Soft Information Forwarding Design for a Two-Way Relaying Channel

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***Abstract*—In this paper we investigate novel soft mutual information forwarding (MIF) protocols in a two-way relay channel (TWRC), where two sources exchange information with the help of an intermediate relay. Based on the estimated signals from the two sources, the relay calculates the soft mutual information, and then broadcasts it to the two sources. In specific, we propose two MIF protocols, namely, network coded MIF (NC-MIF) and superposition coded MIF (SC-MIF), suitable to different channel conditions. The expressions derived for the received signal-to-noise ratio (SNR) at the sources reveal that if both source-to-relay channels are in good conditions, the NC-MIF outperforms the SC-MIF. Otherwise, the SC-MIF is superior to the NC-MIF. For the TWRC with varying channels, we further develop an adaptive scheme, which enables the dynamic switch between the two protocols, depending on the received SNR at the sources. Furthermore, the threshold that determines the switch of the protocols is developed as a close-form expression. Simulation results show that our adaptive scheme outperforms all the existing relaying protocols in the fading channels.**

I. INTRODUCTION

Two classical relay protocols, namely amplify-and-forward (AF) and decode-and-forward (DF), have been widely studied in wireless relaying systems. However, the AF protocol suffers from noise amplification and the DF protocol propagates the erroneous decisions to the destination. Consequently, a new relaying concept based on soft information forwarding (SIF) technologies has been proposed to achieve a better error performance [1]. The SIF based protocols assist the destination to make hard decisions by forwarding the intermediate soft decisions [1–7].

Among the SIF schemes developed for one-way relay channels, a SIF based protocol, namely estimate-and-forward (EF), has been proposed in [5]. It is shown in [5] that the EF protocol maximizes the signal-to-noise ratio (SNR) at the destination and thus gains a better error performance relative to the AF and DF protocols in the one-way single relay channel. Recently, a novel SIF based protocol based on forwarding symbol-wise mutual information at the relay (henceforth referred to as mutual information based forwarding (MIF)) has been proposed [6, 7]. It is shown in [6] that the MIF protocol achieves a better error performance compared with the EF protocol. On the other hand, two-way relay channels (TWRC) have recently attracted a lot of attention [8–14] due to its promising application to contemporary wireless systems. In the TWRC systems, two sources exchange their information via a shared relay. In particular, a network coded EF protocol (refer to as "SBF" protocol) is proposed in [12], where a minimum mean square error (MMSE) estimation of the network coded binary symbols, i.e., the XOR-ed binary symbols from the two sources, are broadcast to the two sources by the relay.

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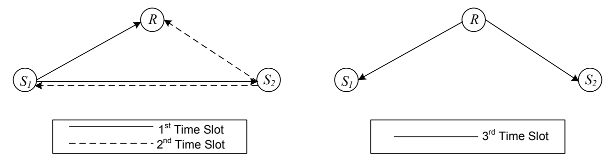

Fig. 1. A two-way relay network.

To the best of our knowledge, no MIF based SIF protocol has been investigated for the TWRC. In this paper, we aim to develop novel SIF protocols for a TWRC under fading conditions. We first review the applications of the conventional AF, DF, and EF protocols in the TWRC. Then we propose two MIF protocols, namely, network coded MIF (NC-MIF) protocol and superposition coded MIF (SC-MIF) protocol. In the NC-MIF, the soft mutual information forwarded by the relay is obtained based on the XOR-ed binary streams from the two sources. In the SC-MIF, the relay first calculates the soft mutual information for each source and then forwards their addition. Next, we derive the received SNR of the two MIF protocols. We have found that if both source-to-relay channels are in good conditions, then the NC-MIF has a better error performance than the SC-MIF. On the other hand, if one or both of the source-to-relay channels are poor, then the SC-MIF outperforms the NC-MIF. Based on these observations, we propose an adaptive scheme that dynamically switch between the SC-MIF and NC-MIF protocols based on the varying channel conditions. In the simulations, AF, DF, and EF protocols are utilized as benchmarks. Simulations show that the proposed two MIF protocols outperform the three conventional protocols. Also, the adaptive scheme between the two MIF protocols can achieve a better performance than either the NC-MIF or the SC-MIF protocol.

II. System Model and Preliminaries

We consider a TWRC as shown in Fig. 1, where the two sources S_1 and S_2 exchange their information with the help of the relay R. It is assumed that all the three nodes operate in a half-duplex mode, i.e., each node transmits and receives in a different time slot. A transmission period is composed of three time slots. In the first time slot S_1 broadcasts its signals to R and S_2, in the second time slot S_2 broadcasts its signals to R and S_1, and in the third time slot R broadcasts the processed signals to both sources to facilitate the information exchange.

In the first and the second time slots, the received signals at the relay and the sources can be expressed as $r_{S_iR} = \sqrt{E_S}h_{S_iR}x_{S_i} + n_{S_iR}$ and $r_{S_iS_j} = \sqrt{E_S}h_{S_iS_j}x_{S_i} + n_{S_iS_j}$, respectively, where E_S is the transmission power at each source, x_{S_i} is the power-normalized symbol transmitted by the source S_i, $i \in \{1, 2\}$, r_{S_iR} is the received signal at the relay R from S_i, $r_{S_iS_j}$ is the received signal at S_j from S_i, $j \in \{1, 2\}, i \neq j$, h_{S_iR} is the channel coefficient between S_i and R, $h_{S_iS_j}$ is the channel coefficient between S_i and S_j, and n_{S_iR} and $n_{S_iS_j}$ are the noise samples at R and S_j, respectively. In the third time slot, the received signal at S_i from R, defined by r_{RS_i}, can be written as $r_{RS_i} = \sqrt{E_R}h_{RS_i}f(r_{S_1R}, r_{S_2R}) + n_{RS_i}$, where E_R is the transmission power at the relay, h_{RS_i} is the channel coefficient between R and S_i, and n_{RS_i} is the noise sample at S_i. In the previous equation, we denote by $f(\cdot)$ the power-normalized relay function based on the received signals r_{S_1R} and r_{S_2R}. Thus, $\sqrt{E_R}f(r_{S_1R}, r_{S_2R})$ is the transmitted signal at R.

We assume that the binary signals 0 and 1 are generated at the sources with equal probability and mapped into BPSK symbols, i.e., $x_{S_i} \in \{\pm 1\}$. The noises at S_i and R have a zero mean and variances σ_S^2 and σ_R^2, respectively. Next, we briefly review that how conventional AF, DF, and EF protocols can be extended to the TWRC. When the AF protocol is applied to the TWRC, the relay superimposes the received signals from the two sources and broadcasts the superposition to the two sources. The relay function is $f_{AF}(r_{S_1R}, r_{S_2R}) = \sqrt{\frac{1}{2(E_S+\sigma_R^2)}}(r_{S_1R} + r_{S_2R})$. As S_i knows its own information, x_{S_i} can be perfectly canceled from r_{RS_i}. After the cancelation operation, S_i combines the result with the received signal $r_{S_jS_i}$ to make a decision on x_{S_j}. In a TWRC with the DF protocol, the relay performs hard decisions on the symbols x_{S_1} and x_{S_2}, and then calculates the network coded symbol based on the hard decisions $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$. We define the network coded symbol as $x_R \triangleq x_{S_1}x_{S_2}$. The hard decision of x_R is then calculated as $\hat{x}_R = \hat{x}_{S_1}\hat{x}_{S_2}$. The relay function can be expressed as $f_{DF}(r_{S_1R}, r_{S_2R}) = \hat{x}_R$. At S_i, r_{RS_i} is multiplied with x_{S_i} to cancel x_{S_i} from r_{RS_i} and $r_{RS_i}x_{S_i}$ will be combined with $r_{S_jS_i}$ to make a hard decision.

We now focus on the EF protocol. According to [12], instead of forwarding $\hat{x}_R$ as in the DF protocol, the relay forwards the MMSE estimation of x_R, i.e., the expectation of $x_{S_1}x_{S_2}$, $\mathcal{E}[x_{S_1}x_{S_2}|r_{S_1R}, r_{S_2R}]$, which is calculated as $\mathcal{E}[x_{S_1}x_{S_2}|r_{S_1R}, r_{S_2R}] = \mathcal{E}[x_{S_1}|r_{S_1R}]\mathcal{E}[x_{S_2}|r_{S_2R}] = \tanh\left(L_{x_{S_1},R}/2\right)\tanh\left(L_{x_{S_2},R}/2\right)$, where $L_{x_{S_i},R} = \ln\frac{p(r_{S_iR}|x_{S_i}=1,h_{S_iR})}{p(r_{S_iR}|x_{S_i}=-1,h_{S_iR})}$ is the log-likelihood ratio (LLR) of the received symbol x_{S_i} at the relay, and $\mathcal{E}[x_{S_i}|r_{S_iR}] = \tanh(L_{x_{S_i},R}/2)$ is the MMSE estimation of x_{S_i}. The received signal r_{RS_i} at S_i from the relay is multiplied with x_{S_i} so as to cancel x_{S_i} from r_{RS_i}, which is then combined with $r_{S_jS_i}$ to make a hard decision on x_{S_j}.

III. Two Novel MIF Protocols

In the MIF protocol proposed in [6] for the one-way relay channels, the relay function is the product of two terms: the first term is the sign (i.e., hard decision) of the received signal at the relay, and the second term is the symbol-wise mutual information (SMI) of the source-to-relay channel between a source symbol x_{S_i} and its corresponding LLR L_i at the relay conditioned on $\lambda_i \triangleq |L_i|$. According to [6, 7], the SMI can be calculated as

$$\begin{aligned}\Theta(\lambda_i) &\triangleq I(x_{S_i}; L_i|\lambda_i)\\ &= \frac{1}{1+e^{\lambda_i}}\log_2\frac{2}{1+e^{\lambda_i}} + \frac{1}{1+e^{-\lambda_i}}\log_2\frac{2}{1+e^{-\lambda_i}}.\end{aligned} \tag{1}$$

We will propose two MIF protocols in the TWRC, namely, NC-MIF and SC-MIF.

A. NC-MIF Protocol

When network coding is applied at a relay R, the relay function can be generally written as $f(r_{S_1R}, r_{S_2R}) = \hat{x}_{S_1}\hat{x}_{S_2}|f(r_{S_1R}, r_{S_2R})|$, where $\hat{x}_{S_i}$ is the hard decision of the symbol x_{S_i} at R. The network coded relay function can be seen as the hard decision of the network coded symbol $\hat{x}_R = \hat{x}_{S_1}\hat{x}_{S_2}$, multiplied with a reliability measurement $|f(r_{S_1R}, r_{S_2R})|$. In the NC-MIF protocol, $|f(r_{S_1R}, r_{S_2R})|$ is calculated based on the SMI of the two source-to-relay channels. Consider the values of $|f(r_{S_1R}, r_{S_2R})|$ in the following three extreme cases: a) When λ_1 and λ_2 go to infinity, we have $|f(r_{S_1R}, r_{S_2R})| = 1$; b) When λ_1 and λ_2 approach zero, we have $|f(r_{S_1R}, r_{S_2R})| = 0$; c) When λ_i goes to infinity and λ_j approaches zero, we have $|f(r_{S_1R}, r_{S_2R})| = 0$. Based on the results of the three cases, we can see that only if both $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ are reliable, then $\hat{x}_R$ is reliable. If any one of $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ is not reliable, then $\hat{x}_R$ is not reliable. That is to say, the reliability measurement of $\hat{x}_R$ should depend on the reliability measurement of both $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$.

Based on the above discussions, we can calculate $\hat{x}_R$'s reliability measurement as $\Theta_1(\lambda_1, \lambda_2) = \Theta(\lambda_1)\Theta(\lambda_2)$, where $\Theta(\lambda_i)$ is the reliability measurement for $\hat{x}_{S_i}$. We use the SMI of $\hat{x}_{S_i}$ as reliability measurement. Thus we have $\Theta(\lambda_i) = I(x_{S_i}; L_{x_{S_i},R}|\lambda_i)$. The relay function is thus expressed as $f_{NC-MIF}(r_{S_1R}, r_{S_2R}) = \sqrt{\frac{1}{\mathcal{E}[\Theta_1^2(\lambda_1,\lambda_2)]}}\hat{x}_{S_1}\hat{x}_{S_2}\Theta_1(\lambda_1, \lambda_2)$.

B. SC-MIF Protocol

As discussed above, the NC-MIF protocol is good only if both $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ are reliable. If at least one source-to-relay channel is poor, then the network coded symbol $\hat{x}_R$ will have a low reliability, which will degrade the error performance of both sources. In such a scenario, network coding is not a sound choice. Instead, we propose the superposition coding based MIF protocol (SC-MIF), in which, the relay function is based on the superposition of the signed SMI of $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$. If we define $\Theta_2(\lambda_1,\lambda_2) \triangleq \hat{x}_{S_1}\Theta(\lambda_1)+\hat{x}_{S_2}\Theta(\lambda_2)$, then the relay function can be expressed as $f_{SC-MIF}(r_{S_1R}, r_{S_2R}) = \sqrt{\frac{1}{\mathcal{E}[\Theta_2^2(\lambda_1,\lambda_2)]}}\Theta_2(\lambda_1,\lambda_2)$.

IV. Performance Analysis and Comparisons

A. Performance Analysis of Two Protocols

We define $\tilde{x}_{S_i} \triangleq \hat{x}_{S_i}\Theta(\lambda_i) = \psi_i(x_{S_i}+e_{S_i})$, where e_{S_i} is the soft noise, and ψ_i denotes the scalar coefficients to make soft noise e_{S_i} uncorrelated to the information symbol x_{S_i}, i.e., $\mathcal{E}[e_{S_i}x_{S_i}] = 0$. According to [5], the coefficient ψ_i is calculated as $\psi_i = \frac{\mathcal{E}[x_{S_i}\tilde{x}_{S_i}]}{\mathcal{E}[x_{S_i}^2]}$.

In the NC-MIF protocol at S_i, we have $r_{RS_i} = \sqrt{E_R}h_{RS_i}f_{NC-MIF-2}(r_{S_1R}, r_{S_2R}) + n_{RS_i}$, which is multiplied with x_{S_i} so as to cancel x_{S_i}. Defining $\beta_{NC} \triangleq \sqrt{E_R/\mathcal{E}\left[(\tilde{x}_{S_1}\tilde{x}_{S_2})^2\right]}$, we have

$$x_{S_i}r_{RS_i} = h_{RS_i}\beta_{NC}\psi_1\psi_2 x_{S_j} + h_{RS_i}\beta_{NC}\psi_1\psi_2 (e_{S_j} + x_{S_i}x_{S_j}e_{S_i} + x_{S_i}e_{S_i}e_{S_j}) + n_{RS_i}x_{S_i}. \quad (2)$$

We define the noise in (2) as $N_{S_i,NC} \triangleq h_{RS_i}\beta_{NC}\psi_1\psi_2(e_{S_j} + x_{S_i}x_{S_j}e_{S_i} + x_{S_i}e_{S_i}e_{S_j}) + n_{RS_i}x_{S_i}$, which can be approximated as a Gaussian noise with the mean value $m_{N_{S_i}}$ and variance $\sigma^2_{N_{S_i}}$ [5]. Since $\mathcal{E}[e_{S_i}x_{S_i}] = 0$, we have $m_{N_{S_i}} = 0$ and $\sigma^2_{N_{S_i}} = h^2_{RS_i}\beta^2_{NC}\psi_1^2\psi_2^2\left(\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}\right) + \sigma^2_S$, where $\sigma^2_{e_{S_i}}$ is the variance of the soft noise e_{S_i}. Thus, the SNR of (2) can be written as

$$\gamma_{NC} = \frac{|h_{RS_i}|^2\beta^2_{NC}\psi_1^2\psi_2^2}{|h_{RS_i}|^2\beta^2_{NC}\psi_1^2\psi_2^2\left(\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}\right) + \sigma^2_S}. \quad (3)$$

In the SC-MIF protocol at S_i, we have $r_{SR_i} = \sqrt{E_R}h_{RS_i}f_{SC-MIF}(r_{S_1R}, r_{S_2R}) + n_{RS_i}$. We define $\beta_{SC} \triangleq \sqrt{E_R/\mathcal{E}\left[(\tilde{x}_{S_1}+\tilde{x}_{S_2})^2\right]}$. To cancel the symbol x_{S_i} at S_i, we subtract $h_{RS_i}\beta_{SC}\psi_i x_{S_i}$ from the received signal. We have

$$r_{RS_i} - h_{RS_i}\beta_{SC}\psi_i x_{S_i} = h_{RS_i}\beta_{SC}\psi_j x_{S_j} + h_{RS_i}\beta_{SC} (\psi_1 e_{S_1} + \psi_2 e_{S_2}) + n_{RS_i}. \quad (4)$$

We define $N_{S_i,SC} \triangleq \beta_{SC}(\psi_1 e_{S_1} + \psi_2 e_{S_2}) + n_{RS_i}$, which is approximated as a Gaussian noise with mean zero and variance $h^2_{RS_i}\beta^2_{SC}(\psi_1^2\sigma^2_{e_{S_1}} + \psi_2^2\sigma^2_{e_{S_2}}) + \sigma^2_S$. Then the SNR of (4) is

$$\gamma_{SC} = \frac{|h_{RS_i}|^2\beta^2_{SC}\psi_j^2}{|h_{RS_i}|^2\beta^2_{SC}\psi_1^2\sigma^2_{e_{S_1}} + |h_{RS_i}|^2\beta^2_{SC}\psi_2^2\sigma^2_{e_{S_2}} + \sigma^2_S}. \quad (5)$$

Before performing a comparison of the two protocols, we introduce the following lemma.

Lemma 1: When the SNR of the source-to-relay channel, i.e., $\rho_{S_iR} \triangleq |h_{S_iR}|^2E_S/\sigma^2_R$, goes to infinity, we have the relation between the soft noise variance $\sigma^2_{e_{S_i}}$ and ρ_{S_iR} as $\sigma^2_{e_{S_i}} \le 2e^{-\rho_{S_iR}}$.

Proof: Please refer to Appendix A.

Based on *Lemma 1*, we present the following theorem with regard to γ_{NC} and γ_{SC}.

Theorem 1: The relation between γ_{NC} and γ_{SC} is given by $\lim_{\substack{\rho_{S_1R}\to\infty \\ \rho_{S_2R}\to\infty}} \frac{\gamma_{NC}}{\gamma_{SC}} = 2$.

Proof: Please refer to Appendix B.

B. Adaptive Scheme for Fading Channels

As discussed before, the relative superiority between the NC-MIF and the SC-MIF depends on the instantaneous channel condition. This leads us to develop an adaptive scheme, which enables the dynamic switch between the two protocols based on the channel conditions. Here, we develop an adaptive scheme to determine which protocol should be used for a given channel condition. By defining $\alpha \triangleq \left|\frac{h_{S_1R}}{h_{S_2R}}\right|^2$, we shall derive the threshold of α, i.e., α_{thr}, based on which, the adaptive scheme can select the protocol with a larger received SNR.

As γ_{NC} represents the received SNR from the relay at S_i in the NC-MIF protocol, we denote by $\gamma_{NC,1}$ the received SNR at S_1 from the relay, and by $\gamma_{NC,2}$ the received SNR at S_2 from the relay. Similarly in the SC-MIF protocol, we denote by $\gamma_{SC,i}$ the received SNR at S_i from the relay. We also denote by $\gamma_{S_jS_i}$ the received SNR at S_i from S_j. Since the LLR combining is utilized at each source, the combined SNR in the NC-MIF at S_i is given by $\gamma_{NC,i} + \gamma_{S_jS_i}$. Then based on the $Q(\cdot)$ function, the equivalent received SNR of the whole system can be written as

$$\Gamma_{sys,NC-MIF} = \left(Q^{-1}\left(\frac{Q(\sqrt{\gamma_{NC,1}+\gamma_{S_2S_1}}) + Q(\sqrt{\gamma_{NC,2}+\gamma_{S_1S_2}})}{2}\right)\right)^2. \quad (6)$$

Similarly, we can obtain the received SNR for the SC-MIF, which is denoted as $\Gamma_{sys,SC-MIF}$. The optimal adaptive scheme should calculate α_{thr} by letting $\Gamma_{sys,NC-MIF} = \Gamma_{sys,SC-MIF}$.

To get a close form of α_{thr}, we approximate $Q(\sqrt{\gamma_{NC,i}+\gamma_{S_jS_i}})$ in (6) by using its Chernoff upper bound $\frac{1}{2}e^{-(\gamma_{NC,i}+\gamma_{S_jS_i})}$. Since it is difficult for the relay to obtain the instantaneous source-to-source channel information, we only use the expectation of $\gamma_{S_1S_2}$ or $\gamma_{S_2S_1}$ in the calculation. Note that $\mathcal{E}[\gamma_{S_1S_2}] = \mathcal{E}[\gamma_{S_2S_1}]$. Therefore, we obtain α_{thr} by letting $\ln\left(\frac{1}{e^{\gamma_{NC,1}}} + \frac{1}{e^{\gamma_{NC,2}}}\right) = \ln\left(\frac{1}{e^{\gamma_{SC,1}}} + \frac{1}{e^{\gamma_{SC,2}}}\right)$. After some manipulations, we have

$$\underbrace{2\left(1 - \frac{\psi_1^2}{\psi_2^2} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S - \psi_1^2\sigma^2_S}{|h_{RS_1}|^2E_R\psi_1^2\psi_2^2}\right)}_{C} e^{-\rho_{S_1R}} = \underbrace{\frac{\sigma^2_S(2\psi_1^2 + 2\psi_1^2\sigma^2_{e_{S_2}} - 1 - \sigma^2_{e_{S_2}})}{|h_{RS_1}|^2E_R\psi_1^2\psi_2^2}}_{D}. \quad (7)$$

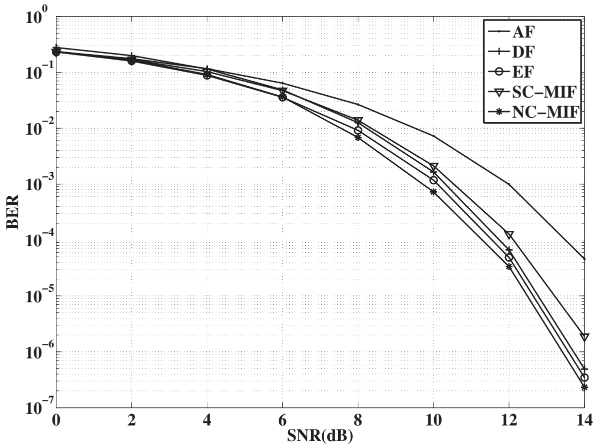

Fig. 2. System BER for Case 1 where h_{S_1R},h_{S_2R}, h_{RS_1}, and h_{RS_2} are equal to one.

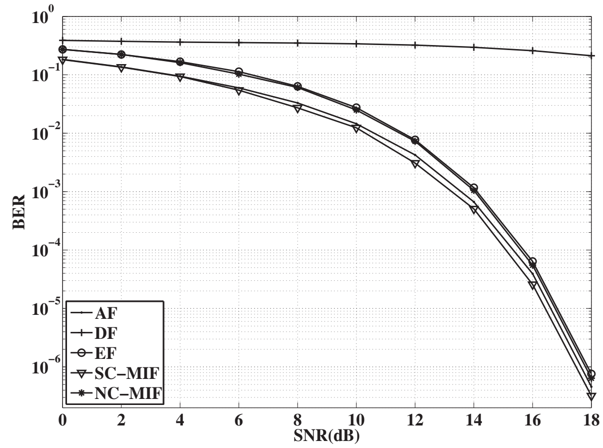

Fig. 3. System BER for Case 2 where $h_{S_1R} = 10$, $h_{S_2R} = 0.1$, $h_{RS_1} = 0.8$, and $h_{RS_2} = 1.2$.

As $\rho_{S_1R} = \alpha\rho_{S_2R}$, we have $e^{-\alpha\rho_{S_2R}} = D/C$. Thus, we obtain $\alpha_{thr} = \rho_{S_2R}^{-1} \ln \frac{C}{D}$. For a given channel realization, we calculate the value $\alpha = \left|\frac{h_{S_1R}}{h_{S_2R}}\right|^2$. If $\alpha < \alpha_{thr}$, then the NC-MIF will have a larger SNR than that of the SC-MIF, and thus the NC-MIF is used at the relay. Otherwise, the SC-MIF has a lager SNR and will be selected at the relay.

V. Simulations

In this section, we assume that $E_S = E_R$ and $\sigma_S^2 = \sigma_R^2$ in all the simulations. Assuming $h_{S_1S_2} = h_{S_2S_1} = 0.6$, we consider the following two cases. Case 1: symmetric AWGN channels with h_{S_1R},h_{S_2R}, h_{RS_1}, and h_{RS_2} being equal to one; Case 2: asymmetric AWGN channels with $h_{S_1R} = 10$, $h_{S_2R} = 0.1$, $h_{RS_1} = 0.8$, and $h_{RS_2} = 1.2$. We investigate the BER performance for the Case 1 and 2. We consider the system BER which is the average value of the two sources' BERs. Fig. 2 shows the system BER of Case 1. We can see that the NC-MIF performs the best among all the protocols, while the AF performs the worst. Note that our NC-MIF is better than the widely used EF protocol. In Case 1, since both channels are good, the network coded symbol $\hat{x}_R$ has a high reliability. Therefore, the network coding based protocols, i.e., the DF, EF, and NC-MIF are better than the superposition coding based protocols, i.e., SC-MIF and AF. Fig. 3 shows the system BER of Case 2. We can see that SC-MIF performs the best, while the DF performs the worst. In Case 2, one source-to-relay channel is in deep fading, which leads to a very low reliability of $\hat{x}_R$ in the network coding based protocols. Therefore, the SC-MIF and AF outperform the DF, EF, and NC-MIF. Particularly in the DF, the errors from $\hat{x}_R$ propagate at the relay, leading to its poor BER performance.

Next, we shall evaluate our adaptive scheme in fading channels. We assume that all the channels are Rayleigh distributed. Since the relay is located between the two sources, the source-to-relay channels should have a larger variance than that of the source-to-source channels. We assume that the channel coefficients h_{S_1R}, h_{S_2R}, h_{RS_1}, and h_{RS_2} have a unit variance, while the channel coefficients $h_{S_1S_2}$ and $h_{S_2S_1}$ have a variance of 0.36. We consider slow fading channels, i.e., the channels keep invariant during one transmission period while varying independently from one period to another. Each source transmits one frame in one period, with the frame length of 1000. In such a system, the maximum diversity gain of the system is two. Fig. 4 shows the system BER over fading channels. We can see that except the DF, which only has the diversity gain of one, the other protocols can achieve the full diversity. The SC-MIF and NC-MIF have a similar BER performance, and they both outperform the EF. The proposed adaptive scheme achieves the best BER performance among all the protocols.

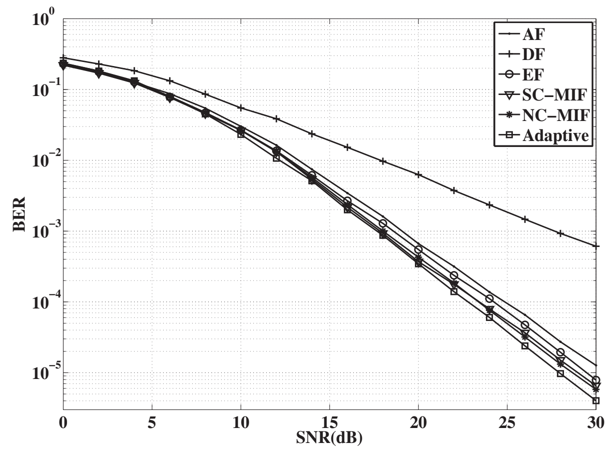

Fig. 4. System BER for Fading Channels.

VI. Conclusion

This paper presents novel mutual information based soft forwarding techniques for a two-way relay channel. We propose two new protocols, namely, NC-MIF and SC-MIF, and develop the expressions for the received SNR of the two protocols. Based on the comparison of the received SNR of the two protocols, we derive an adaptive scheme. This adaptive scheme determines which protocol should be selected for a given channel realization in order to achieve a larger received SNR. In the simulations, we compare our proposed MIF protocols with the conventional AF, DF and EF protocols. Simulations show that our the adaptive scheme perform the best among the existed relaying protocols in the fading channels.

APPENDIX A
PROOF OF LEMMA 1

The event $\tilde{x}_{S_i} = x_{S_i}\Theta(\lambda_i)$ happens with probability $1 - P_{e,x_{S_i}}$, and the event $\tilde{x}_{S_i} = -x_{S_i}\Theta(\lambda_i)$ happens with probability $P_{e,x_{S_i}}$, where $P_{e,x_{S_i}}$ is the detect error probability of x_{S_i} at the relay. We have $\mathcal{E}[\tilde{x}^2_{S_i}] = \mathcal{E}\left[(x_{S_i}\Theta(\lambda_i))^2\right](1-P_{e,x_{S_i}}) + \mathcal{E}\left[(-x_{S_i}\Theta(\lambda_i))^2\right]P_{e,x_{S_i}} = \mathcal{E}\left[(x_{S_i}\Theta(\lambda_i))^2\right]$ and $\mathcal{E}^2[\tilde{x}_{S_i}] = \left(\mathcal{E}\left[x_{S_i}\Theta(\lambda_i)\right](1-P_{e,x_{S_i}}) + \mathcal{E}\left[-x_{S_i}\Theta(\lambda_i)\right]P_{e,x_{S_i}}\right)^2 = (1 - 2P_{e,x_{S_i}})^2\mathcal{E}^2\left[x_{S_i}\Theta(\lambda_i)\right]$. According to [7], the soft noise variance can be written as $\sigma^2_{e_{S_i}} = \frac{\mathcal{E}\left[\Theta^2(\lambda_i)\right]}{(1-2P_{e,x_{S_i}})^2\mathcal{E}^2[\Theta(\lambda_i)]} - 1$. Since $\mathcal{E}\left[\Theta^2(\lambda_i)\right] \geq \mathcal{E}^2\left[\Theta(\lambda_i)\right]$, we can obtain that $\sigma^2_{e_{S_i}} \leq \frac{1}{(1-2P_{e,x_{S_i}})^2} - 1$. We have

$$\begin{aligned} \lim_{\rho_{S_iR}\to\infty}\sigma^2_{e_{S_i}} &\leq \lim_{\rho_{S_iR}\to\infty}\frac{1}{(1-2P_{e,x_{S_i}})^2} - 1 \\ &\overset{(a)}{\leq} \lim_{\rho_{S_iR}\to\infty}\frac{1}{\left(1-e^{-\rho_{S_iR}}\right)^2} - 1 \\ &= \lim_{\rho_{S_iR}\to\infty}\frac{2e^{-\rho_{S_iR}} - e^{-2\rho_{S_iR}}}{1 - 2e^{-\rho_{S_iR}} + e^{-2\rho_{S_iR}}} \\ &= \lim_{\rho_{S_iR}\to\infty} 2e^{-\rho_{S_iR}}, \end{aligned} \tag{8}$$

where (a) is obtained by the Chernoff upper bound of $Q(\cdot)$ function, i.e., $P_{e,x_{S_i}} = Q\left(\sqrt{\rho_{S_iR}}\right) \leq \frac{1}{2}e^{-\rho_{S_iR}}$. Based on (8), we thus complete the proof. ■

APPENDIX B
PROOF OF THEOREM 1

From *Lemma 1* we can see that $\sigma^2_{e_{S_i}}$ reduces exponentially with ρ_{S_iR}. When both ρ_{S_iR} is large enough, the soft noise variance $\sigma^2_{e_{S_i}}$ is approaching zero. Therefore in (3), the item $\sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}$ can be skipped when compared with $\sigma^2_{e_{S_i}}$. Also, when ρ_{S_iR} is large enough, we have $\tilde{x}_{S_i} \approx x_{S_i}$. According to the calculation of ψ_i, we have $\psi_i \to 1$ when $\tilde{x}_{S_i} \to x_{S_i}$. Then we have

$$\begin{aligned} \gamma_{NC} &\approx \frac{1}{\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S}{h^2_{RS_i}\beta^2_{NC}}}, \\ \gamma_{SC} &\approx \frac{1}{\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S}{h^2_{RS_i}\beta^2_{SC}}}. \end{aligned} \tag{9}$$

From (9), we can see that the difference between γ_{NC} and γ_{SC} depends on the difference between β_{NC} and β_{SC}. We focus on the denominators of β_{NC} and β_{SC}. Since $1 > \tilde{x}^2_{S_i} > 0$, we have $\mathcal{E}[(\tilde{x}_{S_1} + \tilde{x}_{S_2})^2] = \mathcal{E}[\tilde{x}^2_{S_1} + \tilde{x}^2_{S_2} + 2\tilde{x}_{S_1}\tilde{x}_{S_2}] = \mathcal{E}[\tilde{x}^2_{S_1} + \tilde{x}^2_{S_2}] > \mathcal{E}[\tilde{x}^2_{S_1}\tilde{x}^2_{S_2}]$, which means that $\beta_{NC} > \beta_{SC}$. Hence, we obtain that $\gamma_{NC} > \gamma_{SC}$ when both ρ_{S_1R} and ρ_{S_2R} are large enough.

When $\rho_{S_iR} \to \infty$, we neglect $\sigma^2_{e_{S_i}}$ in the expressions of γ_{NC} and γ_{SC}. Also, as $\lim_{\rho_{S_iR}\to\infty}\tilde{x}^2_{S_i} = 1$, we obtain that $\lim_{\rho_{S_iR}\to\infty}\frac{\mathcal{E}[\tilde{x}^2_{S_1}+\tilde{x}^2_{S_2}]}{\mathcal{E}[\tilde{x}^2_{S_1}\tilde{x}^2_{S_2}]} = 2$, which indicates that $\lim_{\rho_{S_iR}\to\infty}\frac{\gamma_{NC}}{\gamma_{SC}} = 2$. ■

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