number of cooperative sets increases when our proposed scheme is used. However, we can see that the number of active cooperative sets without outage decreases as the number of cooperative sets increases when the conventional scheme is used because the interference between cooperative sets increases rapidly, as shown in Fig. 4. Although we did not add the simulation results, we also have found that the spectral efficiency of our proposed scheme is at least twice as large as that of the conventional scheme.

We can also find that the outage probability of our proposed scheme is less than $P_{\rm out}$, which is 10%. Moreover, we can see that the number of active cooperative sets without outage is less when the AF scheme is used than when the DF scheme is used because, and when the AF scheme is used, the amount of interference increases due to amplification at relays, as shown in Fig. 5. In this figure, we can see that the probability that the interference caused by other cooperative sets becomes larger than γ is less than 10% for our proposed scheme.

V. Conclusion

Although cooperative transmission has been studied often, a distributed scheduling algorithm that considers cooperative transmission has not hitherto been proposed. To fill this gap, we have proposed a new distributed scheduling algorithm for wireless ad hoc systems with cooperative transmission in which multiple cooperative relays can transmit at the same time by using an AF scheme or a DF scheme. By using our proposed scheme, the probability that the system will suffer an outage can be reduced. Through performance evaluation by simulation, we have shown that the proposed scheme can enhance the performance of ad hoc systems with cooperative transmission.

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Novel Soft Information Forwarding Protocols in Two-Way Relay Channels

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***Abstract*—In this paper, we investigate novel soft mutual information forwarding (MIF) protocols in a two-way relay channel (TWRC), where two sources exchange information with the help of an intermediate relay. Based on the estimated signals from the two sources, the relay calculates the soft mutual information and then broadcasts it to the two sources. In particular, we propose two MIF protocols, namely, network-coded MIF (NC-MIF) and superposition-coded MIF (SC-MIF), which is suitable for different channel conditions. The expressions derived for the received signal-to-noise ratio (SNR) at the receivers reveal that, if both source-to-relay channels are in good condition, the NC-MIF outperforms the SC-MIF. Otherwise, the SC-MIF is superior to the NC-MIF. For the TWRC with varying channels, we further develop an adaptive scheme, which enables the dynamic switch between the two protocols, depending on the received SNR at the sources. Furthermore, the threshold that determines the switch of the protocols is developed as a close-form expression. Simulation results show that our adaptive scheme outperforms other protocols discussed in this paper over fading channels.**

***Index Terms*—Information forwarding, network-coded protocols, relay channels.**

I. Introduction

Two classical relaying protocols, namely, amplify-and-forward (AF) [1] and decode-and-forward (DF) [2], have been widely studied in wireless relaying systems. However, the AF protocol suffers from noise amplification, and the DF protocol propagates the erroneous decisions to the destination. Consequently, a new relaying concept based on soft information forwarding (SIF) technologies has been proposed to achieve better error performance [3]. The SIF-based protocols assist the destination to make hard decisions by forwarding the intermediate soft decisions [3]–[8].

Among the SIF schemes developed for one-way relay channels, a SIF-based protocol, namely, estimate-and-forward (EF), has been proposed in [6]. It is shown in [6] that the EF protocol maximizes the SNR at the destination and thus gains better error performance relative to the AF and DF protocols in the one-way single-relay channel. Recently, a novel SIF-based protocol based on forwarding symbol-wise mutual information (SMI) at the relay [henceforth referred to as mutual information forwarding (MIF)] has been proposed [7], [8]. It is shown in [7] that the MIF protocol achieves better error performance

Manuscript received August 6, 2012; revised December 3, 2012 and January 17, 2013; accepted January 21, 2013. Date of publication February 8, 2013; date of current version June 12, 2013. This work was supported in part by the Australian Research Council under Project DP110104995, Project DP120100405, and Project LP110100110; by the University of Sydney under a bridging support grant; and by the Government of Australia through the Australian Space Research Program. The review of this paper was coordinated by Prof. D. B. da Costa.

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Digital Object Identifier 10.1109/TVT.2013.2246203

compared with the EF protocol. On the other hand, two-way relay channels (TWRCs) have recently attracted much attention [9]–[16] due to its promising application to contemporary wireless systems. In the TWRC systems, two sources exchange their information via a shared relay. In particular, a SIF-based network-coded EF protocol (i.e., the SBF protocol) is proposed in [12], where an MMSE estimation of the network-coded binary symbol, i.e., the XOR-ed binary symbol from the two sources, is broadcast to the two sources by the relay. Apart from the EF-based network coding, there are some non-SIF-based network-coded protocols, e.g., selective DF and analog network coding [14]–[16].[1]

Inspired by the superiority of the MIF to the EF protocol, in this paper, we aim to develop novel MIF-based soft information relaying protocols for a TWRC under fading conditions. To the best of our knowledge, no MIF-based SIF protocol has been investigated for the TWRC. We first review the applications of the conventional AF, DF, and EF protocols in the TWRC. Since the relay receives the two sources' signals in orthogonal channels, it can use either network coding or superposition coding to combine the two signals. We then propose two MIF protocols, namely, network-coded MIF (NC-MIF) protocol and superposition-coded MIF (SC-MIF) protocol. In the NC-MIF, the soft mutual information forwarded by the relay is obtained based on the XOR-ed binary streams from the two sources. In the SC-MIF, the relay first calculates the soft mutual information for each source and then forwards their addition. Then, we derive the received SNR of the two MIF protocols. We have found that, if both source-to-relay channels are in good conditions, the NC-MIF has a better error performance than the SC-MIF. On the other hand, if one or both of the source-to-relay channels are poor, then the SC-MIF outperforms the NC-MIF. Based on these observations, we propose an adaptive scheme that dynamically switches between the SC-MIF and NC-MIF protocols based on the varying channel conditions. In simulations, the AF, DF, and EF protocols are utilized as benchmarks. Simulations show that the two proposed MIF protocols outperform the three conventional protocols. In addition, the adaptive scheme between the two MIF protocols achieves better performance than either the NC-MIF protocol or the SC-MIF protocol. Note that, although we consider the binary phase-shift keying (BPSK) modulation in this paper, our proposed schemes can be easily extended to higher order modulations.

The remainder of this paper is organized as follows. In Section II, we present the model of a TWRC system. In Section III, we propose the NC-MIF and SC-MIF protocols. In Section IV, performance analysis is presented. Adaptive scheme is also developed in Section IV. Section V presents the simulation results, and conclusion is given in Section VI.

II. System Model and Preliminaries

We consider a TWRC, as shown in Fig. 1, where the two sources S_1 and S_2 exchange their information with the help of relay R. It is assumed that all the three nodes operate in half-duplex mode, i.e., each node transmits and receives in a different time slot. A transmission period is composed of three time slots. In the first time slot, S_1 broadcasts its signals to R and S_2. In the second time slot, S_2 broadcasts its signals to R and S_1. In the third time slot, R broadcasts the processed signals to both sources to facilitate the information exchange.

[1]We note that [14] and [15] propose a novel network-coded selective DF (NC-SDF) protocol based on the log-likelihood ratio (LLR) values of the received signals at the relay: The NC-SDF protocol can be seen as a quantized soft information relaying technique, which quantizes the received LLR values into three possible values −1, 0, and 1 and optimizes the quantization thresholds by minimizing the system symbol error rate. In addition, we note that [16] considers the impact of channel estimation errors in the TWRC with analog network coding. We have to mention that channel estimation error in the TWRC is an important issue since the imperfect cancelation of each source's own information will lead to a performance loss. However, in this paper, our major contribution is in the novelty of the proposed soft information relaying techniques in the TWRC. Therefore, we assume that a perfect channel estimation has been done before each transmission in this paper.

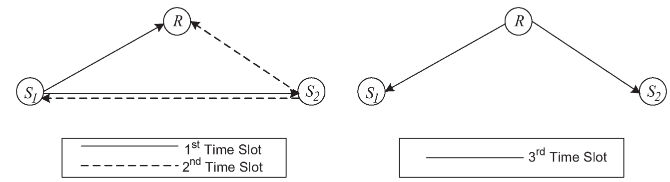

Fig. 1. Two-way relay network.

In the first and second time slots, the received signals at the relay and the sources can be expressed as

$$r_{S_iR} = \sqrt{E_S} h_{S_iR} x_{S_i} + n_{S_iR}$$
$$r_{S_iS_j} = \sqrt{E_S} h_{S_iS_j} x_{S_i} + n_{S_iS_j} \quad (1)$$

respectively, where E_S is the transmission power at each source; x_{S_i} is the power-normalized symbol transmitted by source S_i, $i \in \{1, 2\}$; r_{S_iR} is the received signal at relay R from S_i; $r_{S_iS_j}$ is the received signal at S_j from S_i, $j \in \{1, 2\}$, $i \neq j$; h_{S_iR} is the channel coefficient between S_i and R; $h_{S_iS_j}$ is the channel coefficient between S_i and S_j; and n_{S_iR} and $n_{S_iS_j}$ are the noise samples at R and S_j, respectively. In the third time slot, the received signal at S_i from R, which is defined by r_{RS_i}, can be written as

$$r_{RS_i} = \sqrt{E_R} h_{RS_i} f(r_{S_1R}, r_{S_2R}) + n_{RS_i} \quad (2)$$

where E_R is the transmission power at the relay, h_{RS_i} is the channel coefficient between R and S_i, and n_{RS_i} is the noise sample at S_i. In the earlier equation, we denote by $f(\cdot)$ the power-normalized relay function based on the received signals r_{S_1R} and r_{S_2R}. Thus, $\sqrt{E_R} f(r_{S_1R}, r_{S_2R})$ is the transmitted signal at relay R. The relay function $f(\cdot)$ is determined by the protocol utilized at the relay.

It is assumed that the binary signals 0 and 1 are generated at the sources with equal probability and mapped into the BPSK symbols, i.e., $x_{S_i} \in \{\pm 1\}$. The noise at S_i and R has a zero mean and variances σ_S^2 and σ_R^2, respectively. Next, we briefly review how conventional AF, DF, and EF protocols can be extended to the TWRC.

When the AF protocol is applied to the TWRC, the relay superimposes the received signals from the two sources and broadcasts the superposition to the two sources. The relay function can be written as

$$f_{\mathrm{AF}}(r_{S_1R}, r_{S_2R}) = \sqrt{\frac{1}{2\left(E_S + \sigma_R^2\right)}} (r_{S_1R} + r_{S_2R}). \quad (3)$$

As S_i knows its own information, x_{S_i} can be perfectly canceled from r_{RS_i}. After cancelation, S_i combines the result with the received signal $r_{S_jS_i}$ to make a decision on x_{S_j}.

In a TWRC with the DF protocol, the relay performs hard decisions on the symbols x_{S_1} and x_{S_2} and then calculates the network-coded symbol based on the hard decisions $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$. We define the network-coded symbol as $x_R \triangleq x_{S_1} x_{S_2}$. The hard decision of x_R is then calculated as $\hat{x}_R = \hat{x}_{S_1} \hat{x}_{S_2}$. The relay function can be expressed as

$$f_{\mathrm{DF}}(r_{S_1R}, r_{S_2R}) = \hat{x}_R. \quad (4)$$

At S_i, r_{RS_i} is multiplied with x_{S_i} to cancel x_{S_i} from r_{RS_i}, which is combined with $r_{S_jS_i}$ to make a decision on x_{S_j}.

We now discuss the EF protocol. According to [12], instead of forwarding $\hat{x}_R$ as in the DF protocol, the relay forwards the MMSE estimation of x_R, i.e., the expectation of $x_{S_1}x_{S_2}$, $\mathcal{E}[x_{S_1}x_{S_2}|r_{S_1R}, r_{S_2R}]$, which is calculated as

$$\mathcal{E}[x_{S_1}x_{S_2}|r_{S_1R}, r_{S_2R}] = \mathcal{E}[x_{S_1}|r_{S_1R}]\mathcal{E}[x_{S_2}|r_{S_2R}] = \tanh\left(\frac{L_{x_{S_1},R}}{2}\right)\tanh\left(\frac{L_{x_{S_2},R}}{2}\right) \quad (5)$$

where

$$L_{x_{S_i},R} = \ln\frac{p(r_{S_iR}|x_{S_i}=1, h_{S_iR})}{p(r_{S_iR}|x_{S_i}=-1, h_{S_iR})} \quad (6)$$

is the LLR of the received symbol x_{S_i} at the relay, $\mathcal{E}[x_{S_i}|r_{S_iR}] = \tanh(L_{x_{S_i},R}/2)$ is the MMSE estimation of x_{S_i}, tanh is the hyperbolic function, and $\tanh(x) = e^{2x} - 1/e^{2x} + 1$. Then, the relay function is

$$f_{\mathrm{EF}}(r_{S_1R}, r_{S_2R}) = \frac{\tanh\left(\frac{L_{x_{S_1},R}}{2}\right)\tanh\left(\frac{L_{x_{S_2},R}}{2}\right)}{\sqrt{\mathcal{E}\left[\left|\tanh\left(\frac{L_{x_{S_1},R}}{2}\right)\tanh\left(\frac{L_{x_{S_2},R}}{2}\right)\right|^2\right]}}. \quad (7)$$

The received signal r_{RS_i} at S_i from the relay is multiplied with x_{S_i} to cancel x_{S_i} from r_{RS_i}, which is then combined with $r_{S_jS_i}$ to make a hard decision on x_{S_j}.

III. Two Novel Mutual Information Forwarding Protocols

In the MIF protocol proposed in [7] for the one-way relay channels, the relay function is the product of two terms: The first term is the sign (i.e., hard decision) of the received signal at the relay, and the second term is the SMI of the source-to-relay channel between a source symbol x_S and its corresponding LLR L at the relay conditioned on $\lambda \triangleq |L|$. According to [7] and [8], the SMI is defined as $\Theta(\lambda) \triangleq I(x_S; L|\lambda)$, and

$$\begin{aligned}\Theta(\lambda) &\triangleq I(x_S; L|\lambda)\\ &= \frac{1}{1+e^{\lambda}}\log_2\frac{2}{1+e^{\lambda}} + \frac{1}{1+e^{-\lambda}}\log_2\frac{2}{1+e^{-\lambda}}\\ &= \frac{|L|\cdot e^{|L|}}{1+e^{|L|}}\log_2 e - \log_2(1+e^{|L|}).\end{aligned} \quad (8)$$

Expression (8) involves the calculations of $e^{|L|}$ and one logarithm, which have similar computational complexity as the EF protocol when $|L|$ is large.

With the background information briefly reviewed, we will then describe the two proposed MIF protocols in TWRC, namely, NC-MIF and SC-MIF.

A. NC-MIF Protocol

When network coding is applied at relay R, the relay function can be generally written as

$$f(r_{S_1R}, r_{S_2R}) = \hat{x}_{S_1}\hat{x}_{S_2}|f(r_{S_1R}, r_{S_2R})| \quad (9)$$

where $\hat{x}_{S_i}$ is the hard decision of the symbol x_{S_i} at R. The network-coded relay function can be seen as the hard decision of the network-coded symbol $\hat{x}_R = \hat{x}_{S_1}\hat{x}_{S_2}$, multiplied with reliability measurement $|f(r_{S_1R}, r_{S_2R})|$. In the NC-MIF, $|f(r_{S_1R}, r_{S_2R})|$ is calculated based on the SMI of the two source-to-relay channels.

We will propose two relay functions for the NC-MIF protocol, which are based on two different reliability measurements of the network-coded symbol.

1) NC-MIF-1: We define $\lambda_i \triangleq |L_{x_{S_i},R}|$. The relay function is based on the SMI between the network-coded symbol x_R and the LLRs $L_{x_{S_1},R}$ and $L_{x_{S_2},R}$ conditioned on λ_1, λ_2, i.e.,

$$\begin{aligned}&\Theta_1(\lambda_1, \lambda_2)\\ &\triangleq I(x_R; L_{x_{S_1},R}, L_{x_{S_2},R}|\lambda_1, \lambda_2)\\ &= \sum_{l_1=\pm\lambda_1}\sum_{l_2=\pm\lambda_2} p(l_1|\lambda_1)p(l_2|\lambda_2)I(x_R; l_1, l_2|\lambda_1, \lambda_2)\\ &= \sum_{l_1=\pm\lambda_1}\sum_{l_2=\pm\lambda_2} p(l_1|\lambda_1)p(l_2|\lambda_2)\cdot\\ &\quad\times\sum_{x=\pm1}\underbrace{p(x|l_1, l_2, \lambda_1, \lambda_2)\log\frac{p(x|l_1, l_2, \lambda_1, \lambda_2)}{p(x|\lambda_1, \lambda_2)}}_{A(x)}.\end{aligned} \quad (10)$$

We can see that $\Theta_1(\lambda_1, \lambda_2)$ is used as the reliability measurement for x_R, which is the SMI between x_R and the received signals at the relay. The rationale behind using $\Theta_1(\lambda_1, \lambda_2)$ is that larger mutual information reflects a larger reliability. Without loss of generality, we focus on $x = 1$, and $A(x = 1)$ in (10) can be written as

$$\begin{aligned}&A(x=1)\\ &= p(x=1|l_1, l_2, \lambda_1, \lambda_2)\log\frac{p(x=1|l_1, l_2, \lambda_1, \lambda_2)}{p(x=1|\lambda_1, \lambda_2)}\\ &= \frac{p(l_1, l_2|x=1)p(x=1)}{p(l_1, l_2)}\log\frac{p(l_1, l_2|x=1)p(x=1)}{p(x=1|\lambda_1, \lambda_2)p(l_1, l_2)}\\ &= \underbrace{\frac{C}{4_p(l_1)p(l_2)}}_{B}\log\frac{C}{4_p(l_1)p(l_2)p(x=1|\lambda_1, \lambda_2)}\end{aligned} \quad (11)$$

where

$$C = p(l_1|x_{S_1}=1)p(l_2|x_{S_2}=1) + p(l_1|x_{S_1}=-1)p(l_2|x_{S_2}=-1) \quad (12)$$

and B can be further calculated as

$$B = \left(\frac{1}{(1+e^{l_1})(1+e^{l_2})} + \frac{1}{(1+e^{-l_1})(1+e^{-l_2})}\right). \quad (13)$$

Similarly, we can obtain the expression for $A(x = -1)$. According to [17], we have

$$\begin{aligned}p(\pm\lambda_1|\lambda_1) = p(\pm\lambda_2|\lambda_2) &= \frac{1}{2}\\ p(x=\pm1|\lambda_1, \lambda_2) &= p(x=\pm1).\end{aligned} \quad (14)$$

Then, (10) can be rewritten as

$$\begin{aligned}\Theta_1(\lambda_1, \lambda_2) = &\left(\frac{1}{1+e^{\lambda_1}}\frac{1}{1+e^{-\lambda_2}} + \frac{1}{1+e^{-\lambda_1}}\frac{1}{1+e^{\lambda_2}}\right)\\ &\times\log\left(\frac{2}{1+e^{\lambda_1}}\frac{1}{1+e^{-\lambda_2}} + \frac{2}{1+e^{-\lambda_1}}\frac{1}{1+e^{\lambda_2}}\right)\\ &+\left(\frac{1}{1+e^{\lambda_1}}\frac{1}{1+e^{\lambda_2}} + \frac{1}{1+e^{-\lambda_1}}\frac{1}{1+e^{-\lambda_2}}\right)\\ &\times\log\left(\frac{2}{1+e^{\lambda_1}}\frac{1}{1+e^{\lambda_2}} + \frac{2}{1+e^{-\lambda_1}}\frac{1}{1+e^{-\lambda_2}}\right).\end{aligned} \quad (15)$$

The relay function can be expressed as

$$f_{\mathrm{NC-MIF-1}}(r_{S_1R}, r_{S_2R}) = \frac{\hat{x}_{S_1}\hat{x}_{S_2}\Theta_1(\lambda_1, \lambda_2)}{\sqrt{\mathcal{E}\left[\Theta_1^2(\lambda_1, \lambda_2)\right]}}. \quad (16)$$

2) NC-MIF-2: Consider the values of $\Theta_1(\lambda_1, \lambda_2)$ in the following three extreme cases.

1) When λ_1 and λ_2 simultaneously approach infinity, we have $\Theta_1(\lambda_1, \lambda_2) = 1$.
2) When λ_1 and λ_2 simultaneously approach zero, we have $\Theta_1(\lambda_1, \lambda_2) = 0$.
3) When λ_i approaches infinity and λ_j approaches zero, we have $\Theta_1(\lambda_1, \lambda_2) = 0$.

It is clear that only if both $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ are reliable, then $\hat{x}_R$ is reliable. On the other hand, if any of $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ is not reliable, then $\hat{x}_R$ is not reliable. That is to say, the reliability measurement of $\hat{x}_R$ should depend on the reliability measurements of $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$.

Based on the given discussions, we propose the NC-MIF-2 protocol. In the NC-MIF-2 protocol, we calculate $\hat{x}_R$'s reliability measurement as $\Theta_2(\lambda_1, \lambda_2) = \Theta(\lambda_1)\Theta(\lambda_2)$, where $\Theta(\lambda_i)$ is the reliability measurement for $\hat{x}_{S_i}$. We use the SMI of $\hat{x}_{S_i}$ as its reliability measurement. Thus, we have $\Theta(\lambda_i) = I(x_{S_i}; L_{x_{S_i},R}|\lambda_i)$, which can be calculated according to (8), and

$$\begin{aligned}\Theta_2(\lambda_1, \lambda_2) &= \frac{1}{1+e^{\lambda_1}}\frac{1}{1+e^{\lambda_2}}\log\frac{2}{1+e^{\lambda_1}}\log\frac{2}{1+e^{\lambda_2}} \\ &+ \frac{1}{1+e^{\lambda_1}}\frac{1}{1+e^{-\lambda_2}}\log\frac{2}{1+e^{\lambda_1}}\log\frac{2}{1+e^{-\lambda_2}} \\ &\times \frac{1}{1+e^{-\lambda_1}}\frac{1}{1+e^{\lambda_2}}\log\frac{2}{1+e^{-\lambda_1}}\log\frac{2}{1+e^{\lambda_2}} \\ &+ \frac{1}{1+e^{-\lambda_1}}\frac{1}{1+e^{-\lambda_2}}\log\frac{2}{1+e^{-\lambda_1}}\log\frac{2}{1+e^{-\lambda_2}}. \end{aligned} \tag{17}$$

The relay function is thus expressed as

$$f_{\mathrm{NC-MIF-2}}(r_{S_1R}, r_{S_2R}) = \frac{\hat{x}_{S_1}\hat{x}_{S_2}\Theta_2(\lambda_1, \lambda_2)}{\sqrt{\mathcal{E}\left[\Theta_2^2(\lambda_1, \lambda_2)\right]}}. \tag{18}$$

B. SC-MIF Protocol

As discussed earlier, the NC-MIF protocol is good only if both $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$ are reliable. If at least one source-to-relay channel is poor, then the network-coded symbol $\hat{x}_R$ will have a low reliability, which will degrade the error performance of both sources. In such a scenario, network coding is not a sound choice. Instead, we propose the SC-MIF, in which, the relay function is based on the superposition of the signed SMI of $\hat{x}_{S_1}$ and $\hat{x}_{S_2}$. If we define

$$\Theta_3(\lambda_1, \lambda_2) \triangleq \hat{x}_{S_1}\Theta(\lambda_1) + \hat{x}_{S_2}\Theta(\lambda_2) \tag{19}$$

then the relay function can be expressed as

$$f_{\mathrm{SC-MIF}}(r_{S_1R}, r_{S_2R}) = \sqrt{\frac{1}{\mathcal{E}\left[\Theta_3^2(\lambda_1, \lambda_2)\right]}}\Theta_3(\lambda_1, \lambda_2). \tag{20}$$

IV. Adaptive Mutual Information Forwarding Scheme in Two-Way Relay Channels

A. Two NC-MIF-based Relay Functions

We approximate $\Theta_1(\lambda_1, \lambda_2)$ and $\Theta_2(\lambda_1, \lambda_2)$ for a relatively high-SNR region, where $\lambda_i \gg 1$. Thus, we have $1 + e^{\lambda_i} \approx e^{\lambda_i}$ and $1 + e^{-\lambda_i} \approx 1$. Then, we can rewrite (15) and (17) as

$$\begin{aligned}\Theta_1(\lambda_1, \lambda_2) &\approx \left(\frac{1}{e^{\lambda_1}} + \frac{1}{e^{\lambda_2}}\right)\log\left(\frac{2}{e^{\lambda_1}} + \frac{2}{e^{\lambda_2}}\right) \\ &+ \left(\frac{1}{e^{\lambda_1}e^{\lambda_2}} + 1\right)\log\left(\frac{2}{e^{\lambda_1}e^{\lambda_2}} + 2\right) \\ &\approx \left(\frac{1}{e^{\lambda_1}} + \frac{1}{e^{\lambda_2}}\right)\log\left(\frac{2}{e^{\lambda_1}} + \frac{2}{e^{\lambda_2}}\right) + 1 \\ \Theta_2(\lambda_1, \lambda_2) &\approx \frac{1}{e^{\lambda_1}}\log\frac{2}{e^{\lambda_1}} + \frac{1}{e^{\lambda_2}}\log\frac{2}{e^{\lambda_2}} \\ &+ \frac{1}{e^{\lambda_1}e^{\lambda_2}}\log\frac{2}{e^{\lambda_1}}\log\frac{2}{e^{\lambda_2}} + 1. \end{aligned} \tag{21}$$

respectively. From (21), we can see that $\Theta_1(\lambda_1, \lambda_2) > \Theta_2(\lambda_1, \lambda_2)$. Let us define

$$\Delta\Theta \triangleq \Theta_1(\lambda_1, \lambda_2) - \Theta_2(\lambda_1, \lambda_2). \tag{22}$$

For a large λ_i, we can ignore the item $(1/e^{\lambda_1}e^{\lambda_2})\log(2/e^{\lambda_1})\log(2/e^{\lambda_2})$ in $\Theta_2(\lambda_1, \lambda_2)$ as it is very small compared with $(1/e^{\lambda_i})\log(2/e^{\lambda_i})$. Without loss of generality, we assume $\lambda_1 \geq \lambda_2$. Then, $\Delta\Theta$ can be approximated as

$$\begin{aligned}\Delta\Theta &\approx \left(\frac{1}{e^{\lambda_1}} + \frac{1}{e^{\lambda_2}}\right)\log\left(\frac{2}{e^{\lambda_1}} + \frac{2}{e^{\lambda_2}}\right) \\ &- \frac{1}{e^{\lambda_1}}\log\frac{2}{e^{\lambda_1}} - \frac{1}{e^{\lambda_2}}\log\frac{2}{e^{\lambda_2}} \\ &\overset{(a)}{\approx} \left(\frac{1}{e^{\lambda_1}} + \frac{1}{e^{\lambda_2}}\right)\log\frac{2}{e^{\lambda_2}} - \frac{1}{e^{\lambda_1}}\log\frac{2}{e^{\lambda_1}} - \frac{1}{e^{\lambda_2}}\log\frac{2}{e^{\lambda_2}} \\ &= \frac{(\lambda_1 - \lambda_2)\log e}{e^{\lambda_1}} \end{aligned} \tag{23}$$

where the second approximation (a) is based on the fact that $\log((2/e^{\lambda_1}) + (2/e^{\lambda_2})) \approx \log(2/e^{\lambda_2})$. When λ_1 and λ_2 are large, we can see that $\Theta_1(\lambda_1, \lambda_2)$ and $\Theta_2(\lambda_1, \lambda_2)$ approach 1, whereas $\Delta\Theta$ approaches 0. Therefore, the difference between $\Theta_1(\lambda_1, \lambda_2)$ and $\Theta_2(\lambda_1, \lambda_2)$ can be ignored.

In addition, we note that $\Theta_1(\lambda_1, \lambda_2)$ entails intractability of the performance analysis, whereas by using $\Theta_2(\lambda_1, \lambda_2)$ as the reliability measurement, we can simplify the performance analysis for the both sources. Therefore, we will use $\Theta_2(\lambda_1, \lambda_2)$, i.e., $f_{\mathrm{NC-MIF-2}}(r_{S_1R}, r_{S_2R})$ as the relay function of the NC-MIF protocol in the following performance analysis.

B. Performance Analysis of Two Protocols

We define $\tilde{x}_{S_i} \triangleq \hat{x}_{S_i}\Theta(\lambda_i) = \psi_i(x_{S_i} + e_{S_i})$, where e_{S_i} is the soft noise, and ψ_i denotes the scalar coefficients to make soft noise e_{S_i} uncorrelated to the information symbol x_{S_i}, i.e., $\mathcal{E}[e_{S_i}x_{S_i}] = 0$. According to [6], the coefficient ψ_i is calculated as $\psi_i = (\mathcal{E}[x_{S_i}\tilde{x}_{S_i}]/\mathcal{E}[x_{S_i}^2])$.

In the NC-MIF protocol at S_i, we have

$$r_{RS_i} = \sqrt{E_R}h_{RS_i}f_{\mathrm{NC-MIF-2}}(r_{S_1R}, r_{S_2R}) + n_{RS_i} \tag{24}$$

which is multiplied with x_{S_i} to cancel x_{S_i}. Defining

$$\beta_{\mathrm{NC}} \triangleq \sqrt{\frac{E_R}{\mathcal{E}\left[(\tilde{x}_{S_1}\tilde{x}_{S_2})^2\right]}} \tag{25}$$

we have

$$\begin{aligned}x_{S_i}r_{RS_i} &= h_{RS_i}\beta_{\mathrm{NC}}\psi_1\psi_2x_{S_j} + h_{RS_i}\beta_{\mathrm{NC}}\psi_1\psi_2 \\ &\quad (e_{S_j} + x_{S_i}x_{S_j}e_{S_i} + x_{S_i}e_{S_i}e_{S_j}) + n_{RS_i}x_{S_i}. \end{aligned} \tag{26}$$

We define the noise in (26) as

$$\begin{aligned}N_{S_i,\mathrm{NC}} &\triangleq h_{RS_i}\beta_{\mathrm{NC}}\psi_1\psi_2 \\ &\quad (e_{S_j} + x_{S_i}x_{S_j}e_{S_i} + x_{S_i}e_{S_i}e_{S_j}) + n_{RS_i}x_{S_i} \end{aligned} \tag{27}$$

which can be approximated as a Gaussian noise with the mean value $m_{N_{S_i}}$ and variance $\sigma^2_{N_{S_i}}$ [6]. Since $\mathcal{E}[e_{S_i}x_{S_i}] = 0$, we have

$m_{N_{S_i}} = 0$, and

$$\sigma^2_{N_{S_i}} = h^2_{RS_i}\beta^2_{\mathrm{NC}}\psi_1^2\psi_2^2\left(\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}\right) + \sigma^2_S \quad (28)$$

where $\sigma^2_{e_{S_i}}$ is the variance of the soft noise e_{S_i}. Thus, the SNR of (26) can be written as

$$\gamma_{\mathrm{NC}} = \frac{|h_{RS_i}|^2\beta^2_{\mathrm{NC}}\psi_1^2\psi_2^2}{|h_{RS_i}|^2\beta^2_{\mathrm{NC}}\psi_1^2\psi_2^2\left(\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}\right) + \sigma^2_S}. \quad (29)$$

In the SC-MIF protocol at S_i, we have

$$r_{SR_i} = \sqrt{E_R}h_{RS_i}f_{SC-MIF}(r_{S_1R}, r_{S_2R}) + n_{RS_i}. \quad (30)$$

We define

$$\beta_{\mathrm{SC}} \triangleq \sqrt{\frac{E_R}{\mathcal{E}\left[(\tilde{x}_{S_1} + \tilde{x}_{S_2})^2\right]}}. \quad (31)$$

To cancel the symbol x_{S_i} at S_i, we subtract $h_{RS_i}\beta_{\mathrm{SC}}\psi_i x_{S_i}$ from the received signal. We have

$$r_{RS_i} - h_{RS_i}\beta_{\mathrm{SC}}\psi_i x_{S_i} = h_{RS_i}\beta_{\mathrm{SC}}\psi_j x_{S_j} + h_{RS_i}\beta_{\mathrm{SC}}(\psi_1 e_{S_1} + \psi_2 e_{S_2}) + n_{RS_i}. \quad (32)$$

We define

$$N_{S_i,\mathrm{SC}} \triangleq \beta_{\mathrm{SC}}(\psi_1 e_{S_1} + \psi_2 e_{S_2}) + n_{RS_i}. \quad (33)$$

which is approximated as a Gaussian noise with zero mean and variance $h^2_{RS_i}\beta^2_{\mathrm{SC}}(\psi_1^2\sigma^2_{e_{S_1}} + \psi_2^2\sigma^2_{e_{S_2}}) + \sigma^2_S$. Then, the SNR of (32) is

$$\gamma_{\mathrm{SC}} = \frac{|h_{RS_i}|^2\beta^2_{\mathrm{SC}}\psi_j^2}{|h_{RS_i}|^2\beta^2_{\mathrm{SC}}\psi_1^2\sigma^2_{e_{S_1}} + |h_{RS_i}|^2\beta^2_{\mathrm{SC}}\psi_2^2\sigma^2_{e_{S_2}} + \sigma^2_S}. \quad (34)$$

Before performing a comparison of the two protocols, we introduce the following lemma.

Lemma 1: When the SNR of the source-to-relay channel, i.e., $\rho_{S_iR} \triangleq |h_{S_iR}|^2E_S/\sigma^2_R$, goes to infinity, we have the relationship between the soft noise variance $\sigma^2_{e_{S_i}}$ and ρ_{S_iR} as $\sigma^2_{e_{S_i}} \le 2e^{-\rho_{S_iR}}$.

Proof: See Appendix A.

Based on *Lemma 1*, we present the following theorem with regard to γ_{NC} and γ_{SC}.

Theorem 1: The ratio between γ_{NC} and γ_{SC} is given by $\lim_{\substack{\rho_{S_1R}\to\infty \\ \rho_{S_2R}\to\infty}}(\gamma_{\mathrm{NC}}/\gamma_{\mathrm{SC}}) = 2$.

Proof: See Appendix B.

Based on (26), (29), (32), and (34), we can use the maximal-ratio combining (MRC) method to combine the signals from the S_i-R-S_j link and the S_i-S_j direct link (DL). After the MRC, we make a hard decision for each symbol based on its MRC result.

C. Adaptive Scheme for Fading Channels

As discussed earlier, the relative superiority between the NC-MIF and the SC-MIF depends on the instantaneous channel condition. This leads us to develop an adaptive scheme, which enables the dynamic switch between the two protocols based on the channel conditions. Here, we develop an adaptive scheme to determine which protocol should be used for a given channel condition. By defining $\alpha \triangleq |(h_{S_1R}/h_{S_2R})|^2$, we will derive the threshold of α, i.e., α_{thr}, based on which, the adaptive scheme can select the protocol with a larger received SNR.

As γ_{NC} represents the received SNR from the relay at S_i in the NC-MIF protocol, we denote by $\gamma_{\mathrm{NC},1}$ the received SNR at S_1 from the relay, and by $\gamma_{\mathrm{NC},2}$ the received SNR at S_2 from the relay. Similarly in the SC-MIF protocol, we denote by $\gamma_{\mathrm{SC},i}$ the received SNR at S_i from the relay. We also denote by $\gamma_{S_jS_i}$ the received SNR at S_i from S_j. Since the LLR combining is utilized at each source, the combined SNR in the NC-MIF at S_i is given by $\gamma_{\mathrm{NC},i} + \gamma_{S_jS_i}$. Then, based on the $Q(\cdot)$ function, the equivalent received SNR of the whole system can be written as

$$\Gamma_{\mathrm{sys,NC-MIF}} = \left(Q^{-1}\left(\frac{Q(\sqrt{\gamma_{\mathrm{NC},1} + \gamma_{S_2S_1}}) + Q(\sqrt{\gamma_{\mathrm{NC},2} + \gamma_{S_1S_2}})}{2}\right)\right)^2. \quad (35)$$

Similarly, we can obtain the received SNR for the SC-MIF, which is denoted as $\Gamma_{\mathrm{sys,SC-MIF}}$. The optimal adaptive scheme should calculate α_{thr} by letting

$$\Gamma_{\mathrm{sys,NC-MIF}} = \Gamma_{\mathrm{sys,SC-MIF}}. \quad (36)$$

To obtain a closed form of α_{thr}, we approximate $Q(\sqrt{\gamma_{\mathrm{NC},i} + \gamma_{S_jS_i}})$ in (35) by using its Chernoff upper bound $(1/2)e^{-(\gamma_{\mathrm{NC},i} + \gamma_{S_jS_i})}$. Since it is difficult for the relay to obtain the instantaneous source-to-source channel information, we only use the expectation of $\gamma_{S_1S_2}$ or $\gamma_{S_2S_1}$ in the calculation. Note that $\mathcal{E}[\gamma_{S_1S_2}] = \mathcal{E}[\gamma_{S_2S_1}]$. We obtain α_{thr} by letting

$$\ln\left(\frac{1}{e^{\gamma_{\mathrm{NC},1}}} + \frac{1}{e^{\gamma_{\mathrm{NC},2}}}\right) = \ln\left(\frac{1}{e^{\gamma_{\mathrm{SC},1}}} + \frac{1}{e^{\gamma_{\mathrm{SC},2}}}\right). \quad (37)$$

In the high-SNR region, we further approximate that

$$\ln\left(\frac{1}{e^{\gamma_{\mathrm{NC},1}}} + \frac{1}{e^{\gamma_{\mathrm{NC},2}}}\right) \approx \ln\left(\max\left\{\frac{1}{e^{\gamma_{\mathrm{NC},1}}}, \frac{1}{e^{\gamma_{\mathrm{NC},2}}}\right\}\right)$$
$$\ln\left(\frac{1}{e^{\gamma_{\mathrm{SC},1}}} + \frac{1}{e^{\gamma_{\mathrm{SC},2}}}\right) \approx \ln\left(\max\left\{\frac{1}{e^{\gamma_{\mathrm{SC},1}}}, \frac{1}{e^{\gamma_{\mathrm{SC},2}}}\right\}\right). \quad (38)$$

That is, we let

$$\min\{\gamma_{\mathrm{NC},1}, \gamma_{\mathrm{NC},2}\} = \min\{\gamma_{\mathrm{SC},1}, \gamma_{\mathrm{SC},2}\} \quad (39)$$

to obtain α_{thr}. Without loss of generality, we assume

$$\min\{\gamma_{\mathrm{NC},1}, \gamma_{\mathrm{NC},2}\} = \gamma_{\mathrm{NC},1}$$
$$\min\{\gamma_{\mathrm{SC},1}, \gamma_{\mathrm{SC},2}\} = \gamma_{\mathrm{SC},1}. \quad (40)$$

We compare $\gamma_{\mathrm{NC},1}$ in (29) and $\gamma_{\mathrm{SC},1}$ in (34). Note that

$$\beta^2_{\mathrm{NC}} = \frac{E_R}{\mathcal{E}\left[(x_{S_1} + e_{S_1})^2(x_{S_2} + e_{S_2})^2\right]} = \frac{E_R}{1 + \left(\sigma^2_{e_{S_1}} + \sigma^2_{e_2}\right)}$$
$$\beta^2_{\mathrm{SC}} = \frac{E_R}{\mathcal{E}\left[\left(x_{S_1} + e_{S_1} + x_{S_2} + e^2_{S_2}\right)\right]} = \frac{E_R}{2 + \left(\sigma^2_{e_{S_1}} + \sigma^2_{e_2}\right)}. \quad (41)$$

We make the approximation $\sigma^2_{e_{S_1}} \approx 2e^{-\rho_{S_1R}}$. After some manipulations, we have

$$\underbrace{2\left(1 - \frac{\psi_1^2}{\psi_2^2} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S - \psi_1^2\sigma^2_S}{|h_{RS_1}|^2E_R\psi_1^2\psi_2^2}\right)}_{D}e^{-\rho_{S_1R}} = \underbrace{\frac{\sigma^2_S(2\psi_1^2 + 2\psi_1^2\sigma^2_{e_{S_2}}1 - \sigma^2_{e_{S_2}})}{|h_{RS_1}|^2E_R\psi_1^2\psi_2^2}}_{E}. \quad (42)$$

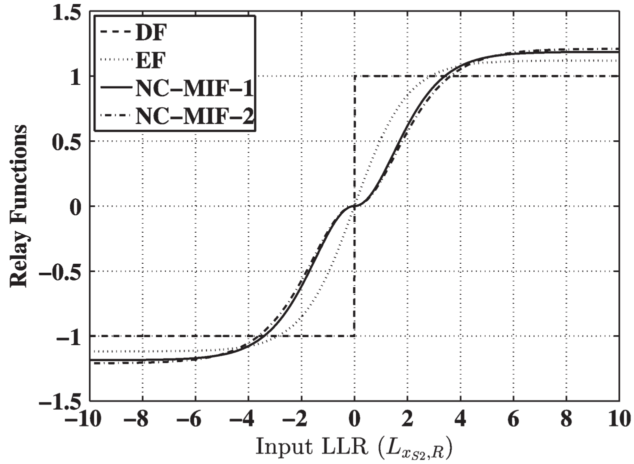

Fig. 2. Relay functions with regard to the input LLR $L_{x_{S_2},R}$ when fixing $L_{x_{S_1},R} = 3$.

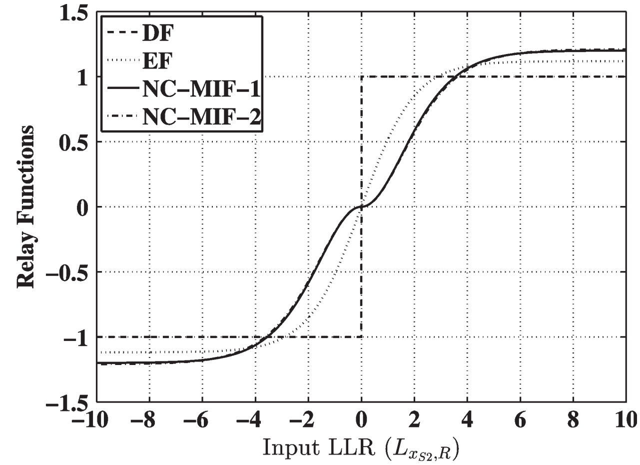

Fig. 3. Relay functions with regard to the input LLR $L_{x_{S_2},R}$ when fixing $L_{x_{S_1},R} = 5$.

As $\rho_{S_1R} = \alpha\rho_{S_2R}$, we have $e^{-\alpha\rho_{S_2R}} = E/D$. Thus, we obtain

$$\alpha_{\rm thr} = \rho_{S_2R}^{-1} \ln \frac{D}{E}. \tag{43}$$

For a given channel realization, we calculate the value $\alpha = |(h_{S_1R}/h_{S_2R})|^2$. If $\alpha < \alpha_{\rm thr}$, then we have $\Gamma_{\rm sys,NC-MIF} > \Gamma_{\rm sys,SC-MIF}$. In addition, if $\alpha > \alpha_{\rm thr}$, we have $\Gamma_{\rm sys,NC-MIF} < \Gamma_{\rm sys,SC-MIF}$. By comparing the instantaneous α with the threshold $\alpha_{\rm thr}$, our adaptive scheme chooses the MIF protocol that has a larger system SNR.

V. Numerical Results and Simulations

In the simulations, it is assumed that $E_S = E_R$ and $\sigma_S^2 = \sigma_R^2$. We first compare the two relay functions of the NC-MIF protocol, i.e., $f_{\rm NC-MIF-1}(r_{S_1R}, r_{S_2R})$ and $f_{\rm NC-MIF-2}(r_{S_1R}, r_{S_2R})$, based on the numerical results from (15) and (17). We fix the received LLR $L_{x_{S_1},R}$ and show the relationship between $L_{x_{S_2},R}$ and the relay functions. Figs. 2 and 3 show the relationship between $L_{x_{S_2},R}$ and the relay functions of the DF, EF, NC-MIF-1, and NC-MIF-2. In Fig. 2, we fix $L_{x_{S_1},R} = 3$, and in Fig. 3, we fix $L_{x_{S_1},R} = 5$. We can see that the gap between the curves of NC-MIF-1 and NC-MIF-2 is very small. In particular, the gap is indistinguishable for $L_{x_{S_1},R} = 5$ in Fig. 3. The numerical results shows that, when $|L_{x_{S_1},R}|$ and $|L_{x_{S_2},R}|$ are large, the difference between the two MIF relay functions can be neglected.

Then, we compare the received SNR of the whole system under various channel conditions. According to Section IV-C, we can calculate the equivalent received SNR of the whole system in the NC-MIF, i.e., $\Gamma_{\rm sys,NC-MIF}$, and in the SC-MIF, i.e., $\Gamma_{\rm sys,SC-MIF}$. Assuming $h_{S_1S_2} = h_{S_2S_1} = 0.6$, we will compare $\Gamma_{\rm sys,NC-MIF}$ and $\Gamma_{\rm sys,SC-MIF}$ in the following two cases: case 1 , which has symmetric additive white Gaussian noise (AWGN) channels with h_{S_1R}, h_{S_2R}, h_{RS_1}, and h_{RS_2} being equal to 1; and case 2, which has asymmetric AWGN channels with $h_{S_1R} = 10$, $h_{S_2R} = 0.1$, $h_{RS_1} = 0.8$, and $h_{RS_2} = 1.2$. Fig. 4 shows $\Gamma_{\rm sys,NC-MIF}$ and $\Gamma_{\rm sys,SC-MIF}$ with regard to the transmit SNR $\rho = (E_S/\sigma_R^2)$. We can see that, in case 1, $\Gamma_{\rm sys,NC-MIF}$ has a gain of 2.2 dB relative to $\Gamma_{\rm sys,SC-MIF}$ when the transmit SNR is 20 dB. Note that this gain will eventually approaches 3 dB when the transmit SNR goes to infinity. This is because that $\gamma_{\rm NC}$ is 3 dB larger than $\gamma_{\rm SC}$ in the high-SNR region, as indicated by *Theorem 1*. In Case 2, the S_1-to-R channel is good, whereas the S_2-to-R channel is poor. We can see in Fig. 4 that the SC-MIF outperforms the NC-MIF in Case 2. As discussed in Section III-B, the worse performance of the NC-MIF is due to low reliability of $\hat{x}_R$ if one channel or both channels are in poor conditions. However, we can

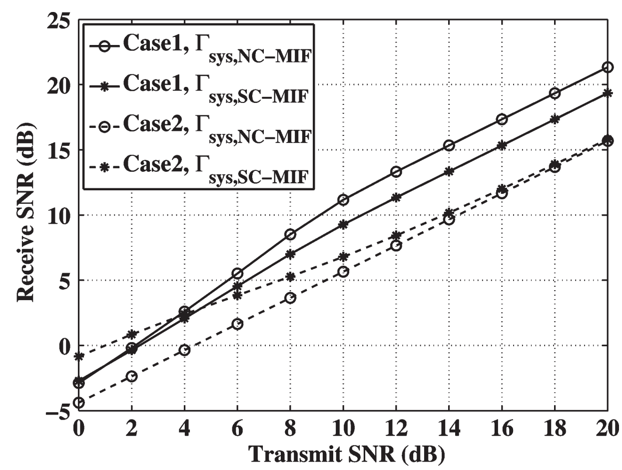

Fig. 4. Equivalent received SNR of the system for the NC-MIF and the SC-MIF protocols in the two cases. Case 1 represents the symmetric channels, and case 2 represents the asymmetric channels.

see that the gap between $\Gamma_{\rm sys,NC-MIF}$ and $\Gamma_{\rm sys,SC-MIF}$ disappears when ρ becomes large enough.

Next, we investigate the bit-error-rate (BER) performance for cases 1 and 2, where the channel gain of the source-to-source link is set to be 0.6 and 0.8. We consider the system BER that is the average value of the two sources' BERs. Fig. 5 shows the system BER of Case 1. We can see that the NC-MIF performs the best among all the protocols, whereas the AF performs the worst. Note that the proposed NC-MIF is better than the widely used EF protocol. In Case 1, since both channels are good, the network-coded symbol $\hat{x}_R$ has high reliability. Therefore, the network-coding-based protocols, i.e., the DF, EF, and NC-MIF are better than the superposition-coding-based protocols, i.e., SC-MIF and AF. Fig. 6 shows the system BER of case 2. We can see that SC-MIF performs the best, whereas the DF performs the worst. In case 2, one source-to-relay channel is in deep fading, which leads to very low reliability of $\hat{x}_R$ in the network-coding-based protocols. Therefore, the SC-MIF and AF outperform the DF, EF, and NC-MIF. Particularly in the DF, the errors from $\hat{x}_R$ propagate at the relay, leading to its poor BER performance.

We use cases 1 and 2 to show that the NC-MIF and SC-MIF outperform each other over different channel conditions. Therefore, the proposed adaptive scheme for the time-varying fading channels should achieve better performance. We now focus on time-varying

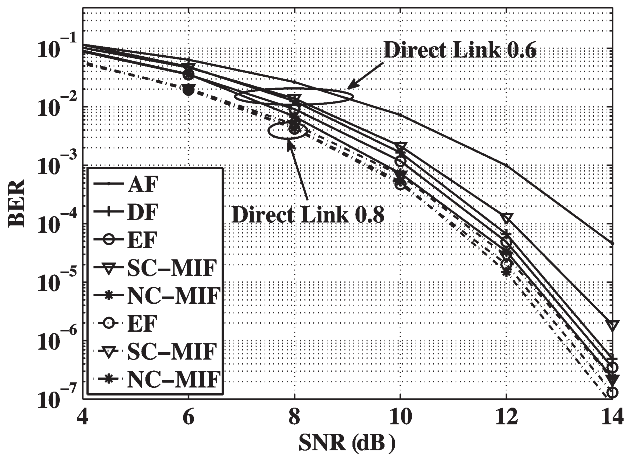

Fig. 5. System BER for Case 1, where h_{S_1R}, h_{S_2R}, h_{RS_1}, and h_{RS_2} are equal to 1. Solid lines represent the BER curves for the TWRC with the channel gain of the DL as 0.6. Dashed lines represent the BER curves for the TWRC with the channel gain of the DL as 0.8.

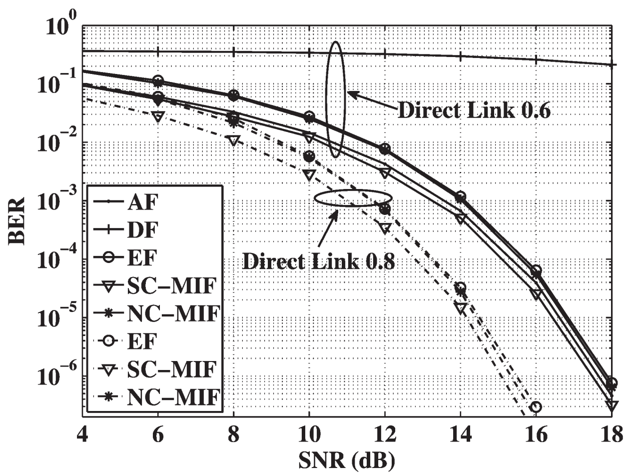

Fig. 6. System BER for Case 2, where $h_{S_1R} = 10$, $h_{S_2R} = 0.1$, $h_{RS_1} = 0.8$, and $h_{RS_2} = 1.2$. Solid lines represent the BER curves for the TWRC with the channel gain of the DL as 0.6. Dashed lines represent the BER curves for the TWRC with the channel gain of the DL as 0.8.

fading channels, where all the channels are assumed to be Rayleigh distributed. In addition, it is assumed that the channel coefficients h_{S_1R}, h_{S_2R}, h_{RS_1}, and h_{RS_2} have a unit variance, whereas the channel coefficients $h_{S_1S_2}$ and $h_{S_2S_1}$ have a variance of either 0.36 or 1. We consider slow-fading channels, i.e., the channels remain invariant during one transmission period while varying independently from one period to another. Each source transmits one frame in one period, with the frame length of 1000 BPSK symbols. In such a system, the maximum diversity gain of the system is 2. Fig. 7 shows the system BER performance. We can see that, except for the DF, which only has the diversity gain of one, the other protocols can achieve full diversity. The SC-MIF and NC-MIF have a similar BER performance, and they both outperform the EF. The proposed adaptive scheme achieves the best BER performance among all the protocols.

VI. Conclusion

This paper has presented novel mutual-information-based soft forwarding techniques for a TWRC. We have proposed two new protocols, namely, NC-MIF and SC-MIF, and developed the expressions for the received SNR of the two protocols. Based on the comparison of the received SNR of the two protocols, we have derived an adaptive

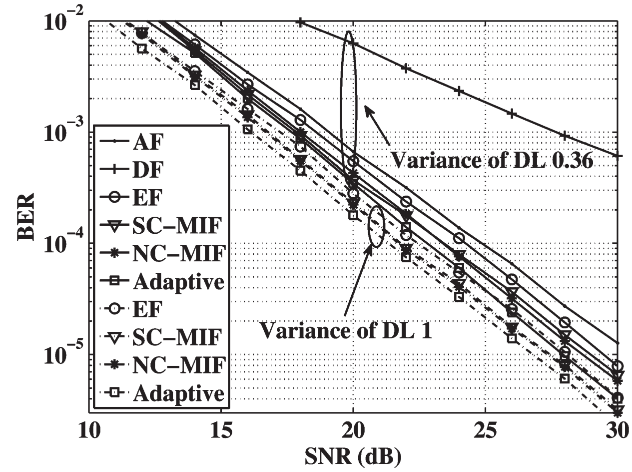

Fig. 7. System BER for fading channels. Solid lines represent the BER curves for the TWRC with the channel variance of the DL as 0.36. Dashed lines represent the BER curves for the TWRC with the channel variance of the DL as 1.

scheme. This adaptive scheme determines which protocol should be selected for a given channel realization to achieve a larger received SNR. In the simulations, we have compared our proposed MIF protocols with the conventional AF, DF and EF protocols. Simulation results show that the proposed adaptive scheme outperforms the other discussed protocols over the fading channels.

Appendix A
Proof of Lemma 1

The event $\tilde{x}_{S_i} = x_{S_i}\Theta(\lambda_i)$ happens with probability $1 - P_{e,x_{S_i}}$, and the event $\tilde{x}_{S_i} = -x_{S_i}\Theta(\lambda_i)$ happens with probability $P_{e,x_{S_i}}$, where $P_{e,x_{S_i}}$ is the detect error probability of x_{S_i} at the relay. We have

$$\begin{aligned}\mathcal{E}[\tilde{x}_{S_i}^2] &= \mathcal{E}\left[(x_{S_i}\Theta(\lambda_i))^2\right](1 - P_{e,x_{S_i}}) + \mathcal{E}\left[(-x_{S_i}\Theta(\lambda_i))^2\right]P_{e,x_{S_i}}\\ &= \mathcal{E}\left[(x_{S_i}\Theta(\lambda_i))^2\right]. \qquad (44)\end{aligned}$$

In addition, we have

$$\begin{aligned}\mathcal{E}^2[\tilde{x}_{S_i}] &= \left(\mathcal{E}\left[x_{S_i}\Theta(\lambda_i)\right](1 - P_{e,x_{S_i}})\mathcal{E}\left[x_{S_i}\Theta(\lambda_i)\right]P_{e,x_{S_i}}\right)^2\\ &= (1 - 2P_{e,x_{S_i}})^2\mathcal{E}^2\left[x_{S_i}\Theta(\lambda_i)\right]. \qquad (45)\end{aligned}$$

According to [8], the soft noise variance can be written as

$$\sigma_{e_{S_i}}^2 = \frac{\mathcal{E}\left[\Theta^2(\lambda_i)\right]}{(1 - 2P_{e,x_{S_i}})^2\mathcal{E}^2\left[\Theta(\lambda_i)\right]} - 1. \qquad (46)$$

Since $\mathcal{E}[\Theta^2(\lambda_i)] \geq \mathcal{E}^2[\Theta(\lambda_i)]$, we can obtain

$$\sigma_{e_{S_i}}^2 \leq \frac{1}{(1 - 2P_{e,x_{S_i}})^2} - 1. \qquad (47)$$

Then, we have

$$\begin{aligned}\lim_{\rho_{S_iR}\to\infty}\sigma_{e_{S_i}}^2 &\leq \lim_{\rho_{S_iR}\to\infty}\frac{1}{(1 - 2P_{e,x_{S_i}})^2} - 1\\ &\overset{(a)}{\leq} \lim_{\rho_{S_iR}\to\infty}\frac{1}{(1 - e^{-\rho_{S_iR}})^2} - 1\\ &= \lim_{\rho_{S_iR}\to\infty}\frac{2e^{-\rho_{S_iR}} - e^{-2\rho_{S_iR}}}{1 - 2e^{-\rho_{S_iR}} + e^{-2\rho_{S_iR}}}\\ &= \lim_{\rho_{S_iR}\to\infty} 2e^{-\rho_{S_iR}} \qquad (48)\end{aligned}$$

where (a) is obtained by the Chernoff upper bound of $Q(\cdot)$ function, i.e., $P_{e,x_{S_i}} = Q(\sqrt{\rho_{S_iR}}) \leq (1/2)e^{-\rho_{S_iR}}$. Based on (48), we thus complete the proof. ■

APPENDIX B
PROOF OF THEOREM 1

From *Lemma 1,* we can see that $\sigma^2_{e_{S_i}}$ exponentially reduces with ρ_{S_iR}. When ρ_{S_1R} and ρ_{S_2R} are large enough, the soft noise variance $\sigma^2_{e_{S_i}}$ is approaching zero. Therefore, the item $\sigma^2_{e_{S_1}}\sigma^2_{e_{S_2}}$ in (29) can be neglected when compared with $\sigma^2_{e_{S_i}}$. In addition, when ρ_{S_iR} is large enough, we have $\tilde{x}_{S_i} \approx x_{S_i}$. According to the calculation of ψ_i, we have $\psi_i \to 1$ when $\tilde{x}_{S_i} \to x_{S_i}$. Then, we have

$$\gamma_{\mathrm{NC}} \approx \frac{1}{\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S}{h^2_{RS_i} \cdot \beta^2_{\mathrm{NC}}}}$$
$$\gamma_{\mathrm{SC}} \approx \frac{1}{\sigma^2_{e_{S_1}} + \sigma^2_{e_{S_2}} + \frac{\sigma^2_S}{h^2_{RS_i} \cdot \beta^2_{\mathrm{SC}}}}. \tag{49}$$

From (49), we can see that the difference between γ_{NC} and γ_{SC} depends on the difference between β_{NC} and β_{SC}. We focus on the denominators of β_{NC} and β_{SC}. Since $1 > \tilde{x}^2_{S_i} > 0$, we have

$$\begin{aligned}\mathcal{E}\left[(\tilde{x}_{S_1} + \tilde{x}_{S_2})\right] &= \mathcal{E}\left[\tilde{x}^2_{S_1} + \tilde{x}^2_{S_2} + 2\tilde{x}_{S_1}\tilde{x}_{S_2}\right] \\ &= \mathcal{E}\left[\tilde{x}^2_{S_1} + \tilde{x}^2_{S_2}\right] > \mathcal{E}\left[\tilde{x}^2_{S_1}\tilde{x}^2_{S_2}\right]\end{aligned} \tag{50}$$

which means that $\beta_{\mathrm{NC}} > \beta_{\mathrm{SC}}$. Hence, we have $\gamma_{\mathrm{NC}} > \gamma_{\mathrm{SC}}$ when both ρ_{S_1R} and ρ_{S_2R} are large enough.

When $\rho_{S_iR} \to \infty$, we neglect $\sigma^2_{e_{S_i}}$ in the expressions of γ_{NC} and γ_{SC}. In addition, as $\lim_{\rho_{S_iR}\to\infty} \tilde{x}^2_{S_i} = 1$, we obtain $\lim_{\rho_{S_iR}\to\infty}(\mathcal{E}[\tilde{x}^2_{S_1} + \tilde{x}^2_{S_2}]/\mathcal{E}[\tilde{x}^2_{S_1}\tilde{x}^2_{S_2}]) = 2$, which indicates that $\lim_{\rho_{S_iR}\to\infty}(\gamma_{\mathrm{NC}}/\gamma_{\mathrm{SC}}) = 2$. ■

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On Max-SINR Receivers for HMT Systems Over a Doubly Dispersive Channel

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***Abstract*—In this paper, a novel receiver for hexagonal multicarrier transmission (HMT) systems based on a maximizing signal-to-interference-plus-noise ratio (Max-SINR) criterion is proposed. Theoretical analyses show that there is a timing offset between the prototype pulses of the proposed Max-SINR receiver and a traditional projection receiver. Meanwhile, the timing offset should be matched to the channel scattering factor of a doubly dispersive (DD) channel. The closed-form timing offset expressions of the prototype pulses for the Max-SINR HMT receiver over the DD channel with different channel scattering functions are derived. Simulation results show that the proposed Max-SINR receiver outperforms the traditional projection scheme and obtains an approximation to the theoretical upper bound SINR performance. Consistent with the SINR performance improvement, the bit error rate (BER) performance of the HMT system has been also further improved by using the proposed Max-SINR receiver. Meanwhile, the SINR performance of the proposed Max-SINR receiver is robust to the channel delay spread estimation errors.**

***Index Terms*—Doubly dispersive (DD) channel, hexagonal multicarrier transmission (HMT) system, maximizing signal-to-interference-plus-noise-ratio (Max-SINR) receiver.**

Manuscript received June 28, 2012; revised September 24, 2012 and November 26, 2012; accepted January 5, 2013. Date of publication January 11, 2013; date of current version June 12, 2013. This work was supported in part by the Jiangsu Province Natural Science Foundation under Grant BK2012055 and Grant BK2011002 and in part by the Young Scientists Preresearch Fund of PLA University of Science and Technology. This paper was presented in part at the 2012 IEEE Global Communications Conference, Anaheim, CA, USA, December 3–7, 2012. The review of this paper was coordinated by Dr. A. J. Al-Dweik.

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Digital Object Identifier 10.1109/TVT.2013.2239674