A Contract-Based Incentive Mechanism for Data Caching in Ultra-Dense Small-Cells Networks

Shunfeng Chu[1], Jun Li[1], Tingting Liu[1,2], Feng Shu[1]
[1]School of Electrical and Optical Engineering, Nanjing University of Science and Technology, Nanjing, China 210094
[2]Nanjing Institute of Technology, Nanjing, China 211167
Email: shunfeng.chu@njust.edu.cn, jleesr80@gmail.com, liutt@njit.edu.cn, shufeng@njust.edu.cn

***Abstract*—Wireless caching is an efficient mechanism for reducing downloading delay and reducing the traffic pressure over backhual channels by caching some popular content, e.g., video clips, in small base stations (SBSs). In this paper, we consider a commercialized small-cell caching system consisting of a network service provider (NSP), several video retailers (VRs) and mobile users (MUs). The NSP leases its SBSs to VRs in order to earn profits, while the VRs store popular videos into the lent SBSs, thereby gaining profits from providing better services to the MUs. We conceive the system within the framework of contract theory by designing the optimal quality-price contract. We establish the profit function of NSP and VRs and solve the profit maximization problem through contract theory. Numerical results validate the effectiveness of our incentive mechanism for the system.**

***Index Terms*—Small-cell caching, cellular networks, stochastic geometry, incentive mechanism, Contract Theory.**

I. Introduction

Wireless data traffic is expected to increase exponentially in the next few years driven by a staggering growth of mobile users (MU) and their bandwidth-hungry mobile applications. The redundancy of data transmissions can be reduced by locally storing popular content, known as caching, into the memory of intermediate network nodes, effectively forming a local caching system [1]. The local caching brings content closer to the MUs and alleviates redundant data transmissions via redirecting the downloading requests to the intermediate nodes. Generally, wireless data caching consists of two stages: data placement and data delivery [2]. In the data placement stage, popular data are cached into local storages during off-peak time, while in the data delivery stage, requested contents are delivered from the local caching system to the MUs.

As small-cell embedded architectures will be prevailing in future cellular networks, known as heterogeneous networks (HetNet) [3], caching relying on small-cell base stations (SBS), namely, small-cell caching, is a promising trend for the HetNets. In [4], a small-cell caching scheme, called 'Femtocaching', is proposed for a cellular network embedded with SBSs, where the data placement at the SBSs is optimized in a centralized manner for reducing the transmission delay imposed. However, [4] considers an idealized system, where neither the interference nor the impact of wireless channels is taken into account. In [5], the small-cell caching is investigated in the context of stochastic networks. The average performance is developed via stochastic geometry theory [6], [7], where the distribution of network nodes are modeled by Poisson point process (PPP). However, the caching strategy in [5] assumes that the SBSs always cache the same content.

From above discussions, considering the data placement issue is important for current research. However, the caching model combine many issues instead of data placement. From a commercial standpoint, considering the topics such as pricing on video streaming, renting local storages, is more interesting. As video transmissions dominate the mobile data traffic, we consider a commercialized caching system which is consisted of video retailers (VR), network service providers (NSP) and MUs. The VRs buy the right to the videos and publish the videos on their web. The NSPs are typically operators of cellular networks, because of the control of network facilities, such as macro-cell base stations and SBSs.

In such a commercialized caching system, the VRs provide video streaming services to MUs to earn profits. As the central servers of the VRs, which store the popular videos, are usually located at backbone networks and far away from the MUs, an efficient solution is to locally cache these videos for reducing the transmission latency, thereby attracting more customers. These local caching demands raised by the VRs offer the NSPs profitable opportunities from leasing their resources, i.e., the S-BSs. Besides, by reducing redundant video transmissions over SBSs' back-haul channels, the NSPs can save considerable costs. Under this circumstances, both the VRs and NSPs are the beneficiaries from the local caching system. However, each participant is selfish and wishes to maximize its own benefit, leading to a competition and optimization problem.

In this paper, we research on a commercialized caching prototype within a contract theoretic framework. The system consists of an NSP and multiple VRs, where the NSP, as the monopolist in the market in charge of the trading resource, i.e., SBSs, wishes to lease its SBSs to the VRs for the purpose of making profits. To comply with the future trend in 5G, we consider the ultra-dense deployment of the SBSs, i.e., the number of the SBSs is much higher than that of the MUs. The main contributions of the paper are as follow. First of all, by modeling the MUs and SBSs using two independent PPPs, we develop the probability expression of direct downloading. Then, we formulate contracts betweeen VRs and NSP and state the feasibility of the contract. Next, we solve the optimization problem by a series of transformation. Finally, we provide several results for the pricing and SBSs allocation scheme.

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II. System Model

We consider a signal commercial small-cell caching model, which is consisted of only one NSP, V VRs, and some MUs. Denote by $\mathbf{L}$ the NSP, by $\boldsymbol{\mathcal{V}} = \{\mathcal{V}_1, \mathcal{V}_2, \cdots, \mathcal{V}_V\}$ the set of the VRs, and by $\mathcal{M}$ one of the MUs. In such a system, the VRs negotiate with the NSP for renting part of its SBSs and caching popular videos. Both the NSP and each VR aim for maximizing their profits.

A. Network Model

Let us consider multiple SBSs owned by $\mathbf{L}$ equipping with a fixed volume of storage for caching Q video files. We assume that the SBSs transmit over the channels that are orthogonal to those of the macro-cell base stations to avoid the interference incurred by the macro-cell base stations. The distribution of SBSs can be regarded as a homogeneous PPP (HPPP) Φ of intensity λ. Here, the intensity λ means the number of the SBSs per unit area. And the distribution of the MUs also can be regarded as an independent HPPP Ψ of intensity ζ.

The wireless down-link channels spanning from the SBSs to the MUs are independent and identically distributed (*i.i.d.*), and modeled as the combination of path-loss and Rayleigh fading. Without loss of generality, we just analyze a typical MU located at the origin. The path-loss between an SBS located at x and the typical MU can be expressed as $\|x\|^{-\alpha}$, where α means the path-loss exponent. The channel gain of the Rayleigh fading can be denoted by h_x, where $h_x \sim \exp(1)$. The noise at a MU is Gaussian distributed with a variance σ^2. To alleviate interference among the densely deployed SBSs, we adopt the dynamic on-off architecture [8], where an SBS will switch to the idle mode, i.e., turn off its radio transmissions, if there is no MU associated with it for video downloading.

B. Preference and Affiliation

We now consider the preference distribution, i.e., the distribution of request probabilities, among the popular videos to be cached. Denote by $\boldsymbol{\mathcal{F}} = \{\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_F\}$ the file set consisting of F video files, where each video file contains an individual movie or video clip that is frequently requested by MUs. The popularity distribution of $\boldsymbol{\mathcal{F}}$ is represented by a vector $\mathbf{p} = [p_1, p_2, \cdots, p_N]$. Generally, $\mathbf{p}$ can be modeled by the Zipf distribution [9] as

$$p_f = \frac{1/f^{\beta}}{\sum_{j=1}^{F} 1/j^{\beta}}, \quad \forall f, \tag{1}$$

where the exponent β is a positive value, characterizing the video popularity. From Eq. (1), a video file with a smaller f corresponds to a higher popularity.

At the same time, the MUs have imbalanced affiliations with regard to the V VRs, i.e., some VRs have more mobile customers than others. The affiliation distribution among the VRs is denoted by $\mathbf{q} = [q_1, q_2, \cdots, q_V]$, where q_v, $v = 1, \cdots, V$, represents the probability that an MU is affiliated with $\mathcal{V}_v$. The affiliation distribution $\mathbf{q}$ can also be modeled by the Zipf distribution. Hence, we have

$$q_v = \frac{1/v^{\gamma}}{\sum_{j=1}^{V} 1/j^{\gamma}}, \quad \forall v, \tag{2}$$

where γ is a positive value, characterizing the preference of the VRs.

III. Analysis on Caching Performance

There are three stages for caching. In the first stage, the VRs purchase the copyrights of popular videos from video producers and publish them on their web-sites.

In the second stage, upon obtaining the popular videos, the VRs negotiate with the NSP for renting its SBSs. As $\mathbf{L}$ leases its SBSs to multiple VRs, we denote by $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_V]$ the intensity vector of the rented SBSs, where λ_v represents the intensity of the SBSs that are rented by $\mathcal{V}_v$, $\forall v$. We assume that the SBSs rented by each VR are uniformly distributed. Hence, the SBSs that are allocated to $\mathcal{V}_v$ can be modeled as a "thinned" HPPP Φ_v with intensity λ_v. We have $\sum_{v=1}^{V} \lambda_v \leq \lambda$. For simplicity, we assume that each of the rented SBSs will be required to cache the Q most popular video clips $\mathcal{F}_1, \cdots, \mathcal{F}_Q$.

We view the SBSs rented by $\mathcal{V}_v$ combined with the MUs affiliated with $\mathcal{V}_v$ as the v-th tier, namely, Tier-v, where the intensities of these SBSs and MUs are λ_v and $q_v\zeta$, respectively. In the dynamic on-off architecture, some of the SBSs may be activated for transmitting data, while other SBSs may be in an idle mode. We denot by $\mathcal{E}_v^{\text{active}}$ the event that an SBS in Tier-v is active, and its probability can be approximately expressed by [10]

$$\Pr(\mathcal{E}_v^{\text{active}}) \approx 1 - \left(1 + \frac{q_v\zeta}{3.5\lambda_v}\right)^{-3.5} \tag{3}$$

We assume that the SBSs from $\mathcal{V}_v$ are allocated with a power P_v. Hence, the received signal-to-interference-plus-noise ratio (SINR) at the typical MU from an SBS in Φ_v located at x can be expressed as

$$\rho(x) = \frac{P_v h_x \|x\|^{-\alpha}}{I_1 + I_2 + \sigma^2}, \tag{4}$$

where

$$I_1 = \sum_{v' \neq v} \sum_{x' \in \Phi_{v'}} \Pr(\mathcal{E}_{v'}^{\text{active}}) P_{v'} h_{x'} \|x'\|^{-\alpha} \tag{5}$$

is the interference imposed by the SBSs from other $V-1$ tiers, while

$$I_2 = \sum_{x' \in \Phi_v \setminus x} \Pr(\mathcal{E}_{v'}^{\text{active}}) P_v h_{x'} \|x'\|^{-\alpha} \tag{6}$$

is the interference from other SBSs in the same tier.

The typical MU is considered to be "covered" by an SBS located at x as long as $\rho(x)$ is no lower than a pre-set SINR threshold δ, i.e., $\rho(x) \geq \delta$. Generally, an MU can be covered by multiple SBSs. Note that the SINR threshold δ defines the highest delay of downloading a video file.

In the third stage, the MUs start to download videos from nearby SBSs. When an MU $\mathcal{M}$ affiliated with $\mathcal{V}_v$ requires a video clip from $\mathcal{V}_v$, it searches the SBSs in Φ_v and tries to connect to the nearest SBS that covers $\mathcal{M}$. Provided that such an SBS exists, the MU $\mathcal{M}$ will obtain this video directly from this SBS, and we thereby define this event by $\mathcal{E}_v^{\text{cover}}$. Otherwise, $\mathcal{M}$ will be redirected to the central servers of $\mathcal{V}_v$ for downloading the requested file.

Based on stochastic geometry, the probability of the event $\mathcal{E}_v^{\text{cover}}$, i.e., the event that an MU in Tier v can successfully download a video file from its nearby SBS, can be expressed, according to the result in [11], as

$$\begin{aligned}\Pr(\mathcal{E}_v^{\text{cover}}) = &\sum_{f=1}^{Q} p_f \cdot \\ &\int_0^\infty \prod_{v'} \exp\left(-\pi \Pr(\mathcal{E}_{v'}^{\text{active}})\lambda_{v'} \left(\frac{P_{v'}}{P_v}\right)^{\frac{2}{\alpha}} C(\delta,\alpha)\, z^2\right) \\ &\exp\left(-\pi \Pr(\mathcal{E}_v^{\text{active}})\lambda_v A(\delta,\alpha) z^2\right) \\ &\exp(-\frac{z^\alpha \delta}{P_v}\sigma^2)\pi\lambda_v \exp\left(-\pi\lambda_v z^2\right)\, \mathrm{d}z^2,\end{aligned} \tag{7}$$

where we have

$$\begin{aligned} A(\delta,\alpha) &\triangleq \frac{2\delta}{\alpha-2}\, {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\delta\right), \\ C(\delta,\alpha) &\triangleq \frac{2}{\alpha}\delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1-\frac{2}{\alpha}\right). \end{aligned} \tag{8}$$

Furthermore, ${}_2F_1(\cdot)$ in the function $A(\delta,\alpha)$ is the hypergeometric function and the Beta function in $C(\delta,\alpha)$ is formulated as $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$.

Generally, since the power of interference in a network is much greater than that of the noise, we can assume that $\frac{\sigma^2}{P_v}$ goes to zero. Due to the SBSs densely deployed in our system, we also assume $\lambda_v \gg q_v\zeta$. Then we can simplify $\Pr(\mathcal{E}_v^{\text{active}})$ in Eq. (3) as $\Pr(\mathcal{E}_v^{\text{active}}) \approx \frac{q_v\zeta}{\lambda_v}$.

Since SBSs usually transmit with the same power in a cellular network, we can obtain a more concise form of $\Pr(\mathcal{E}_v^{\text{cover}})$ as

$$\Pr(\mathcal{E}_v^{\text{cover}}) = \frac{\sum_{f=1}^{Q} p_f \lambda_v}{\lambda_v + q_v A(\delta,\alpha)\zeta + (1-q_v)C(\delta,\alpha)\zeta}. \tag{9}$$

IV. Contract-based Problem Formulation

In our contract-based caching system, the NSP is in charge of the trading resources, i.e., the SBSs. Hence, trading on the SBSs is a monopoly market, where the NSP, as the monopolist, dominates the trading process, and the VRs act as the consumers.

A. Monopolist Model

We can denote the set of the V SBS groups by vector $\mathbf{\Phi} \triangleq \{\Phi_1, \cdots, \Phi_V\}$, in which, Φ_v, $\forall v$ represents the SBS group rented by $\mathcal{V}_v$. So we utilize the intensity λ_v of Φ_v to represent its quality. We assume that if a VR $\mathcal{V}_v$ rents the SBSs in Φ_v with the quality of λ_v, a payment $\pi(\lambda_v)$ will be charged by the NSP for a unit period. Consequently, we denote by $\mathbf{\Lambda} \triangleq \{\lambda_1, \cdots, \lambda_V\}$ the quality set, and by $\mathbf{\Pi} \triangleq \{\pi(\lambda_1), \cdots, \pi(\lambda_V)\}$ the price set for commodity set $\mathbf{\Phi}$.

We now investigate the profit of the NSP, S_v^{NSP}, gained by leasing its SBSs in Φ_v, and we have

$$S_v^{\text{NSP}}(\lambda_v) = \pi(\lambda_v) - s^{\text{cost}}\lambda_v, \tag{10}$$

where s^{cost} is the cost for the maintaining each SBS by the NSP during a unit period. The overall profit of the NSP can thus be expressed by $S^{\text{NSP}} = \sum_{v=1}^{V} S_v^{\text{NSP}}(\lambda_v)$.

The objective of the NSP is to maximize its profit via optimizing the qualities of its resources and the corresponding prices charged. Obviously, a rational NSP will not accept a negative S^{NSP}, and thus it always makes $\pi(\lambda_v) \geq s^{\text{cost}}\lambda_v$ when optimizing $\mathbf{\Lambda}$ and $\mathbf{\Pi}$.

B. Consumer Model

We assume that each VR prefers a higher allocation of SBSs for achieving a better coverage on its affiliated MUs, since the revenue of $\mathcal{V}_v$, $R(\lambda_v)$, stems from providing fast and local video downloading services to these MUs. We have

$$\begin{aligned} R(\lambda_v) &= c q_v \zeta \Pr(\mathcal{E}_v^{\text{cover}}) s^{\text{cache}} \\ &= \frac{(c\zeta s^{\text{cache}} \sum_{f=1}^{Q} p_f) q_v \lambda_v}{\lambda_v + q_v A(\delta,\alpha)\zeta + (1-q_v)C(\delta,\alpha)\zeta}, \end{aligned} \tag{11}$$

where c is the average number of video downloading requests of each MU during a unit period, and thus the item $c q_v \zeta \Pr(\mathcal{E}_v^{\text{cover}})$ represents the overall quantity of video downloading from $\mathcal{V}_v$'s local caching system per unit period. Furthermore, s^{cache} in Eq. (11) is the extra fee charged by $\mathcal{V}_v$ on an MU for providing local downloading of a video.

To facilitate the analysis, we further classify the VRs into different types according to the Zipf distribution. That is, the type of $\mathcal{V}_v$ is represented by the corresponding probability q_v, and the vector $\mathbf{q}$ can be viewed as the set of all types. Thus, there are overall V types, with each type containing one VR. From the second equation of Eq. (11), $R(\lambda_v)$ can also be viewed as a function of q_v.

Accordingly, we can verify $\frac{\partial R(\lambda_v, q_v)}{\partial \lambda_v} > 0$ and $\frac{\partial R(\lambda_v, q_v)}{\partial q_v} > 0$. It means that the VR $\mathcal{V}_v$ prefers a higher λ_v, to achieve a greater revenue $R(\lambda_v)$ and given a λ, a higher type has more profit than a lower one. By further calculations, we obtain $\frac{\partial^2 R(\lambda_v, q_v)}{\partial \lambda_v^2} < 0$ which means $R(\lambda_v, q_v)$ is a concave function of λ_v.

In addition, the VR $\mathcal{V}_v$ needs to pay for renting the SBSs, which is $\pi(\lambda_v)$ per unit period. Therefore, the net profit of the VR $\mathcal{V}_v$ can be expressed as

$$S^{\text{VR}}(\lambda_v, q_v) = R(\lambda_v, q_v) - \pi(\lambda_v). \tag{12}$$

Each VR is selfish and rational, whose objective is to maximize its profit during the resource trading.

C. Contracts Formulation

A feasible contract is a set of quality-price combinations, in which the VR $\mathcal{V}_v$ with the type $q_v, \forall v$, prefers the product with quality λ_v at price π_v to any other product with a different quality. To be specific, each VR finds it in its own interest to buy the product assigned to its type, which is called incentive compatible (IC), i.e.,

$$R(\lambda_v, q_v) - \pi_v \geq R(\lambda_{v'}, q_v) - \pi_{v'}, \quad \forall v' \neq v. \tag{13}$$

Additionally, each VR is assumed to be rational to not buy a product without a positive profit, i.e.,

$$R(\lambda_v, q_v) - \pi_v \geq 0, \quad \forall v. \tag{14}$$

This property is referred to as individual rationality (IR).

A feasible contract must satisfy the IC and IR constraints, and any contract satisfying the IC and IR must be feasible. The overall profit of the NSP can be written as

$$S^{\text{NSP}} = \sum_{v=1}^{V} \pi_v - s^{\text{cost}} \lambda_v. \tag{15}$$

The optimal contract, denoted by $\{(\lambda_v^\star, \pi_v^\star), \forall v\}$, is thus defined as a feasible contract that maximizes the profit of the NSP. We have

$$\{(\lambda_v^\star, \pi_v^\star), \forall v\} = \arg\max_{\lambda_v, \pi_v} \sum_{v=1}^{V} \pi_v - s^{\text{cost}} \lambda_v \tag{16}$$

subject to the IC and IR constraints in Eqs. (13) and (14), respectively.

V. Optimal Contract Design

In our contract, each consumer type containing only one VR specifies one quality-price contract item, and these consumer types satisfy $q_1 > q_2 > \cdots > q_V$ according to the Zipf distribution. We consider the incomplete information scenario, where the NSP is only aware of the Zipf distribution parameter γ, while it does not know the type value of a given VR. The quality-price contract items can be denoted as $\{(\lambda_v, \pi_v),\ v = \{1, 2, \cdots, V\}\}$.

A. Feasibility of Contract

The profit maximization problem in Eq. (16) is nontrivial, which can, be solved by first simplifying the IR and IC constraints before the optimization.

Due to page limit, all the proofs for the following lemmas will be presented in the journal version.

Lemma 1: Regarding the optimal contract in the incomplete information scenario, IR constraint can be replaced by

$$\frac{(c\zeta s^{\text{cache}} \sum_{f=1}^{Q} p_f) q_V \lambda_V}{\lambda_V + q_V A(\delta, \alpha)\zeta + (1 - q_V) C(\delta, \alpha)\zeta} - \pi_V = 0. \tag{17}$$

given that the IC constraint holds.

Lemma 2: If all the $V(V-1)$ constraints in the IC are satisfied, then the monotonicity constraint will hold, i.e., $\lambda_{v_1} \geq \lambda_{v_2}$ if and only if the user type $q_{v_1} \geq q_{v_2}$.

Lemma 3: For any type $q_{j_1} \geq q_{j_2}$ and $\lambda_{v_1} \geq \lambda_{v_2}$, the revenue function satisfies the following condition:

$$R(\lambda_{v_1}, q_{j_1}) - R(\lambda_{v_1}, q_{j_2}) \geq R(\lambda_{v_2}, q_{j_1}) - R(\lambda_{v_2}, q_{j_2}). \tag{18}$$

Lemma 4: (LDICs: Local Downward Incentive Constraints) If the LDICs are satisfied for the type $q_v, \forall v$, i.e.,

$$R(\lambda_v, q_v) - \pi_v \geq R(\lambda_{v+1}, q_v) - \pi_{v+1} \tag{19}$$

with $\lambda_v \geq \lambda_{v+1}$, then the IC constraint will hold for any $v_1 \leq v_2$, i.e.,

$$R(\lambda_{v_1}, q_{v_1}) - \pi_{v_1} \geq R(\lambda_{v_2}, q_{v_2}) - \pi_{v_2}. \tag{20}$$

Lemma 5: (LUICs: Local Upward Incentive Constraints) If the LUICs are satisfied for the type $q_v, \forall v$, i.e.,

$$R(\lambda_v, q_v) - \pi_v \geq R(\lambda_{v-1}, q_v) - \pi_{v-1} \tag{21}$$

with $\lambda_v \leq \lambda_{v-1}$, then the IC constraint will hold for any $v_1 \geq v_2$, i.e.,

$$R(\lambda_{v_1}, q_{v_1}) - \pi_{v_1} \geq R(\lambda_{v_2}, q_{v_1}) - \pi_{v_2}. \tag{22}$$

Lemma 6: If the profit of the NSP is maximized, i.e., the contract is at the optimum, then the LDICs must satisfy the following condition:

$$R(\lambda_v, q_v) - \pi_v = R(\lambda_{v+1}, q_v) - \pi_{v+1}. \tag{23}$$

B. Optimality of Contract

According to the profit expression of the NSP, the profit maximization problem, subject to the IC and IR constraints for all types, can be written as

$$\begin{aligned} &\max_{(\lambda_v, \pi_v)} \sum_{v=1}^{V} (\pi_v - s^{\text{cost}} \lambda_v) \\ &\text{s.t. } R(\lambda_v, q_v) - \pi_v \geq 0, \text{ (IR)} \\ &R(\lambda_v, q_v) - \pi_v \geq R(\lambda_{v'}, q_v) - \pi_{v'}, \text{ (IC)} \\ &\forall v, \quad \forall v' \neq v. \end{aligned} \tag{24}$$

Based on the series lemmas in the previous subsection, the above profit maximization problem can be further represented as follows:

$$\begin{aligned} &\max_{(\lambda_v, \pi_v)} \sum_{v=1}^{V} (\pi_v - s^{\text{cost}} \lambda_v) \\ &\text{s.t. } R(\lambda_V, q_V) - \pi_V = 0, \\ &\lambda_v \geq \lambda_{v'} \text{ iff } q_v \geq q_{v'}, \\ &R(\lambda_v, q_v) - \pi_v = R(\lambda_{v+1}, q_v) - \pi_{v+1}, \\ &\forall v, \quad \forall v' \neq v. \end{aligned} \tag{25}$$

From the first and third conditions of the above problem, we can get the following expression,

$$S^{\mathrm{NSP}}=\sum_{v=1}^{V}(\pi_v-s^{\mathrm{cost}}\lambda_v)$$
$$=\sum_{v=1}^{V}\left(R(\lambda_V,q_V)+\sum_{k=v}^{V}w_k-s^{\mathrm{cost}}\lambda_v\right),$$

where

$$\begin{cases} w_v=0 & v=V \\ w_v=R(\lambda_v,q_v)-R(\lambda_{v+1},q_v) & v=1,2,...,V-1. \end{cases} \tag{26}$$

Furthermore, by defining $q_0=0$ for notation simplification, the profit S^{NSP} can be further written as

$$S^{\mathrm{NSP}} = \sum_{v=1}^{V}(vR(\lambda_v,q_v)-(v-1)R(\lambda_v,q_{v-1})-s^{\mathrm{cost}}\lambda_v). \tag{27}$$

By introducing the function

$$S_v \triangleq vR(\lambda_v,q_v)-(v-1)R(\lambda_v,q_{v-1})-s^{\mathrm{cost}}\lambda_v, \tag{28}$$

we can find that S_v is only related to λ_v and independent of other qualities $\lambda_{v'}$, $\forall v'\neq v$. Thus for any $v\in\{1,2,...,V\}$, the optimal quality λ_v^* can be obtained by maximizing each of S_v separately, i.e.,

$$\lambda_v^*=\arg\max_{\lambda_v} S_v$$
$$=\arg\max_{\lambda_v}(vR(\lambda_v,q_v)-(v-1)R(\lambda_v,q_{v-1})-s^{\mathrm{cost}}\lambda_v). \tag{29}$$

According to the above problem, we can get a solution $\lambda_v^\star$, $v\in\{1,2,...,V\}$ for the problem. In addition, we need to check whether these solutions satisfy monotonicity condition. If these solution satisfy the monotonicity condition, they can be regarded as our optimal solutions. However, if they do not satisfy the condition, the obtained solution $\lambda_v^\star$ must be adjusted by "Bunching and Ironing" algorithm. So, we introduce a important lemma as follow.

Lemma 7: Let $X_1(x)$ and $X_2(x)$ be concave functions on x. If $x_1^{'}\geq x_2^{'}$ where $x_1^{'}=\arg\max_{x_1}X_1(x_1)$ and $x_2^{'}=\arg\max_{x_2}X_2(x_2)$, then $\hat{x}_1=\hat{x}_2$ where

$$\{\hat{x}_1,\hat{x}_2\}=\arg\max_{x_1,x_2}\sum_{i=1}^{2}X_i(x_i),\ \text{s.t. } x_1\leq x_2 \tag{30}$$

We can refer to [12] for the detailed proof of this lemma. By Lemma 7, we can adjust the obtained solution which does not satisfy the monotonicity condition. According to the solution λ_v^* and Eq. (25), we can get the corresponding optimal price π_v^* as follows:

$$\pi_v^*=R(\lambda_V^*,q_V)+\sum_{k=v}^{V}w_k^*, \tag{31}$$

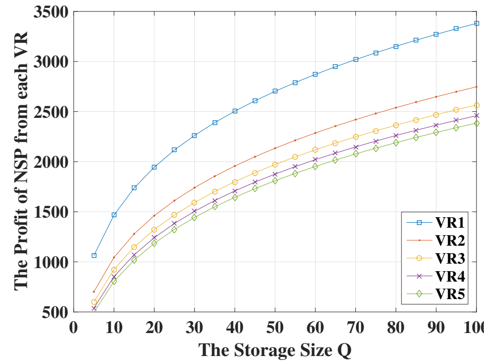


Fig. 1. The profit of NSP from each VR π_v with respect to the storage size Q and the type q_v of each VR.

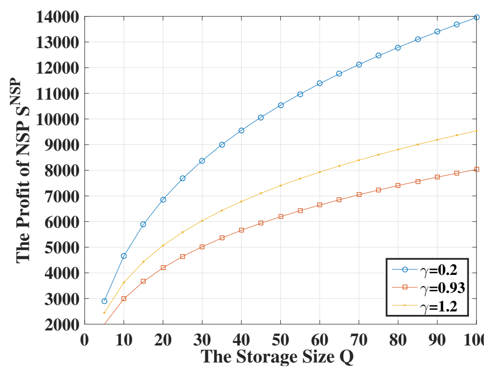


Fig. 2. The profit of NSP S^{NSP} with respect to the storage size Q and the Zipf distribution parameter γ of VR.

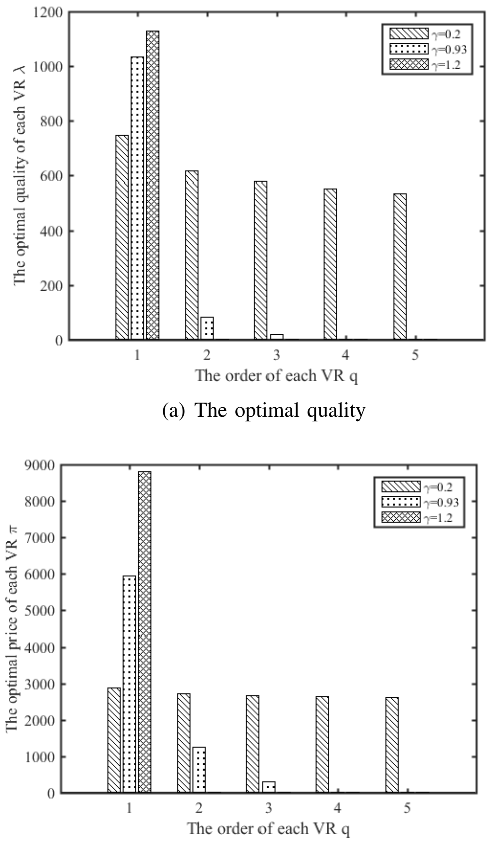
where

$$\begin{cases} w_v^*=0 & v=V \\ w_v^*=R(\lambda_v^*,q_v)-R(\lambda_{v+1}^*,q_v) & v=1,2,...,V-1. \end{cases} \tag{32}$$

VI. NUMERICAL RESULTS

In this section, we conduct numerical simulations to validate the performance of our scheme in commercial small-cell caching system. We assume that there are five VRs and the type q of each VR is subject to Zipf distribution. In Fig. 1, we compare the profit of NSP gained from each VR π_v from the five VRs with five different type as we vary the storage size Q. From the simulation results we can see that, as the storage size Q increases, the profit of NSP gained from each VR π_v is increasing as well. The reason for this phenomenon is that larger storage size Q means more successful download files. Therefore, NSP will gain more profit for a larger storage size in each SBS. Within these VRs, q_1 means the largest type, while q_5 is the smallest type. From the figure, we can see that a VR with less type contributes less to the profit of the NSP.

In Fig. 2, we analyze the impact of VR's Zipf distribution parameter γ versus the storage size Q on the total profit of NSP S^{NSP}. We choose three different values of γ and compare



(a) The optimal quality

(b) The optimal price

Fig. 3. The optimal quality and optimal price with respect to the type of each VR and the Zipf distribution parameter γ of VR.

the results for the three values $\gamma = 0.2, 0.93, 1.2$. We can also see that the profit of NSP S^{NSP} is increasing as Q increases.

Fig. 3 demonstrates the optimal quality and the optimal price of each type VR with varying the parameter γ, where γ are set as 0.2, 0.93, and 1.2, respectively. As expected, the higher type leads to more quality λ than the lower type in Fig. 3(a). The optimal quality of first type with $\gamma = 1.2$ is larger than the value whit $\gamma = 0.2$ and 0.93. But for other types, we get the opposite conclusion. The reason for this phenomenon is that the lager γ means higher first type q_1 of VR. From the Fig. 3(a), we can also see that q_4 and q_5 equal 0 when $\gamma = 0.93$ and q_2, q_3, q_4, q_5 equal 0 when $\gamma = 1.2$. This indicates that the lower the type is more inclined to buy the lower the quality, even the quality is 0. Fig. 3(b) demonstrates the optimal price of each type VR with varying the parameter γ, where γ are set as 0.2, 0.93, and 1.2. Since the price is corresponding to the quality, the trend in Fig. 3(b) is similar to Fig. 3(a).

VII. Conclusions

In this paper, we considered a commercial small-cell caching system consisting of an NSP and multiple VRs, where the NSP leases its SBSs to the VRs for gaining profits and for reducing the costs of back-haul channel transmissions, while the VRs, after storing popular videos to the rented SBSs, can provide faster transmissions to the MUs, hence gaining more profits. We first modeled the MUs and SBSs using two independent PPPs with the aid of stochastic geometry, and developed the probability expression of direct downloading. Then, based on the probability derived, we formulated a contract and illustrated the feasibility of the contract for maximizing the profit of the NSP. Next, we solved the optimization problem by a series of transformations based on Contract Theory. Finally, we provided several numerical results for showing that the proposed schemes are effective in both pricing and SBSs allocation.

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