Optimum Power Allocation for LDPC coded Soft Forwarding Scheme in Wireless Networks

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***Abstract*—This paper proposes a new power efficient multilevel threshold based soft quantization (MLT-SQ+PA) scheme for a multiple access relay system (MARS). In the proposed MLT-SQ+PA protocol, the relay computes the reliabilities, expressed as log-likelihood ratios (LLRs), of the received signals from the two sources. We provide an analytical closed form expression for the optimized power allocation factor at the relay. The relay evaluates the LLRs of the network-coded packet, quantizes and scales these using the optimum power allocation factor, forwarding the resulting "*quantized soft symbols*" to the destination. Compared to competing schemes, the performance of our system is superior in terms of BER when the same amount of channel state information (CSI) is exploited.**

I. Introduction

Cooperative communication for wireless networks provide an improved transmit diversity and spectral efficiency [1]. The two most frequently used relaying protocols are decode-and-forward (DF) and amplify-and-forward (AF). In DF, errors at the relay can be corrected and thus error propagation can be avoided under good source-relay channel conditions. In contrast, the AF protocol suffers from the noise amplification.

A newly proposed and promising relay protocol called *soft information relaying* (SIR) has gained significant attention [2]-[5]. In [2], the authors studied the SIR together with distributed turbo coding (DTC). In this method, the reliability of the recursively re-encoded soft bits depend strongly on the least reliable input bits, causing a decaying LLR profile. The work in [2] models the effective noise introduced by the relay as Gaussian noise. A disadvantage of these unquantized soft forwarding schemes is that a complex and not quite accurate model must be adopted for the equivalent noise (comprising contributions from both channel and relay operations), in order to form LLRs at the destination. This is an inherent problem which originates due to the unquantized nature of the signal transmitted from the relay.

An LLR-threshold based soft quantize-and-forward protocol was presented in [6]-[7]. In these works, the relay evaluates the reliabilities of the received symbols (from each source) and selects the corresponding level to send to the destination. When the symbol LLR is sufficiently large, the relay transmits the hard decision for the symbol; otherwise it is silent. Therefore, these works [6]-[7] may be regarded as a soft forwarding scheme based on three-level LLR quantization in simple relay networks; however, the extension to multilevel quantization and a multiple access relay is not trivial.

The idea of *multilevel* threshold based soft quantization (MLT-SQ+PA) protocol using multiple thresholds which are optimized for minimum overall bit error rate (BER) at the destination in a multiple access relay system used in this paper is based on our previous work in [5]. Our contribution in this paper mainly concentrate on the optimum power allocation factor which minimises the BER at the destination and this improves the diversity and coding gain. The simulation results demonstrate that the proposed MLT-SQ+PA scheme efficiently mitigates error propagation in a power efficient manner when compared to the benchmark schemes.

II. System Model

In this paper, we consider a four-terminal topology as shown in Fig. 1, but this can be extended for any relay topology. It is assumed that there is a direct transmission from the sources to the destination. In the system description and analysis, we will consider un-coded MARS. The sources[1] S_1 and S_2 transmit an uncoded frame, of length N, of BPSK modulated symbols in the first and second time slots respectively; these are received both by the relay and the destination. In the third time slot, the relay aids the destination by transmitting a network coded message based on the signals received in the first and second time slots. We assume that all nodes have only one antenna working in a half-duplex mode.

We denote[2] by h_{iR}, h_{iD}, and h_{RD} where $i \in \{S_1, S_2\}$, the channel coefficients between i and R, between i and D, and between R and D, respectively. The corresponding distances between nodes are denoted by d_{iR}, d_{iD}, and d_{RD} respectively. We assume that h_{iR}, h_{iD}, and h_{RD} are independent and identically Rayleigh distributed. The channel gains are related to the corresponding distances by the attenuation exponent γ ,

[1]Unless otherwise stated, in this work S, R, D stand for source, relay, and destination respectively. Throughout the paper all vectors are taken to be row vectors. Also, vectors and matrices are denoted by bold letters and the i^{th} element by an italic letter. We use regular letters to denote scalars (including random variables). For a random variable x, we use $\mathbb{E}[x]$ to denote the expected value of x. The soft information corresponding to symbol a is represented by $\tilde{a}$. The notation $\text{sgn}(\cdot)$ indicates the sign of the variable in the bracket.

[2]Unless otherwise stated, in this work we assume $i \in \{S_1, S_2\}$ and $j \in \{1, 2, \ldots, N\}$.

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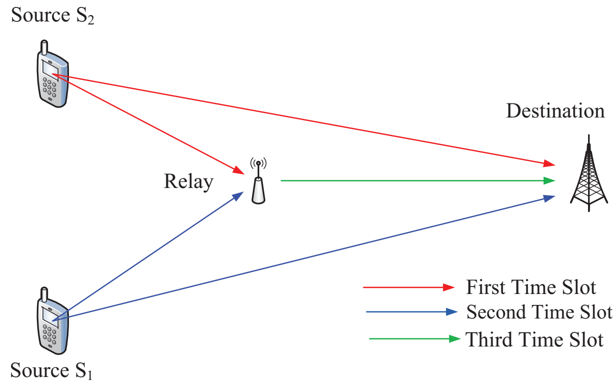

Figure 1. The proposed multiple access relay system in half-duplex mode.

i.e., $\lambda_{iR} = 1/(d_{iR})^{\gamma}$, $\lambda_{iD} = 1/(d_{iD})^{\gamma}$, and $\lambda_{RD} = 1/(d_{RD})^{\gamma}$ respectively. We consider quasi-static fading channels, i.e., the channel coefficients are constant during a frame of transmission, and change independently from one frame to another. In each time slot, the source bit $u_{i,j} \in \{0, 1\}$ is mapped to a BPSK symbol $x_{i,j} \in \{-1, 1\}$ via the mapping $0 \mapsto +1$, $0 \mapsto -1$. The received signals at the relay and destination corresponding to the source i in the jth time slot are given by $y_{iR,j} = \sqrt{P_i} h_{iR} x_{i,j} + n_{iR,j}$, $y_{iD,j} = \sqrt{P_i} h_{iD} x_{i,j} + n_{iD,j}$, where each vector $n_{iR,j}$ and $n_{iD,j}$ contains i.i.d. real Gaussian random variables, each having zero mean and same variance $\sigma^2 = N_0/2$ and P_i is the transmit power constraint from node i. Here we assume $P_1 = P_2 = 1$.

When the network coding operation is employed, in general, a relay can transmit the network coded symbols, i.e., XOR, between two bits is equivalent to the multiplication of the corresponding BPSK symbols, the network coded symbol $x_{R,j}$ can be obtained via $x_{R,j} = \hat{x}_{1,j}\hat{x}_{2,j}$, where $\hat{x}_{i,j}$ is the hard decision of $x_{i,j}$ at the relay, to the destination in order to achieve diversity (BPSK modulation being assumed for retransmission), at a power of P_R. In the regime of low source-relay SNR, $u_{i,j}$ can be detected incorrectly, and forwarding the hard decision can result in erroneous symbols being propagated to the destination. In our proposed MLT-SQ+PA scheme, the relay performs a soft decision on the received signals. It computes the appropriate soft information by computing each LLR $L_{i,j} = \ln\left(\frac{p(x_{i,j}=+1|y_{iR,j})}{p(x_{i,j}=-1|y_{iR,j})}\right) = \frac{2h_{iR}}{\sigma^2} y_{iR,j}$. The relay computes the LLR value $L_{R,j}$ for each network coded symbol $x_{R,j}$ after detection at the relay as follows:

$$L_{R,j} = 2\tanh^{-1}\left(\tanh\left(L_{S_1,j}/2\right)\tanh\left(L_{S_2,j}/2\right)\right) \quad (1)$$

In the third time slot, if the LLR value in (1) shows that the confidence regarding the decision is high, i.e., the absolute LLR value is larger than the lowest preset optimal threshold, then a corresponding soft quantized value is forwarded to the destination $\tilde{x}_{R,j} = f(L_{R,j})$, where the function $f(.)$ will be elucidated in Section III.

III. Multilevel Threshold based Soft Quantization Scheme

In this section, we will focus on the relay processing, i.e., MLT-SQ+PA and re-transmission. The LLR $L_{R,j}$ of the jth soft network coded bit as given by (1) is quantized according to

$$\tilde{x}_{R,j} = \begin{cases} \mathrm{sgn}(L_{R_j}) & |L_{R_j}| \geq |L_1| \\ \frac{2}{3}\mathrm{sgn}(L_{R_j}) & |L_2| \leq |L_{R_j}| < |L_1| \\ \frac{1}{3}\mathrm{sgn}(L_{R_j}) & |L_3| \leq |L_{R_j}| < |L_2| \\ 0 & |L_{R_j}| < |L_3|. \end{cases} \quad (2)$$

The key advantage of the proposed soft symbol mapping[3] proposed in [5] is that there is a finite number of possible transmit levels rather than an infinite number as would be produced by the $\tanh$ function mapping; this facilitates exact LLR computation at the destination and avoids heuristic approaches as used in the literature. The signal transmitted by the relay to the destination is received as $y_{RD,j} = \sqrt{P_R}\sqrt{\alpha} h_{RD}\tilde{x}_{R,j} + n_{RD,j}$, where $n_{RD,j}$ is a Gaussian noise with zero mean and variance $\sigma^2 = N_0/2$, $j = 1, 2......N$, P_R is the relay transmit power and it is set to be $P_R = 1$, here the factor α is chosen to satisfy the transmit power constraint at the relay and this we will explain later. The instantaneous channel state information (CSI) of h_{iD} and h_{RD} are assumed to be available at the relay during the course of optimization of quantization levels.

A. Derivation of Optimal Multilevel Thresholds

In this section, we present the optimal individual thresholds in order to minimize the BER at the destination as first presented in [5]. In general, most of reported approaches in the literature, use only a single positive threshold (3 levels) [6]-[7]. In this analysis, we consider three positive thresholds (7 levels) as in [5]. As presented in [5], $\varepsilon_{c,k}$ denoted by the event that the magnitude of bit LLR $|L_{R,j}|$ lies above the kth threshold and $\tilde{x}_{R,j}$ has the *correct* sign, i.e., $\mathrm{sgn}(\tilde{x}_{R,j}) = \mathrm{sgn}(x_{R,j})$ where we define $x_{R,j} = x_{1,j}x_{2,j}$. Similarly, we denote by $\varepsilon_{e,k}$ the event that the magnitude of bit LLR $|L_{R,j}|$ lies above the kth threshold and $\tilde{x}_{R,j}$ has the *incorrect* sign, i.e., $\mathrm{sgn}(\tilde{x}_{R,j}) \neq \mathrm{sgn}(x_{R,j})$. Finally, the event ε_s represents the event that $|L_{R,j}|$ is smaller than the smallest threshold L_3, i.e., the relay is silent. These events can be illustrated as follows:

$$\begin{array}{lccl} \varepsilon_{c,1} & : & |L_{R,j}| \geq |L_1|, & \mathrm{sgn}(\tilde{x}_{R,j}) = \mathrm{sgn}(x_{R,j}), \\ \varepsilon_{e,1} & : & |L_{R,j}| \geq |L_1|, & \mathrm{sgn}(\tilde{x}_{R,j}) \neq \mathrm{sgn}(x_{R,j}), \\ \varepsilon_{c,2} & : & |L_2| \leq |L_{R,j}| < |L_1|, & \mathrm{sgn}(\tilde{x}_{R,j}) = \mathrm{sgn}(x_{R,j}), \\ \varepsilon_{e,2} & : & |L_2| \leq |L_{R,j}| < |L_1|, & \mathrm{sgn}(\tilde{x}_{R,j}) \neq \mathrm{sgn}(x_{R,j}), \\ \varepsilon_{c,3} & : & |L_3| \leq |L_{R,j}| < |L_2| & \mathrm{sgn}(\tilde{x}_{R,j}) = \mathrm{sgn}(x_{R,j}), \\ \varepsilon_{e,3} & : & |L_3| \leq |L_{R,j}| < |L_2|, & \mathrm{sgn}(\tilde{x}_{R,j}) \neq \mathrm{sgn}(x_{R,j}), \\ \varepsilon_s & : & |L_{R,j}| < |L_3|. & \end{array} \quad (3)$$

The average BER at the destination $P_{error,i}$ for source i is expressed as $P_{error,i} =$

[3]Now we have 3 positive, 3 negative, and the zero level and it makes total of 7 symmetrical quantization levels. In each quantized symbol is spaced by $1/3$ to the next adjacent level.

$\sum_{k=1}^{3}\left[P_{i,k}^{(c)}\Pr(\varepsilon_{c,k})+P_{i,k}^{(e)}\Pr(\varepsilon_{e,k})\right]+P_i^{(s)}\Pr(\varepsilon_s)$, where $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$ respectively indicate the bit error rate at the destination: where the relay transmits the kth quantization level and this has the *correct* sign; where the relay transmits the kth quantization level and this has the *incorrect* sign; and where the relay stays silent. The average BER of two sources at the destination is represented by $P_{error} = \frac{1}{2}(P_{error,1}+P_{error,2})$. Consequently, we concentrate on determining $\{\Pr(\varepsilon_{c,k})\}$ and $\{\Pr(\varepsilon_{e,k})\}$ for $k=1,2,3$, and $\Pr(\varepsilon_s)$.

The set of optimal thresholds $\{L_k^*\}$ which minimize the overall BER at the destination as first presented[4] in [5] can be expressed as

$$\begin{aligned} L_1^* &= \ln\left[\frac{P_1^{(e)}-P_2^{(e)}}{P_2^{(c)}-P_1^{(c)}}\right], \quad L_2^* = \ln\left[\frac{P_2^{(e)}-P_3^{(e)}}{P_3^{(c)}-P_2^{(c)}}\right], \\ L_3^* &= \ln\left[\frac{P_3^{(e)}-P^{(s)}}{P^{(s)}-P_3^{(c)}}\right], \end{aligned} \tag{4}$$

where $P_k^{(c)}=\frac{1}{2}\left(P_{1,k}^{(c)}+P_{2,k}^{(c)}\right)$, $P_k^{(e)}=\frac{1}{2}\left(P_{1,k}^{(e)}+P_{2,k}^{(e)}\right)$, and $P^{(s)}=\frac{1}{2}\left(P_1^{(s)}+P_2^{(s)}\right)$. The probabilities $P_k^{(c)}$, $P_k^{(e)}$ and $P^{(s)}$, which appear in (4), are derived in the next section. We omit the proof here and is already presented in [5].

B. Error Probability Analysis

As presented in (4), the optimal thresholds L_k^* are based on $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$, which in turn are based on the channel realizations h_{1D}, h_{2D} and h_{RD}. We will compute these probabilities when perfect CSI (i.e., knowledge of h_{1D}, h_{2D} and h_{RD}) is available at the relay. The received LLR values at the destination corresponding to the source transmissions are $L_{iD,j} = \frac{2h_{iD}}{\sigma^2}y_{iD,j}$. The evaluation of LLR corresponding to the relay $L_{RD,k,j}$ is similar to [5] and omitted here for space consideration. The extrinsic LLR for bit $x_{i,j}$, determined from the network coding operation between [5] $x_{\bar{i},j}$ and $\tilde{x}_{R,j}$, is represented by $L_{iD,k,j}^E = 2\tanh^{-1}\left(\tanh\left(L_{\hat{i}D,j}/2\right)\tanh\left(L_{RD,k,j}/2\right)\right)$. The combined LLR at the destination, denoted by $L_{D,j}$, is computed as $L_{D,j}=L_{iD,j}+L_{iD,k,j}^E$. The received LLRs at the destination, i.e., $L_{iD,j}$, $L_{RD,j}$, and $L_{iD,k,j}^E$, given $h_{i,D}$ and h_{RD}, are approximately Gaussian distributed with their variances being twice the absolute value of their means [9]. The mean of $L_{iD,j}$ can be approximated as $m_{L_{iD}} = \mathbb{E}(L_{iD,j}) \triangleq \frac{2h_{iD}^2 x_{i,j}}{\sigma^2}$ and the mean value of $L_{RD,k,j}$ can be approximated as $m_{L_{RD,k}} = \mathbb{E}(L_{RD,k}|s_k) \triangleq \frac{2h_{RD}^2 s_k^2 \mathrm{sgn}(x_{R,j})}{\sigma_{RD}^2}$ where s_k is the soft quantization level as given in (3).

Then the mean value of the extrinsic LLR $L_{iD,k,j}^E$, denoted by $m_{L_{iD,k}^E}$, can be calculated by employing the function $\phi(z)$ where $z\in[0,\infty)$ introduced in [9]. We obtain the following mean value for $L_{iD,k,j}^E$ as

$$\begin{aligned} m_{L_{iD,k}^E} &= x_{i,j}\mathrm{sgn}\left(\tilde{x}_{R,j}\right)\phi^{-1}\Big(\phi\left(|m_{L_{iD,j}^E}|\right) \\ &+\phi\left(|m_{L_{RD,k}^E}|\right)-\phi\left(|m_{L_{iD}}|\right)\phi\left(|m_{L_{RD,k}}|\right)\Big). \end{aligned} \tag{5}$$

Due to space consideration we have curtailed the derivation of the following error probabilities and are appear in [7].

In case of $\varepsilon_{c,k}$, obtain the error probability of $x_{i,j}$ as

$$P_{i,k}^{(c)} = Q\left(\sqrt{\frac{|m_{L_{iD}}|+|m_{L_{iD,k}^E}|}{2}}\right). \tag{6}$$

In case of $\varepsilon_{e,k}$, the value of $P_{i,k}^{(e)}$ is then

$$\begin{cases} Q\left(\sqrt{\frac{(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)^2}{2(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)}}\right), & |m_{L_{iD}}|>|m_{L_{iD,k}^E}|, \\ 1-Q\left(\sqrt{\frac{(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)^2}{2(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)}}\right), & |m_{L_{iD}}|\le|m_{L_{iD,k}^E}|. \end{cases} \tag{7}$$

When the relay is silent, we have $P_{i,k}^{(s)} = Q\left(\sqrt{|m_{L_{x_iD}}|/2}\right)$. By use of (6),(7) and $P_{i,k}^{(e)}$ we can compute the optimum positive values of three-level thresholds.

IV. Power Allocation

In the following section, we consider a further enhancement to the propose MLT-SQ system. We introduce an optimised threshold based power allocation factor α_k. This caters for efficient power allocation of each threshold level k where $1<k<n$. We minimize the average BER $\mathbb{E}(P_{error})$, and obtain the optimal α_k with the constraint $0<\alpha_k\leqslant 1$, when the statistical CSI is available at the relay.

In the following, we will explain how the power allocation factor α is optimized to facilitate for each transmission. This outline feature makes the proposed MLT-SQ scheme more power efficient. Therefore, we refer this proposed system as MLT-SQ+PA. The analysis of power allocation is started by investigating the statistical CSI based error probabilities of the proposed system.

The statistical CSI based error probabilities can be computed by averaging the event probabilities $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$ over h_{iD}^2 and h_{RD}^2 respectively. The PDF of h_{iD}^2 and h_{RD}^2 can be written as $P_{h_{iD}^2}(h) = \frac{1}{\lambda_{iD}}\exp(-\frac{h}{\lambda_{iD}})$ and $P_{h_{RD}^2}(h) = \frac{1}{\lambda_{RD}}\exp(-\frac{h}{\lambda_{RD}})$ respectively. Alternatively, we can define $m_{L_{iD,k}^{(c)}} \triangleq |m_{L_{iD}}|+|m_{L_{iD,k}^E}|$, $m_{L_{iD,k}^{(e)}} \triangleq \frac{(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)^2}{(|m_{L_{iD}}|+|m_{L_{iD,k}^E}|)}$, and $m_{L_{iD}^{(s)}} \triangleq |m_{L_{iD}}|$. We average event probabilities $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$ over variables $m_{L_{iD,k}^{(c)}}$, $m_{L_{iD,k}^{(e)}}$ and $m_{L_{iD}^{(s)}}$. The PDF's of these three variables are given in the following lemma.

Lemma 1. *The PDF of $m_{L_{iD},k}^{(c)}$ can be expressed as*

$$p_{m_{L_{iD},k}^{(c)}}(m) = \frac{l_i\hat{l}_i}{l-\hat{l}_i}\left(\exp(-\widehat{l}_i m)-\exp\left(-l_i m\right)\right), \tag{8}$$

[4]The computational steps have been curtailed due to space consideration and are available in [5].

[5]The subscript $\bar{i}$ refers to the opposite source when source i is under consideration.

where $l_i = \frac{1}{\frac{2}{\sigma^2}\lambda_{iD}}$ *and* $\hat{l}_i = \frac{1}{\frac{2}{\sigma^2}\lambda_{\bar{i}D}} + \frac{1}{\frac{2s_k^2\alpha_k}{\sigma^2}\lambda_{RD}}$*. The PDF of* $m^{(e)}_{L_{iD},k}$ *can be represented as*

$$p_{L^{(e)}_{iD,k}}(m) = \frac{l_i^2\hat{l}_i^2}{2(l_i^2-\hat{l}_i^2)}\int_0^\infty \frac{u^2}{m}(\exp(-\hat{l}_i u^2 - l_i\sqrt{mu} - \exp(-l_i u^2 - l_i\sqrt{mu}))du, \tag{9}$$

Finally, the PDF of $m^{(s)}_{L_{iD},k}$ *can be expressed as* $p_{L^{(s)}_{iD,k}}(m) = l_i \exp(-l_i m)$.

Proof: The proof of this lemma is given in Appendix A. ■

On the basis of the Lemma 1, we can analyse the statical CSI based error probabilities, i.e. the expectations of $P^{(c)}_{i,k}$, $P^{(e)}_{i,k}$, and $P^{(s)}_i$. The expectation of $P^{(c)}_{i,k}$ is calculated as

$$\begin{aligned}\mathbb{E}(P^{(c)}_{i,k}) &= \frac{l_i}{l_i-\hat{l}_i}\int_0^\infty Q\left(\sqrt{m/2}\right)\hat{l}_i\exp\left(-\hat{l}_i m\right)dm \\ &-\frac{\hat{l}}{l_i-l_i}\int_0^\infty Q\left(\sqrt{m/2}\right)l_i\exp(-l_i m)\,dm \\ &= \frac{l_i}{2\left(l_i-\hat{l}_i\right)}\left(1-\sqrt{\frac{1}{1+4\hat{l}_i}}\right) - \frac{\hat{l}_i}{2\left(l_i-\hat{l}_i\right)}\left(1-\sqrt{\frac{1}{1+4l_i}}\right).\end{aligned} \tag{10}$$

The calculation of the expectation of $P^{(e)}_{i,k}$ is rather involved. We start the computation by calculating the probability $\Pr\left(|m_{L_{iD}}| > |m_{L^E_{iD}}|\right)$. By use of (**??**) in Appendix 2, first, we can start the computation of $\Pr\left(|m_{L_{iD}}| - |m_{L^E_{iD}}|\right)$ as follows

$$p(|m_{L_{iD}}| - |m_{L^E_{iD,k}}|) = \frac{l_i\hat{l}_i}{l_i+\hat{l}_i}\exp\left(-l_i(|m_{L_{iD}}| - |m_{L^E_{iD,k}}|)\right) \tag{11}$$

Then, in order to compute $p(|m_{L_{iD}}| - |m_{L^E_{iD,k}}| > 0)$, we can integrate (11) from 0 to ∞ and then we have

$$\Pr\left(|m_{L_{iD}}| > |m_{L^E_{iD,k}}|\right) = \frac{\hat{l}_i}{l_i - \hat{l}_i}. \tag{12}$$

Now we can obtain the expectation of $\mathbb{E}(P_{i,e,k})$ as shown in [6]

$$\begin{aligned}\mathbb{E}(P^{(e)}_{i,k}) &= \int_0^\infty Q\left(\frac{\sqrt{m}}{2}\right)p_{m_{L_{iD,e}}}(m)dm \\ &\Pr\left(|m_{L_{iD}}| > |m_{L^E_{iD,k}}|\right) + \left(1-\int_0^\infty Q\left(\frac{\sqrt{m}}{2}\right)\right. \\ &\left. p_{m_{L_{iD,e}}}(m)dm\right)\Pr\left(|m_{L_{iD}}| \le |m_{L^E_{iD,k}}|\right) \\ &= \frac{l_i}{l_i+\hat{l}_i} - \frac{l-\hat{l}_i}{l_i+\hat{l}_i}\int_0^\infty Q\left(\frac{\sqrt{m}}{2}\right)p_{m_{L_{iD,e}}}(m)dm \\ &= \frac{l_i}{l_i+\hat{l}_i} - \frac{l^2\hat{l}_i^2}{\left(l_i+\hat{l}_i\right)^2}\left(\psi\left(\frac{1}{4},\hat{l}_i,l_i\right) - \psi\left(\frac{1}{4},l_i,\hat{l}_i\right)\right),\end{aligned} \tag{13}$$

where the function $\psi(.)$ is illustrated in Equation (A-4) of [6]. The expectation of $\mathbb{E}(P^{(s)}_i)$ is calculated as

$$\begin{aligned}\mathbb{E}(P^{(s)}_i) &= \int_0^\infty Q\left(\sqrt{m/2}\right)\hat{l}_i\exp(-l_i m)\,dm, \\ &= \frac{1}{2}\left(1-\sqrt{\frac{1}{1+4l_i}}\right).\end{aligned} \tag{14}$$

Now we can further simplify the expected values of the event probabilities (10), (13) and (14) when the statistical CSI available at the relay. In order to simplify these terms, we make the following approximations when the SNR $\rho = 1/\sigma^2$ is large enough. By utilizing the property of the function $Q(.)$, we can simplify Eq. (10) to be

$$\begin{aligned}\mathbb{E}(P^{(c)}_{i,k}) &\approx \frac{3\left(s_k^2\lambda_{RD}\alpha_k + \lambda_{\bar{i}D}\right)}{4\rho^2\lambda_{iD}s_k^2\alpha_k\lambda_{RD}\lambda_{\bar{i}D}}, \\ \mathbb{E}(P^{(s)}_i) &= \frac{1}{2\lambda_{iD}\rho}.\end{aligned} \tag{15}$$

As given in [8], by making use of the property of $Q(.)$ function, it is easy to obtain a good approximation for $\mathbb{E}(P^{(e)}_{i,k})$ from (13) as follows

$$\mathbb{E}(P^{(e)}_{i,k}) = \frac{l_i}{l_i+\hat{l}_i} = \frac{\alpha_k s_k^2 l_{\bar{i}D} l_{RD}}{\alpha_k s_k^2 l_{\bar{i}D} l_{RD} + \alpha_k s_k^2 l_{iD} l_{RD} + l_{\bar{i}D} l_{iD}}. \tag{16}$$

In the following theorem, we investigate the optimization of power allocation factor α_k at the relay to enhance the error performance at the destination.

Theorem 1. *The optimized power allocation factor for the proposed network coded MLT-SQ+PA scheme is given by the following generic form*

$$\alpha_k = \begin{cases} 1, & \alpha_{k,1} \le 0, or\ \alpha_{k,1} > 1, \\ \alpha_{k,1}, & 0 < \alpha_{k,1} \le 1, \end{cases} \tag{17}$$

where
$$\alpha_{k,1} = \frac{L_{1D}L_{2D}/s_k^2}{\sqrt{\frac{4\left(\Pr(\varepsilon_{e,k})\right)}{3\left(\Pr(\varepsilon_{c,k})\right)}\rho L_{1D}L_{2D}L_{RD} - L_{1D}L_{RD} - L_{2D}L_{RD}}}.$$

Proof: The proof of this theorem is given in Appendix B. ■

As explained in (17), when the source-relay channels are good, we have $\alpha_{k,1} < 0$ or $\alpha_{k,1} > 0$and therefore the relay should transmit with its full power, i.e., $\alpha_k = 1$. When the source-relay channels are poor, i.e., $0 < \alpha_{k,1} \le 1$, the relay should transmit with less power, i.e., $\alpha_k = \alpha_{k,1}$.

V. Simulation results and Discussion

In the simulations, first, we use uncoded BPSK signalling with a frame length of $N = 10,000$. All channels are assumed to exhibit quasi-static fading, i.e., the channel coefficients h_{1D}, h_{2D}, h_{RD}, h_{1R} and h_{2R} are constant for each transmission phase and change independently from one phase to the next. Next, we provide error rate performance for the LDPC coded MLT-SQ+PA system together with other competing schemes. Here, the LDPC codes at both of the sources have a fixed code rate of $1/2$ and the same degree profile. The relevant dimensions of the parity-check matrix are $N = 816$ and $K = 408$.

Here we consider the case of a symmetric MARC system, where the relay is located midway between the two sources and the destination, i.e., $d_{S_1R} = d_{S_2R} = d_{RD} = 0.5$. The distances between the sources and the destination are

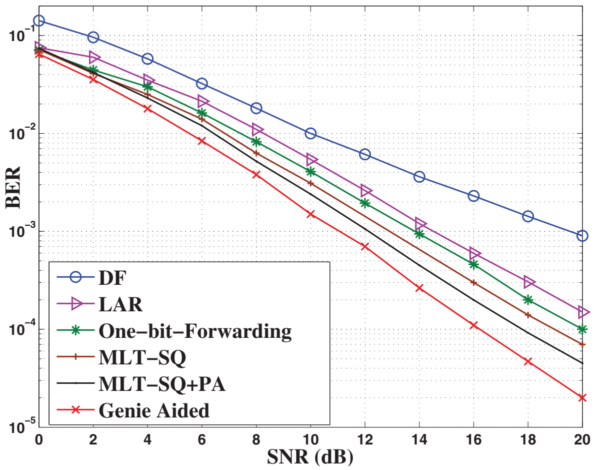

Figure 2. Error rate performance of the proposed MLT-SQ+PA scheme, together with that of competing schemes, in a quasi-static fading environment.

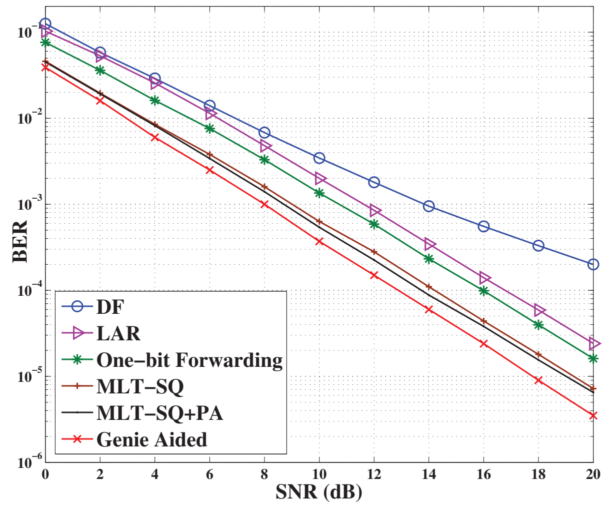

Figure 3. Error rate performance of the proposed LDPC coded MLT-SQ+PA scheme, together with that of competing schemes, in a quasi-static fading environment.

normalized to unity, i.e., $d_{S_1D} = d_{S_2D} = 1$. The attenuation exponent was chosen to be $\gamma = 2$.

All the schemes presented in Fig. 3 are LDPC coded schemes and Fig. 2 are uncoded. In this work, we assume three positive threshold levels, i.e., L_1^*, L_2^*, L_3^* (this will form seven threshold levels by symmetry including zero level). Note that we have adopted LDPC coded DF and uncoded DF in Fig. 2 and Fig. 3 respectively. As shown in the simulation, it has the worst error performance as compared to other schemes and it does not achieve the full diversity order of 2.

As lower bound for BER, we assume that, for any given frame, the relay transmission is error free[6]. This assumption gives us a lower bound for the proposed scheme and is known as the 'Genie Aided' protocol. We have also demonstrated the simulation result for link-adaptive regeneration (LAR) in [7], a power scaling scheme and the relay located power scalar w is given by $w = \min\left(\frac{\gamma_{SR,min}}{\gamma_{RD}}, 1\right)$ where γ_{SR} and γ_{RD} are the channel SNR of $S-R$ and $R-D$ links respectively. We have adopted it according to the proposed MARS with physical layer network coded system as $w = \min\left(\frac{\min\left(h_{S_1R}^2, h_{S_2R}^2\right)}{h_{RD}^2}, 1\right)$. This scheme adapts the power for each transmitted block according to the channel information. Also note that the LAR protocol has a lower computational complexity than our proposed scheme MLT-SQ+PA.

We have also presented the simulation results for one-bit forwarding (MLT-SQ with a single positive level). In this technique, we have three threshold levels. If the amplitude of the relay received reliability $|L_{x_{R,j}}|$ is higher than the threshold value L_T^*, then the relay transmits the symbol $\mathrm{sgn}(L_{x_{R,j}})$, i.e., $+1$ or -1; otherwise it stays silent (this forms three threshold levels). As shown in the simulation, it has improved BER performance over uncoded DF and LAR. As can be seen from the simulation results, the proposed MLT-SQ scheme has superior BER performance over the three-level forwarding case (one-bit forwarding). It also shows in the simulation results, that the proposed MLT-SQ scheme displays better performance over the uncoded DF, one-bit forwarding and LAR. It also noted that the proposed MLT-SQ+PA scheme has better BER performance over the MLT-SQ. The proposed MLT-SQ+PA scheme yields considerable gains in the regime of low source-relay SNR, since here erroneous detection at the relay results in the forwarding of incorrect symbols to the destination.

[6]Note that we do not plot the analytical results here.

VI. Conclusion

We developed a novel optimized power allocation factor for soft quantization scheme (MLT-SQ+PA) based on cooperative network coding in a multiple access relay system. We present the BER analysis of the proposed MLT-SQ+PA scheme and used this to optimize power allocation factor at the destination in order to minimize the BER at the destination. The simulation results confirm the optimality of the proposed MLT-SQ+PA scheme over other relevant competing schemes.

Appendix A
Proof of Lemma 1

Let consider $a = \frac{2h_{iD}^2}{\sigma^2}$, $b = \frac{2h_{\bar{i}D}^2}{\sigma^2}$, and $c = \frac{2\alpha_k s_k^2 h_{RD}^2}{\sigma^2}$. The PDFs of these three variables are as follows: $p_a(a) = \frac{1}{2h_{iD}^2/\sigma^2}\exp\left(-\frac{a}{\frac{2}{\sigma^2}l_{iD}}\right)$, $p_b(b) = \frac{1}{2h_{\bar{i}D}^2/\sigma^2}\exp\left(-\frac{b}{\frac{2}{\sigma^2}l_{\bar{i}D}}\right)$, and $p_c(c) = \frac{1}{2h_{RD}^2/\sigma^2}\exp\left(-\frac{c}{\frac{2\alpha_k s_k^2}{\sigma^2}l_{RD}}\right)$. As illustrated in [10], if b and c are independent exponential random variables, the PDF of $h = \min(b, c)$ is given by $f_h(h) = f_b(h) + f_c(h) - f_b(h)F_c(h) - F_b(h)f_c(h)$, where $f_b(h) = f_c(h) = \upsilon\exp(-\upsilon h)$, and $F_b(h) = F_c(h) = 1 - \exp(-\upsilon h)$ where υ. Then, we obtain the PDF $p_h(h)$ as follows $p_h(h) = \hat{l}_i\exp(-\hat{l}_i h)$ where $\hat{l}_i = \left(\frac{1}{\frac{2}{\sigma^2}l_{\bar{i}D}} + \frac{1}{\frac{2\alpha_k s_k^2}{\sigma^2}l_{iD}}\right)$.

To simplify the notations, we use $l_i = \frac{1}{\frac{2}{\sigma^2} l_{iD}}$. It is straightforward to obtain the PDF of $m_{L_{iD},s}$ by viewing a as $m_{L_{iD},s}$. Now we derive PDF of exponential random variable $w = a + h$. As given in [10], we have $p_w(w) = \frac{l_i \hat{l}_i}{l_i - \hat{l}_i} \left(\exp\left(-\hat{l}_i w\right) - \exp\left(-l_i w\right) \right)$. Then, we focus on the derivation of the PDF of $z = \frac{(a-h)^2}{a+h}$. To obtain the PDF of z, we start the derivation by obtaining the PDF $t = a - h$, and we have $p_t(t) = \frac{l_i \hat{l}_i}{l_i + \hat{l}_i} \exp\left(-l_i t\right)$. Now we express z in terms of t as $z = t^2/w$. From [10], [6], we can now express the PDF of z as follows $p_t(t) = \frac{\left(l_i \hat{l}_i\right)^2}{l_i + \hat{l}_i} \int_0^\infty \left(\frac{u^2}{\sqrt{t}}\right) \left(\exp\left(-\hat{l}_i u^2 - l_i \sqrt{t} u\right) - \exp\left(-l_i u^2 - \hat{l}_i \sqrt{t} u\right)\right) du.$ Now, we can obtain the PDFs of the variables $m_{L_{iD},c}$ and $m_{L_{iD},e}$, by viewing the variable w as $m_{L_{iD},c}$ and t as $m_{L_{iD},e}$. This completes the proof.

Appendix B
Proof of Theorem 2

We start the analysis by averaging the P_{error} over h_{1D}, h_{2D} and h_{RD} and we have: $\mathbb{E}(P_{error}) = \sum_{k=1}^{n} \mathbb{E}(P_k^{(c)}) \Pr(\varepsilon_{c,k}) + \sum_{k=1}^{n} \mathbb{E}(P_k^{(e)}) \Pr(\varepsilon_{(e,k)}) + \mathbb{E}(P^{(s)}) \Pr(\varepsilon_s)$. where $P_k^{(c)} = \frac{1}{2}\left(P_{1,k}^{(c)} + P_{2,k}^{(c)}\right)$, $P_k^{(e)} = \frac{1}{2}\left(P_{1,k}^{(e)} + P_{2,k}^{(e)}\right)$, and $P^{(s)} = \frac{1}{2}\left(P_1^{(s)} + P_2^{(s)}\right)$. We obtain the optimal power allocation factor α_k by minimizing the average $\mathbb{E}(P_{error}) = \frac{1}{2}\left(\mathbb{E}(P_{error,1} + \mathbb{E}(P_{error,2})\right.$; solving the $\frac{\partial \mathbb{E}(P_{error})}{\partial \alpha_k} = 0$ under the constraint $0 < \alpha_{k,1} < 1$. Since the $\Pr(\epsilon_{c,k})$, $\Pr(\epsilon_{e,k})$ and $\Pr(\varepsilon_s)$ are constant with respect to α_k, the derivation of the optimal power allocation factor α_k can be simplified. This allows us to obtain the following close form expression for the optimal α_k.

In the sequel, we optimize the α_k by taking into account the power constraint $0 < \alpha_k \leq 1$. With the help of the approximation provided in (15) and (16), we derive the optimal power allocation factor α_k as follows. At the first step let the $\frac{\partial \mathbb{E}(P_{error})}{\partial \alpha_k} = 0$, and then we have

$$\frac{4 \Pr(\varepsilon_{e,k}) \left(\rho \alpha_k s_k^2 L_{1D} L_{2D} L_{RD}\right)^2}{3 \Pr(\varepsilon_{c,k})} = \left(\alpha_k s_k^2 (L_{1D} L_{RD} + L_{2D} L_{RD}) + L_{1D} L_{2D}\right)^2 \tag{18}$$

where $\Pr(\varepsilon_{c,k}) = \frac{1}{2}(\Pr(\varepsilon_{c,k,1}) + \Pr(\varepsilon_{c,k,2}))$ and $\Pr(\varepsilon_{e,k}) = \frac{1}{2}(\Pr(\varepsilon_{e,k,1}) + \Pr(\varepsilon_{e,k,2}))$. This leads to two solutions for α_k as given below

$$\begin{aligned} \alpha_{k,1} &= \frac{L_{1D} L_{2D}/s_k^2}{\sqrt{\frac{4 \Pr(\varepsilon_{e,k})}{3 \Pr(\varepsilon_{c,k})}} \rho L_{1D} L_{2D} L_{RD} - L_{1D} L_{RD} - L_{2D} L_{RD}} \\ \alpha_{k,2} &= \frac{-L_{1D} L_{2D}/s_k^2}{\sqrt{\frac{4 \Pr(\varepsilon_{e,k})}{3 \Pr(\varepsilon_{c,k})}} \rho L_{1D} L_{2D} L_{RD} - L_{1D} L_{RD} - L_{2D} L_{RD}} \end{aligned} \tag{19}$$

Depending on the $\Pr(\epsilon_{c,k})$, $\Pr(\epsilon_{e,k})$, channel gain of $R - D$ link and ρ, the optimal power allocators $\alpha_{k,1}$ and $\alpha_{k,2}$ can be positive or negative. Next, we investigate the optimal power allocation α_k while taking into account the following two cases $\alpha_{k,1} \leq 0$ or $\alpha_{k,1} > 0$ and $\alpha_{k,2} > 0$. In case 1, i.e., $\alpha_{k,1} \leq 0$, we first relax the power constraint i.e., $0 < \alpha_{k,1} < 1$, and have the following two factors (i) $\mathbb{E}(P_{e,k})$ is a monotonically increasing function (MIF) of α_k when $\alpha_{k,2} < \alpha_k < \alpha_{k,1}$ (ii) $\mathbb{E}(P_{i,e,k})$ is a monotonically decreasing function (MDF) of α_k if $\alpha_k < \alpha_{k,2}$ or $\alpha_k > \alpha_{k,1}$. Then we consider the optimal α_k with the constraint $0 < \alpha_k < 1$. With this constraint, $\mathbb{E}(P_{e,k})$ is an MDF of α_k. Therefore, the optimal power allocation is obtained as $\alpha_k = 1$ when $\alpha_{k,1} \leq 0$.

In case 2, i.e., $\alpha_{k,1} > 0$, we first relax the power constraint, i.e., $0 < \alpha_{k,1} \leq 1$ and have the following two facts (i) $\mathbb{E}(P_{e,k})$ is a MIF of α_k when $\alpha_{k,2} \geq \alpha_k$ or $\alpha_k > \alpha_{k,1}$ (ii) $\mathbb{E}(P_{i,e,k})$ is a MDF of α_k if $\alpha_{k,1} \geq \alpha_k > \alpha_{k,2}$. Then we optimize α_k with the constraint $0 < \alpha_k \leq 1$. We can also see that when $\alpha_{k,1} > 1$ the expectation of error probability $\mathbb{E}(P_{e,k})$ is an MDF of α_k in the constraint of $0 < \alpha_{k,1} \leq 1$. Therefore the optimal power allocation can be obtained as $\alpha_k = 1$ when $\alpha_{k,1} > 1$. When the $\alpha_{k,1}$ is in the range of $0 < \alpha_{k,1} \leq 1$, $\mathbb{E}(P_{e,k})$ is an MDF of α_k when $0 < \alpha_k \leq \alpha_{k,1}$, and $\mathbb{E}(P_{i,e,k})$ is a MIF of α_k when $1 \geq \alpha_k > \alpha_{k,1}$. Thus, we can deduce the optimal power allocation as $\alpha_k = \alpha_{k,1}$ when $0 < \alpha_{k,1} \leq 1$.

References

[1] J. Laneman, D. Tse, and G. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[2] Y. Li, B. Vucetic, T. F. Wong, and M. Dohler, "Distributed turbo coding with soft information relaying in multihop relay networks," *IEEE Journal on Sel. Areas in Comm.*, vol. 24, pp. 2040–2050, Nov. 2006.

[3] Dushantha N. K. Jayakody, M. F. Flanagan, "LDPC Coding with Soft Information Relaying in Cooperative Wireless Networks," *IEEE Wireless Communications and Networking Conference (WCNC 2013)*, Shanghai, China, pp. 4317-4322, April 2013.

[4] Dushantha N. K. Jayakody and M. F. Flanagan. "A Soft Decode-Compress-Forward Relaying Scheme for Cooperative Wireless Networks". *24th IEEE Annual Symposium on Personal, Indoor and Mobile Radio Communications Conference, (PIMRC 2013)* -WDN, London, UK, September 2013.

[5] Dushantha N. K. Jayakody, Jun Li and M. F. Flanagan. "A Multilevel Soft Quantize-and-Forward Scheme for Multiple Access Relay Systems". *25th IEEE Annual Symposium on Personal, Indoor and Mobile Radio Communications Conference, (PIMRC 2014)*, Washington DC, USA, September 2014.

[6] H. V. Khuong and H. Y. Kong, "LLR-based decode-and-forward protocol for relay networks and closed-form BER expressions," *IEICE Trans. Fundamentals*, vol. E89A, no. 6, pp. 1832–1841, Jun. 2006.

[7] W. Guan, K. J. R. Liu, "Mitigating error propagation for wireless network coding," *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3632–3643, Oct. 2012.

[8] M. K. Simon and M.-S. Alouini, "A unified approach to the performance analysis of digital communication over generalized fading channels," *Proceedings of the IEEE*, vol. 86, no. 9, pp. 1860–1877, Sep. 1998.

[9] S.-Y. Chung, T. J. Richardson, and R. L. Urbanke, "Analysis of sumproduct decoding of low-density parity-check codes using a Gaussian approximation," *IEEE Trans. Inform. Theory*, vol. 47, no. 2, pp. 657– 670, Feb. 2001.

[10] A. Papoulis and S. U. Pillai, Probability, *Random Variables and Stochastic Process*. 4th ed. McGraw Hill, 2002.

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