}(z) = F_{\gamma_{SR_m}}(u) + F_{\gamma_{R_m D}}(v) - F_{\gamma_{SR_m}, \gamma_{R_m D}}(u, v) = 1 - \left(1 - \Phi\left(a_0 - a_1 (z/\gamma)^{-a_2/\lambda}\right)\right)^2 \quad (12)$$

where $F_{\gamma_{SR_m}, \gamma_{R_m D}}(x, y)$ is the CDF of the joint distribution of γ_{SR_m} and $\gamma_{R_m D}$.

According to Fig. 1, since the distance among relay nodes is designed much farther than the effective communication scope of a PLC channel, these relaying nodes work independently. Thus, from the information provided by (6), (8), and (12), the CDF of γ_o is given as

$$F_{\gamma_o}(x) = \prod_{m=1}^{M} F_{\gamma_{SR_m D}}(x) = \left(1 - \left(1 - \Phi\left(a_0 - a_1 (x/\gamma)^{-a_2/\lambda}\right)\right)^2\right)^M. \quad (13)$$

B. Channel Capacity

The capacity of a PLC channel subject to the Bernoulli–Gaussian noise is strictly derived by [6], where the Bernoulli–Gaussian noise channel is considered as a special case with two Gaussian states. Thus, Dubey *et al.* [7] derive the capacity of the PLC channel with perfect knowledge of the channel as well as the noise states, given as

$$C(\gamma_o) = \sum_{j=0}^{1} p_j \log_2(1 + \gamma_o \alpha_j) \quad (14)$$

where $p_0 = 1 - p$ and $p_1 = p$. By defining $\eta \triangleq \frac{\sigma_I^2}{\sigma_G^2}$, we have

$$\alpha_0 = 1 + p\eta \quad \alpha_1 = \frac{1 + p\eta}{1 + \eta}. \quad (15)$$

In a high SNR regime, both $\gamma_o \alpha_0$ and $\gamma_o \alpha_1$ are far larger than one. Thus, the capacity can be approximated as

$$C(\gamma_o) \approx \Theta(p) + \frac{\ln \gamma_o}{\ln 2} \quad (16)$$

where $\Theta(p) = (1 - p)\log_2(\alpha_0) + p\log_2(\alpha_1)$. Now, the average capacity can be expressed as

$$\overline{C} \triangleq \mathbb{E}_{\gamma_0}(C(\gamma_0)) = \int_0^\infty C(x)\, \mathrm{d}F_{\gamma_o}(x) \quad (17)$$

where $\mathbb{E}(\cdot)$ represents the expectation function. Substituting (13) and (16) in (17), we have

$$\overline{C} = \Theta(p) + \frac{1}{\ln 2}\int_0^\infty \ln x \times \mathrm{d}\left(1 - \left(1 - \Phi\left(a_0 - a_1 (x/\gamma)^{-a_2/\lambda}\right)\right)^2\right)^M. \quad (18)$$

By defining $s = a_0 - a_1 \left(\frac{x}{\gamma}\right)^{-\frac{a_2}{2\lambda}}$, we can arrive at

$$\overline{C} = \Theta(p) + \frac{2M}{\sqrt{2\pi}\ln 2}\int_{-\infty}^{\infty}\left(\ln\gamma + \frac{\lambda}{a_2}\ln\frac{a_1}{a_0 - s}\right) \times \left(1 - (1 - \Phi(s))^2\right)^{M-1}(1 - \Phi(s))\, e^{-\frac{s^2}{2}}\mathrm{d}s. \quad (19)$$

To obtain the closed form of the capacity, we take a series of steps to obtain a simplified approximation of (19). First, to deal with the item $\left(1 - (1 - \Phi(s))^2\right)^{M-1}$ in (19), we utilize the Taylor expansion

$$(1 + t)^\xi = \sum_{n=0}^{N} \frac{\xi! t^n}{(\xi - n)! n!} \quad (20)$$

where N is a positive integer and we have $N \to \infty$. Let $t = -(1 - \Phi(s))^2$ and $\xi = M - 1$. Since $(1 - \Phi(s))^2 < 1$, we can neglect the high-order items, by choosing a limited value of N from the right-hand side of (20), to obtain an approximated closed form. The accuracy of this approximation depends on the value of N. Based on (20), we rewrite (19) as

$$\bar{C} = \Theta(p) + \frac{2}{\sqrt{2\pi}\ln 2}\int_{-\infty}^{\infty}\left(\ln\gamma + \frac{\lambda}{a_2}\ln\frac{a_1}{a_0 - s}\right) \times \sum_{n=0}^{N} T_n (1 - \Phi(s))^{2n+1} e^{-\frac{s^2}{2}}\mathrm{d}s \quad (21)$$

where $T_n = \frac{M!(-1)^n}{(M-1-n)!n!}$.

Second, according to [10], we have the approximation

$$\int_{-\infty}^{\infty} f(x)\, e^{-x^2} dx \approx \sum_{k=1}^{K} w_k f(x_k) \quad (22)$$

where K is the order of the Hermite polynomial, set to be 20 in this letter, $w_i = \frac{2^{k-1} k! \sqrt{\pi}}{k^2 (H_{k-1}(x_k))^2}$ is the weight, and x_k is the kth zero of the Hermite polynomial.

At last, we can obtain the closed form of the average capacity from (21) and (22), given as

$$\bar{C} = \Theta(p) + \frac{2}{\sqrt{\pi}\ln 2}\sum_{k=1}^{K}\sum_{n=0}^{N} w_k T_n \times \left(\ln\gamma + \frac{\lambda}{a_2}\ln\frac{a_1}{a_0 - \sqrt{2}s_k}\right)\left(1 - \Phi\left(\sqrt{2}s_k\right)\right)^{2n+1}. \quad (23)$$

C. Outage Probability

The outage probability, defined as the probability that the transmission rate R is greater than the channel mutual information $I(x_S; y_D | h_{SR_i}, h_{R_i D})$, is considered as an important measure to quantify the service of PLC networks.

According to (14), the outage probability can be written as

$$P_{\text{out}} = \Pr\left(\sum_{j=0}^{1} p_j \log_2(1 + \alpha_j \gamma_o) < R\right) \simeq F_{\gamma_o}\left(2^R \alpha_0^{-p_0} \alpha_1^{-p_1}\right). \quad (24)$$

After substituting (13) into (24), we can directly obtain the following closed-form expression of outage probability as

$$P_{\text{out}} = \left(1 - \left(1 - \Phi\left(a_0 - a_1\left(\frac{2^R \alpha_0^{-p_0} \alpha_1^{-p_1}}{\gamma}\right)^{-\frac{a_2}{\lambda}}\right)\right)^2\right)^M. \quad (25)$$

IV. Simulation Results

In this section, we present the numerical results of our ODFR scheme, and compare them with the Monte Carlo simulation ones. The fading parameters for the PLC channels are set to $\sigma = (6\lambda/\ln 10)$ dB, and $\mu = -\sigma^2$. Also, the parameter p is set to 3.27×10^{-3} and we set the parameters in (9) as $a_0 = 8.75$, $a_1 = 15.58$, and $a_2 = 0.0438$ with [9].

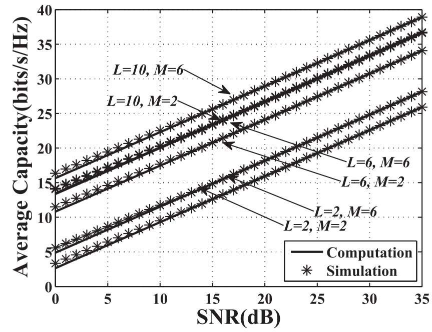

Fig. 2. Comparison of the average capacity between our computational results and the simulation ones with different relay numbers M and different PLC subchannel numbers L.

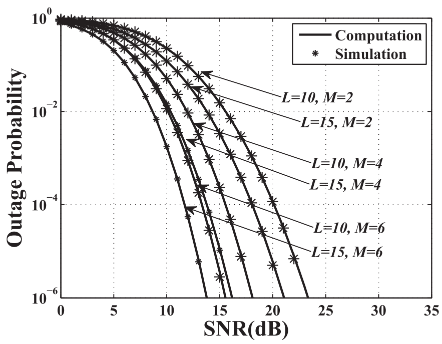

Fig. 3. Comparison of the outage probability between our computational results and the simulation ones with different relay numbers M and different PLC subchannel number L.

Fig. 2 demonstrates the channel capacity $\overline{C}$ against γ of the PLC system. The numerical results are obtained by (23). From Fig. 2, we observe that the numerical results are almost consistent with the simulation ones, with a small performance gap. This is due to the limited value of N used in the Taylor expansion. The capacity increases linearly with the horizontal axis (i.e., γ in dB). This can be verified by (23), where $\overline{C}$ is a linear function of $\ln \gamma$. We can also observe that with the increase in the number of the PLC subchannels, the capacity dramatically increases due to the enlarging of the SNR. Interestingly, the number of the subchannels has a more obvious influence on the capacity than on the relay numbers.

Fig. 3 shows the curves of the outage probability P_{out} for the transmission rate $R = 10$ bit/s/Hz versus the transmission SNR γ for the PLC system with our ODFR scheme, as shown in Fig. 1. The numerical curves are obtained by using the closed-form expression given in (25). We consider the cases with different values of the relay numbers, i.e., $M = 2, 4, 6$. It follows that the analytical results and the simulation results are consistent. Also, the outage performance is strengthened when more relay nodes and more subchannels are involved.

V. Conclusion

In this letter, we focused on the performance analysis of our ODFR scheme in the PLC network. By exploiting the statistical characteristics of the log-normal sum fading and noise model rooted in the PLC channels, we formulated the CDF of received SNR. Based on this derived CDF, we further investigated the outage probability and the approximate capacity with closed forms. Monte Carlo simulations were shown to match our analytical results and also verified the effectiveness of the proposed ODFR scheme.

References

[1] A. Chaudhuri and M. R. Bhatnagar, "Optimised resource allocation under impulsive noise in power line communications," *IET Commun.*, vol. 8, no. 7, pp. 1227–1229, 2014.

[2] M. Sebeck and G. Bumiller, "A network management system for powerline communications and its verification by simulation," in *Proc. IEEE Int. Symp. Power Line Commun.*, 2000, pp. 225–232.

[3] L. Lampe, R. Schober, and S. Yiu, "Distributed space-time coding for multihop transmission in power line communication networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 7, pp. 1389–1400, 2007.

[4] A. M. Tonello, F. Versolatto, and S. D. Alessandro, "Opportunistic relaying in in-home PLC networks," in *Proc. Global Commun. Conf.*, Altanta, GA, USA, 2010, pp. 1–5.

[5] V. B. Balakirsky and A. J. HanVinck, "Potential performance of PLC systems composed of several communication links," in *Proc. IEEE Int. Conf. Power Line Commun. Appl.*, Vancouver, BC, Canada, 2005, pp. 6–8.

[6] M. Hosseini, G. Hodtani, and M.Kakhki, "Capacity analysis of power line communication point-to-point and relay channels," *Trans. Emerg. Telecommun. Technol.*, vol. 27, pp. 200–217, 2016.

[7] A. Dubey, R. K. Mallik, and R. Schober, "Performance analysis of a power line communication system employing selection combining in correlated lognormal channels and impulsive noise communications," *IET Commun.*, vol. 8, no. 7, pp. 1072–1082, 2014.

[8] L. Fan, N. Yang, T. Duong, M. Elkashlan, and G. Karagiannidis, "Exploiting direct links for physical layer security in multiuser multirelay networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 3856–3867, Jun. 2016.

[9] N. C. Beaulieu and F. Rajwan, "Highly accurate simple closed-form approximations to lognormal sum distributions and densities," *IEEE Commun. Lett.*, vol. 8, no. 12, pp. 709–712, Dec. 2004.

[10] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed. New York, NY, USA: Dover, 1970.