Optimal Multichannel Slotted ALOHA for Deadline-Constrained Unicast Systems

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***Abstract*—This paper considers multichannel slotted ALOHA for unicast in an ad hoc network. Under a deadline-constrained scenario that a packet is required to be transmitted to an intended recipient within a strict delivery deadline D (in unit of slot) and retransmission is not allowed, we obtain the optimal channel selection probabilities and the optimal transmission probability that maximize the reliability. In particular, the throughput maximization can be studied as the case of $D = 1$. Furthermore, by noting that there is a tradeoff between reliability and throughput when $D > 1$, we obtain the optimal schedule that maximizes the reliability subject to a throughput constraint and the one that maximizes the throughput subject to a reliability constraint.**

***Index Terms*—Ad hoc networks, delivery deadline, multichannel slotted ALOHA, unicast.**

I. Introduction

As a generalization of slotted ALOHA to support random access over a number of orthogonal channels, multichannel slotted ALOHA has been widely used in various communication systems, spanning from satellite networks [1]–[4], cellular networks [5] to Internet of Things [6]. For these uses, many mission-critical applications are deadline-constrained, that is, every packet has limited lifetime and needs to be sent within a strict delivery deadline. Relevant examples include target tracking in battlefield and health monitoring in medical care.

Definitely, reliable and timely delivery is important for the success of these missions. Furthermore, with regard to different QoS guarantees, there are two different but complementary design goals that have been considered in the past and recent studies. The first one is to maximize the throughput under a requirement that a certain reliability should be met prior to a specified deadline [1]–[4], and the second is to maximize the deadline-constrained reliability subject to a certain throughput constraint [7]–[9]. In this paper, we aim to find the solutions of channel selection probabilities and transmission probability to these constrained optimization problems for unicast services in an ad hoc network, where each user transmits directly to some other, but cannot transmit and receive simultaneously. This scenario is usual in machine-to-machine communications [10].

Related work differs from our work in assumptions of the network architecture. [1]–[4], [11] considered multichannel uplink to a sink, and [7]–[9] considered single-channel broadcast. Similar to ours, [12] focused on unicast in an ad hoc network, but the model therein requires every user to be assigned an unchanged role as a transmitter or a receiver. Note that both [11] and [12] only considered the throughput maximization issue without a reliability constraint.

The rest of this paper is organized as follows. After describing the system model in Section II, in Section III we derive the optimal schedule that maximizes the reliability within a delivery deadline. Based on this result, in Section IV we obtain the optimal schedules for the constrained optimization problems considered. Simulation results are provided in Section V to verify our analysis. Section VI concludes this paper.

II. System Model

Consider M half-duplex users in a fully connected ad hoc network with N orthogonal channels, which admit the same time-slotted structure. We assume that each user always has an individual data stream to send to an intended recipient. At every slot, each user is allowed to transmit a packet according to a probability τ, but otherwise keeps silent as a potential receiver that can listen to all the channels. For each transmission, only one channel is selected, and each user chooses channel n with the probability p_n for $n = 1, 2, \ldots, N$. To avoid uninteresting cases, we consider $M > N \geq 1$.

Every packet is neither acknowledged nor retransmitted at the medium access control layer although it may be eventually resubmitted by a higher level protocol, since an acknowledgement would incur costly overhead and waiting time for a short packet. Every packet is required to be transmitted within a specific delivery deadline D ($D \geq 1$) (in unit of slot), which is defined as the time duration between the moment of its arrival at the head of the queue and the completion of its transmission. A packet transmitted on a channel is successfully received if and only if it is not interfered by other packets transmitted on the same channel, and the intended recipient keeps silent.

III. Optimal Schedule

Let $\mathbf{V}_N$ be the collection of all vectors with N non-negative entries that add up to one. For a given number $\tau \in [0, 1]$ and a given vector

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$\mathbf{p} = (p_1, p_2, \ldots, p_N) \in \mathbf{V}_N$, the reliability of a unicast packet $R_D(\tau, \mathbf{p})$, which is defined as the probability that a unicast packet is successfully received within the delivery deadline D, can be obtained as

$$R_D(\tau, \mathbf{p}) = \sum_{n=1}^{N} p_n \sum_{k=1}^{D} (1-\tau)^{k-1}\tau(1-\tau)(1-\tau p_n)^{M-2}$$
$$= (1-\tau)\big(1-(1-\tau)^D\big)\sum_{n=1}^{N} p_n(1-\tau p_n)^{M-2}. \quad (1)$$

In particular, the individual throughput T defined as the time average of the number of successfully received packets that are transmitted by a user, can be written as $T = R_1(\tau, \mathbf{p})$.

For given integers $M > N \geq 1$ and $D \geq 1$, we aim to find the optimal probability vector $\mathbf{p}_D^{\text{opt}}$ and the optimal transmission probability τ_D^{opt} that maximize $R_D(\tau, \mathbf{p})$, i.e.,

$$\left(\tau_D^{\text{opt}}, \mathbf{p}_D^{\text{opt}}\right) = \arg\max_{\tau\in[0,1], \mathbf{p}\in\mathbf{V}_N} R_D(\tau, \mathbf{p}). \quad (2)$$

A. Optimal Channel Selection Probabilities

We first present the following property that will be useful.

Lemma 1: Let $x_1, x_2, \ldots, x_N$ be N variables on the interval $[0, \frac{1}{M-1}]$ under the condition that their summation is fixed. Then $\sum_{n=1}^{N} x_n(1-x_n)^{M-2}$ achieves its maximum only when $x_1 = x_2 = \ldots = x_N$.

Proof: As the case $N = 1$ is trivial, we consider $N \geq 2$ as below. Assume that $\sum_{n=1}^{N} x_n(1-x_n)^{M-2}$ can achieve its maximum when $x_1 = x_1^*, x_2 = x_2^*, \ldots, x_N = x_N^*$. Suppose to the contrary that there exist indexes s and t such that $x_s^* \neq x_t^*$. Let $m = x_s^* + x_t^*$ and view

$$y(1-y)^{M-2} + (m-y)(1-m+y)^{M-2} \quad (3)$$

as a function of y on the interval $[0, \frac{1}{M-1}]$. The derivative of (3) with respect to y is

$$\Big(1-(M-1)y\Big)(1-y)^{M-3}$$
$$- \Big(1-(M-1)(m-y)\Big)(1-m+y)^{M-3}. \quad (4)$$

As $M > N \geq 2$ and $0 \leq y, m-y \leq \frac{1}{M-1}$, (4) is equal to zero when $y = \frac{m}{2}$, larger than zero when $0 \leq y < \frac{m}{2}$, and smaller than zero when $\frac{m}{2} < y \leq \frac{1}{M-1}$. This implies $x_s^*(1-x_s^*)^{M-2} + x_t^*(1-x_t^*)^{M-2} < (x_s^* + x_t^*)(1 - \frac{x_s^*+x_t^*}{2})^{M-2}$, which contradicts to that $\sum_{n=1}^{N} x_n(1-x_n)^{M-2}$ can achieve the maximum when $x_s = x_s^*$, $x_t = x_t^*$. We conclude that there do not exist indexes s and t such that $x_s^* \neq x_t^*$, as desired. ∎

We are now ready to find $\mathbf{p}_D^{\text{opt}}$.

Theorem 2: For an integer $D \geq 1$, $\mathbf{p}_D^{\text{opt}} = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$.

Proof: As $R_D(0, \mathbf{p}) = R_D(1, \mathbf{p}) < R_D(\tau, \mathbf{p})$ for $0 < \tau < 1$, we only consider $\tau \in (0, 1)$ here. Define

$$f_1(\tau) := \frac{(1-\tau)\big(1-(1-\tau)^D\big)}{\tau}$$
$$f_2(\tau, p_n) := \tau p_n(1-\tau p_n)^{M-2} \text{ for } n = 1, 2, \ldots, N.$$

Obviously, we have $R_D(\tau, \mathbf{p}) = f_1(\tau)\sum_{n=1}^{N} f_2(\tau, p_n)$. To prove $\mathbf{p}_D^{\text{opt}} = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$, by Lemma 1, it suffices to show that, when $R_D(\tau, \mathbf{p})$ achieves its maximum, one has

$$0 \leq \tau p_n \leq \frac{1}{M-1} \quad \text{for all } n. \quad (5)$$

We first investigate the monotonicity of $f_1(\tau)$ and $f_2(\tau, p_n)$. Taking the derivative of $f_1(\tau)$ with respect to τ derives that

$$\frac{\mathrm{d}}{\mathrm{d}\tau} f_1(\tau) = \frac{(1-\tau)^D + D\tau(1-\tau)^D - 1}{\tau^2} < 0. \quad (6)$$

Meanwhile, for each n, by viewing $f_2(\tau, p_n)$ as a single variable function of τp_n on the interval $(0, 1)$, it is easy to see from its first derivative that $f_2(\tau, p_n)$ increases with τp_n when $\tau p_n < \frac{1}{M-1}$, achieves the maximum when $\tau p_n = \frac{1}{M-1}$, and decreases with τp_n when $\tau p_n > \frac{1}{M-1}$.

Now, we are ready to show (5). Let $(\tau^*, \mathbf{p}^*) = (\tau_D^{\text{opt}}, \mathbf{p}_D^{\text{opt}})$. Here $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_N^*)$. We consider the following two cases according to the value of τ^*.

First, consider the case that $\tau^* > \frac{N}{M-1}$. Since $\sum_{n=1}^{N} p_n^* = 1$, there exists t such that $p_t^* \geq \frac{1}{N}$ and, thus, $\tau^* p_t^* > \frac{1}{M-1}$. By the monotonicity of $f_2(\tau, p_n)$ on τp_n, we have $f_2(\tau^*, p_t^*) < f_2(\frac{N}{M-1}, \frac{1}{N})$, and then $\sum_{n=1}^{N} f_2(\tau^*, p_n^*) < \sum_{n=1}^{N} f_2(\frac{N}{M-1}, \frac{1}{N})$. By the monotonicity of $f_1(\tau)$ in (6), we also have $f_1(\tau^*) < f_1(\frac{N}{M-1})$ as $\tau^* > \frac{N}{M-1}$. This concludes

$$f_1(\tau^*)\sum_{n=1}^{N} f_2(\tau^*, p_n^*) < f_1\left(\frac{N}{M-1}\right)\sum_{n=1}^{N} f_2\left(\frac{N}{M-1}, \frac{1}{N}\right)$$

which is a contradiction to $(\tau^*, \mathbf{p}^*) = (\tau_D^{\text{opt}}, \mathbf{p}_D^{\text{opt}})$.

Second, consider the case that $\tau^* \leq \frac{N}{M-1}$. Suppose to the contrary that there exists an index t such that $\tau^* p_t^* > \frac{1}{M-1}$. Since $\sum_{n=1}^{N} \tau^* p_n^* \leq \frac{N}{M-1}$, there exists s, $s \neq t$, such that $\tau^* p_s^* < \frac{1}{M-1}$. Now, pick a positive number ξ such that $\tau^*(p_t^* - \xi) \geq \frac{1}{M-1}$ and $\tau^*(p_s^* + \xi) \leq \frac{1}{M-1}$. By the monotonicity of $f_2(\tau, p_n)$ on τp_n, we have

$$f_1(\tau^*)\sum_{n=1}^{N} f_2(\tau^*, p_n^*) < f_1(\tau^*) \cdot$$
$$\left(\left(\sum_{n=1, n\neq s,t}^{N} f_2(\tau^*, p_n^*)\right) + f_2(\tau^*, p_t^* - \xi) + f_2(\tau^*, p_s^* + \xi)\right)$$

which contradicts to $(\tau^*, \mathbf{p}^*) = (\tau_D^{\text{opt}}, \mathbf{p}_D^{\text{opt}})$. Therefore, we have $\tau^* p_n^* \leq \frac{1}{M-1}$ for all n, as desired. ∎

B. Optimal Transmission Probability

By (2) and Theorem 2, we have

$$\tau_D^{\text{opt}} = \arg\max_{\tau\in[0,1]} R_D(\tau) \quad (7)$$

in which $R_D(\tau) := (1-\tau)\big(1-(1-\tau)^D\big)(1-\frac{\tau}{N})^{M-2}$.

By the extreme value theorem, we know τ_D^{opt} exists, since that $R_D(\tau)$ is continuous on $[0, 1]$. By adopting the notation

$$H_1(\tau) := \frac{D(1-\tau)^D}{1-(1-\tau)^D}, \quad H_2(\tau) := 1 + \frac{(1-\tau)(M-2)}{N-\tau}$$

the derivative of $R_D(\tau)$ with respect to τ can be written as

$$\frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) = \left(1 - \frac{\tau}{N}\right)^{M-2}\left(1-(1-\tau)^D\right)\left(H_1(\tau) - H_2(\tau)\right). \quad (8)$$

Define an interval $\mathcal{I} := [1 - (\frac{M-1}{M-1+D})^{\frac{1}{D}}, 1 - (\frac{1}{D+1})^{\frac{1}{D}}]$. We first provide a key property of τ_D^{opt}.

Lemma 3: For an integer $D \geq 1$, τ_D^{opt} is a solution of

$$H_1(\tau) - H_2(\tau) = 0 \quad (9)$$

and lies on the interval $\mathcal{I}$.

Proof: As $R_D(0) = R_D(1) < R_D(\tau)$ if $0 < \tau < 1$ and $R_D(\tau)$ is continuous on $[0,1]$, it is easy to see that $R_D(\tau)$ has a local maximum at τ_D^{opt}, which lies on the interval $(0,1)$. Since $R_D(\tau)$ is differentiable on the interval $(0,1)$, we by The Fermat's Theorem know τ_D^{opt} is a solution of $\frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) = 0$.

As $1 \le N < M$, $D \ge 1$, it is easy to see that $1 \le H_2(\tau) \le M-1$ when $\tau \in (0,1)$. Then, from (8) we obtain that

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) > 0, & \text{if } 0 < \tau < 1 - \left(\frac{M-1}{M-1+D}\right)^{\frac{1}{D}} \\ \frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) < 0, & \text{if } 1 - \left(\frac{1}{1+D}\right)^{\frac{1}{D}} < \tau < 1. \end{cases} \tag{10}$$

Since $R_D(\tau)$ is differentiable on the interval $(0,1)$, by the intermediate value theorem, the solutions of $\frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) = 0$ on $(0,1)$ must lie in $\mathcal{I}$. As $\left(1 - \frac{\tau}{N}\right)^{M-2}\left(1 - (1-\tau)^D\right)$ in (8) is always larger than zero, we further find τ_D^{opt} is a solution of $H_1(\tau) - H_2(\tau) = 0$ on the interval $\mathcal{I}$. ∎

In what follows, we will show that (9) has a unique solution on the interval $\mathcal{I}$.

Theorem 4: For an integer $D \ge 1$, (9) has a unique solution on the interval $\mathcal{I}$, denoted by τ^{**} and $\tau_D^{\text{opt}} = \tau^{**}$.

Proof: Suppose there are two distinct solutions τ_a, τ_b $(\tau_a < \tau_b)$ to the (9) in $\mathcal{I}$. Obviously, $H_1(\tau)$ and $H_2(\tau)$ are both continuous on the closed interval $[\tau_a, \tau_b]$, and differentiable on the open interval (τ_a, τ_b). By Cauchy's mean value theorem, then there exists some $\tau_c \in (\tau_a, \tau_b)$, such that $\frac{\mathrm{d}}{\mathrm{d}\tau} H_1(\tau)\Big|_{\tau=\tau_c} = \frac{\mathrm{d}}{\mathrm{d}\tau} H_2(\tau)\Big|_{\tau=\tau_c}$.

Furthermore, as $\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} H_2(\tau) = \frac{-2(M-2)(N-1)}{(N-\tau)^3} \le 0$ and $\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} H_1(\tau) = \frac{\tau^2(1-\tau)^D\left(D + D(1-\tau)^D + (1-\tau)^D - 1\right)}{\left(1-(1-\tau)^D\right)^3 (1-\tau)^2} > 0$, we have $\frac{\mathrm{d}}{\mathrm{d}\tau} H_1(\tau) > \frac{\mathrm{d}}{\mathrm{d}\tau} H_2(\tau)$ when $\tau_c < \tau < 1$. As $H_1(\tau_b) = H_2(\tau_b)$, then there exists some τ_d on the interval $(1 - (\frac{1}{D+1})^{\frac{1}{D}}, 1)$, such that $H_1(\tau_d) > H_2(\tau_d)$. However, one can deduce from (10) that $H_1(\tau) < H_2(\tau)$ when $1 - (\frac{1}{D+1})^{\frac{1}{D}} < \tau < 1$. A contradiction occurs. Hence, (9) has a unique solution τ^{**} in $\mathcal{I}$, which by Lemma 3 promises $\tau_D^{\text{opt}} = \tau^{**}$. ∎

IV. Constrained Optimal Schedule

Clearly, there is some tradeoff involved in reliability and throughput if $D > 1$. Let $R_D^{\max}$ be the maximum of $R_D(\tau, \mathbf{p})$. For an integer $D > 1$, in this section we investigate the optimal schedule that maximizes $R_D(\tau, \mathbf{p})$ while satisfying a given individual throughput requirement ϵ $(0 \le \epsilon \le R_1^{\max})$, i.e.,

$$\left(\tau_D^{\text{opt}}(\epsilon), \mathbf{p}_D^{\text{opt}}(\epsilon)\right) = \arg\max_{\substack{\tau\in[0,1] \\ \mathbf{p}\in\mathbf{V}_N}} R_D(\tau, \mathbf{p}) \quad \text{s.t. } R_1(\tau, \mathbf{p}) \ge \epsilon \tag{11}$$

and the optimal schedule that maximizes $R_1(\tau, \mathbf{p})$ under a given reliability constraint η $(0 \le \eta \le R_D^{\max})$, i.e.,

$$\left(\tau_1^{\text{opt}}(\eta), \mathbf{p}_1^{\text{opt}}(\eta)\right) = \arg\max_{\substack{\tau\in[0,1] \\ \mathbf{p}\in\mathbf{V}_N}} R_1(\tau, \mathbf{p}) \quad \text{s.t. } R_D(\tau, \mathbf{p}) \ge \eta. \tag{12}$$

We first obtain $\mathbf{p}_D^{\text{opt}}(\epsilon)$ and $\mathbf{p}_1^{\text{opt}}(\eta)$ as below.

Theorem 5: Consider an integer $D > 1$ and numbers $\epsilon \in [0, R_1^{\max}]$, $\eta \in [0, R_D^{\max}]$. Then, $\mathbf{p}_D^{\text{opt}}(\epsilon) = \mathbf{p}_1^{\text{opt}}(\eta) = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$.

Proof: Let $(\tau^*, \mathbf{p}^*) = (\tau_D^{\text{opt}}(\epsilon), \mathbf{p}_D^{\text{opt}}(\epsilon))$. Suppose that $\tau^* > \frac{N}{M-1}$. By (1) and the proof of Theorem 2, we find

$$R_1\left(\frac{N}{M-1}, \left(\frac{1}{N}, \ldots, \frac{1}{N}\right)\right) > R_1(\tau^*, \mathbf{p}^*) \ge \epsilon$$

$$R_D\left(\frac{N}{M-1}, \left(\frac{1}{N}, \ldots, \frac{1}{N}\right)\right) > R_D(\tau^*, \mathbf{p}^*)$$

which contradict to $(\tau^*, \mathbf{p}^*) = (\tau_D^{\text{opt}}(\epsilon), \mathbf{p}_D^{\text{opt}}(\epsilon))$. Hence, we have $\tau^* \le \frac{N}{M-1}$. Then, if $\mathbf{p}^* \ne (\frac{1}{N}, \ldots, \frac{1}{N})$, by (1) and the proof of Theorem 2, we have

$$R_1\left(\tau^*, \left(\frac{1}{N}, \ldots, \frac{1}{N}\right)\right) > R_1(\tau^*, \mathbf{p}^*) \ge \epsilon$$

$$R_D\left(\tau^*, \left(\frac{1}{N}, \ldots, \frac{1}{N}\right)\right) > R_D(\tau^*, \mathbf{p}^*)$$

which also lead to a contradiction. This concludes that $\mathbf{p}^* = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ as desired.

The proof that $\mathbf{p}_1^{\text{opt}}(\eta) = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ is similar to the above and, thus, is omitted. ∎

By (11), (12), and Theorem 5, we know

$$\tau_D^{\text{opt}}(\epsilon) = \arg\max_{\tau\in[0,1]} R_D(\tau) \quad \text{s.t. } R_1(\tau) \ge \epsilon \tag{13}$$

$$\tau_1^{\text{opt}}(\eta) = \arg\max_{\tau\in[0,1]} R_1(\tau) \quad \text{s.t. } R_D(\tau) \ge \eta. \tag{14}$$

The uniqueness of τ_D^{opt} and the monotonicity of $R_D(\tau)$ provided in the proof of Lemma 3 jointly yield the following.

Corollary 6: For an integer $D \ge 1$, $R_D(\tau)$ admits a unimodal distribution with peak at $\tau = \tau_D^{\text{opt}}$.

Define $\mathcal{I}_{\eta,D} := \{\tau \in [0,1] : R_D(\tau) \ge \eta\}$ for an integer $D > 1$. Define $\mathcal{I}_{\epsilon,1} := \{\tau \in [0,1] : R_1(\tau) \ge \epsilon\}$. Note that both of them are closed intervals by Corollary 6. In what follows, we will show that both $\tau_D^{\text{opt}}(\epsilon)$ and $\tau_1^{\text{opt}}(\eta)$ are unique.

Theorem 7: Consider an integer $D > 1$ and numbers $\epsilon \in [0, R_1^{\max}]$, $\eta \in [0, R_D^{\max}]$. Then, we have $\tau_D^{\text{opt}}(\epsilon) = \max\{\tau_D^{\text{opt}}, \min \mathcal{I}_{\epsilon,1}\}$ and $\tau_1^{\text{opt}}(\eta) = \min\{\tau_1^{\text{opt}}, \max \mathcal{I}_{\eta,D}\}$.

Proof: As $R_D(\tau) = R_1(\tau)(1-(1-\tau)^D)/\tau$ and $D > 1$, we obtain $\frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau) < \frac{1-(1-\tau)^D}{\tau} \cdot \frac{\mathrm{d}}{\mathrm{d}\tau} R_1(\tau)$ and then $\frac{\mathrm{d}}{\mathrm{d}\tau} R_D(\tau)\big|_{\tau=\tau_1^{\text{opt}}} < 0$. By Corollary 6, $R_D(\tau)$ is strictly increasing on the interval $(0, \tau_D^{\text{opt}}]$ and strictly decreasing on the interval $[\tau_D^{\text{opt}}, 1)$. Hence, we have $\tau_D^{\text{opt}} < \tau_1^{\text{opt}}$.

If $\tau_D^{\text{opt}} \in \mathcal{I}_{\epsilon,1}$, we have $\min \mathcal{I}_{\epsilon,1} \le \tau_D^{\text{opt}}$ and $\tau_D^{\text{opt}}(\epsilon) = \tau_D^{\text{opt}}$. Otherwise, by $\tau_D^{\text{opt}} < \tau_1^{\text{opt}}$ and the fact that τ_1^{opt} must lie in $\mathcal{I}_{\epsilon,1}$, we have $\min \mathcal{I}_{\epsilon,1} > \tau_D^{\text{opt}}$. Since $R_D(\tau)$ is strictly decreasing on the right of τ_D^{opt}, $\tau_D^{\text{opt}}(\epsilon) = \min \mathcal{I}_{\epsilon,1}$. In either case, we conclude that $\tau_D^{\text{opt}}(\epsilon) = \max\{\tau_D^{\text{opt}}, \min \mathcal{I}_{\epsilon,1}\}$.

If $\tau_1^{\text{opt}} \in \mathcal{I}_{\eta,D}$, we have $\max \mathcal{I}_{\eta,D} \ge \tau_1^{\text{opt}}$ and $\tau_1^{\text{opt}}(\eta) = \tau_1^{\text{opt}}$. Otherwise, by $\tau_D^{\text{opt}} < \tau_1^{\text{opt}}$ and the fact that τ_D^{opt} must lie in $\mathcal{I}_{\eta,D}$, we have $\max \mathcal{I}_{\eta,D} < \tau_1^{\text{opt}}$. Since $R_1(\tau)$ is strictly increasing on the left of τ_1^{opt}, $\tau_1^{\text{opt}}(\eta) = \max \mathcal{I}_{\eta,D}$. In either case, we conclude that $\tau_1^{\text{opt}}(\eta) = \min\{\tau_1^{\text{opt}}, \max \mathcal{I}_{\eta,D}\}$. ∎

V. Simulation Results

In this section, we verify the previous analysis and compare different cases via simulation. Following the system model described in Section II, we implemented the multichannel ALOHA in Monte Carlo simulations using MATLAB programm. Each simulation point represents the value averaged over ten independent simulation runs, each lasting for 10^6 time slots.

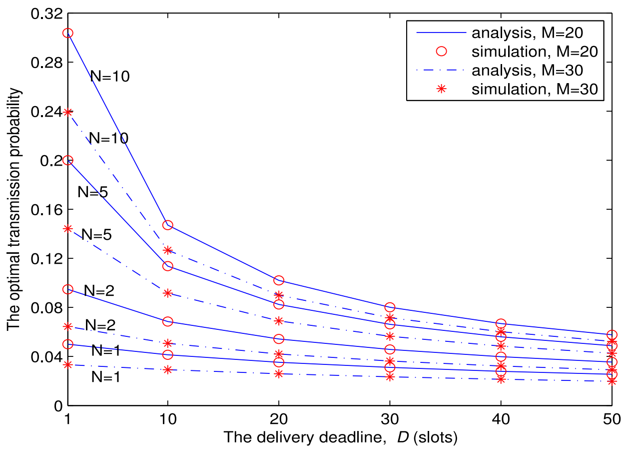

Fig. 1. Optimal transmission probabilities as a function of D for $M = 20, 30$ and $N = 1, 2, 5, 10$.

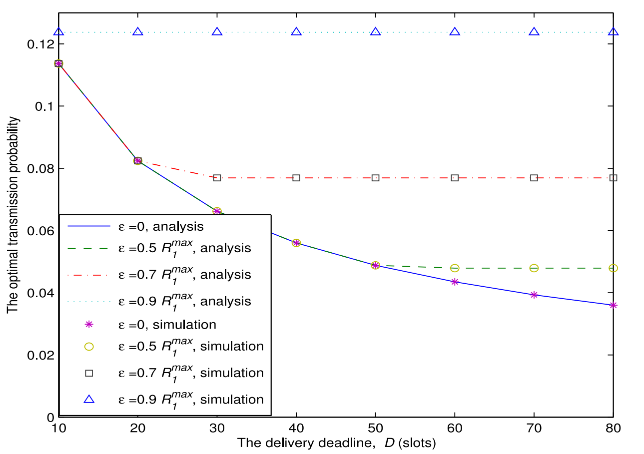

Fig. 2. Optimal transmission probability for maximizing reliability subject to a throughput constraint ϵ as a function of D for $M = 20$, $N = 5$.

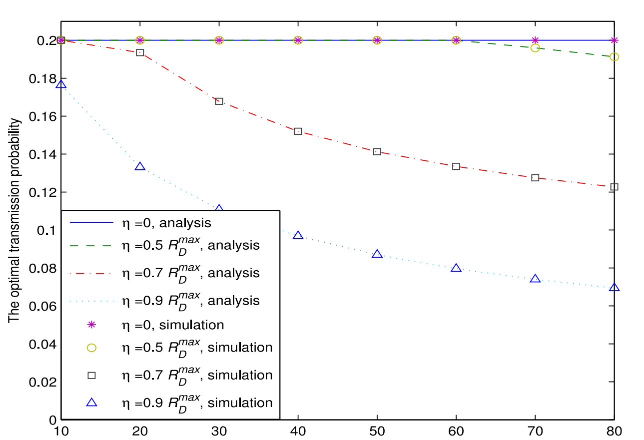

Fig. 3. Optimal transmission probability for maximizing throughput subject to a reliability constraint η as a function of D for $M = 20$, $N = 5$.

Fig. 1 shows $\tau_D^{\rm opt}$ in a variety of operating conditions. We see that analytical results match well with simulation results in all the scenarios. Given $M = 20, N = 5$, Fig. 2 shows $\tau_D^{\rm opt}(\epsilon)$ under the throughput constraint ϵ. We see that $\tau_D^{\rm opt}(\epsilon)$ is closer to $\tau_D^{\rm opt}$, when ϵ is smaller. We further note that $\tau_D^{\rm opt}(\epsilon)$ for some positive ϵs keeps unchanged when D is larger than a certain value, as $\tau_D^{\rm opt}(\epsilon)$ is only determined by $\mathcal{I}_{\epsilon,1}$ in these cases. Fig. 3 shows $\tau_1^{\rm opt}(\eta)$ under a reliability constraint η. We see $\tau_1^{\rm opt}(\eta)$ is closer to $\tau_1^{\rm opt}$ when D or η is smaller. It is also observed that $\tau_1^{\rm opt}(\eta)$ is only determined by $\mathcal{I}_{\eta,D}$ when D or η is larger than a certain value.

VI. Conclusion

This paper considered multichannel slotted ALOHA for deadline-constrained unicast in ad hoc networks. We found the optimal schedule that maximizes the reliability under a throughput constraint, and the one that maximizes the throughput under a reliability constraint. These results facilitate the parameters design for different QoS requirements. If we relax the ideal channel assumption to consider asymmetric channel conditions, finding the optimal schedule would become more complicated, and is an interesting avenue for future work.

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