Capacity Performance of Relay Beamformings for MIMO Multirelay Networks With Imperfect $\mathcal{R}$-$\mathcal{D}$ CSI at Relays

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***Abstract*—In this paper, we consider a dual-hop multiple-input–multiple-output (MIMO) wireless relay network in the presence of imperfect channel state information (CSI), in which a source–destination pair, both equipped with multiple antennas, communicates through a large number of half-duplex amplify-and-forward (AF) relay terminals. We investigate the performance of three linear beamforming schemes when the CSI of the relay-to-destination ($\mathcal{R}$-$\mathcal{D}$) link is not perfect at the relay nodes. The three efficient linear beamforming schemes are based on the matched-filter (MF), zero-forcing (ZF) precoding, and regularized ZF (RZF) precoding techniques, which utilize the CSI of both the $\mathcal{S}$-$\mathcal{R}$ channel and the $\mathcal{R}$-$\mathcal{D}$ channel at the relay nodes. By modeling the $\mathcal{R}$-$\mathcal{D}$ CSI error at the relay nodes as independent complex Gaussian random variables, we derive the ergodic capacities of the three beamformers in terms of instantaneous signal-to-noise ratio. Using the Law of Large Number, we obtain the asymptotic capacities, upon which the optimized MF-RZF is derived. Simulation results show that the asymptotic capacities match the respective ergodic capacities very well. Analysis and simulation results demonstrate that the optimized MF-RZF outperforms MF and MF-ZF for any power of the $\mathcal{R}$-$\mathcal{D}$ CSI error.**

***Index Terms*—Beamforming, capacity, channel state information (CSI), multiple-input–multiple-output (MIMO) relay.**

I. Introduction

RELAY communications can extend the coverage of wireless networks and improve spatial diversity of cooperative systems. Meanwhile, the multiple-input–multiple-output (MIMO) technique is well verified to provide significant improvement in spectral efficiency and link reliability because of the multiplexing and diversity gains [1], [2]. Combining the relaying and MIMO techniques can make use of both advantages to increase the data rate in the cellular edge and extend the network coverage.

MIMO relay networks have been extensively investigated in [3]–[8]. In addition, MIMO multirelay networks have been studied in [9]–[12]. In [9], the authors showed that the corresponding network capacity scales as $C = (M/2)\log(K) + O(1)$, where M is the number of antennas at the source and $K \to \infty$ is the number of relays. The authors also proposed a simple protocol to achieve the upper bound as $K \to \infty$ when perfect channel state information (CSI) of both source-to-relay ($\mathcal{S}$-$\mathcal{R}$) and relay-to-destination ($\mathcal{R}$-$\mathcal{D}$) channels is available at the relay nodes. When CSI is not available at the relays, a simple amplify-and-forward (AF) beamforming protocol is proposed at the relays, but the distributed array gain is not obtained. In [10], a linear relaying scheme based on minimum mean square error fulfilling the target SNRs on different substreams is proposed, and a power-efficient relaying strategy is derived in closed form for a MIMO multirelay network. In [11], [12], the authors designed three relay beamforming schemes based on matrix triangularization, which have superiority over the conventional zero-forcing (ZF) and AF beamformers. The proposed beamforming scheme can fulfill both intranode gain and distributed array gain.

However, most of the works only consider perfect CSI to design beamformers at the relays or successive interference cancelation (SIC) matrices at the destination. For the multirelay networks, imperfect CSI of the $\mathcal{R}$-$\mathcal{D}$ channel is a practical consideration [9]. In particular, knowledge for the CSI of $\mathcal{R}$-$\mathcal{D}$ channels at relays will result in large delay and significant training overhead, because the CSI of $\mathcal{R}$-$\mathcal{D}$ channels at the relays is obtained through feedback links to multiple relays [13].

For the works on imperfect CSI, the ergodic capacity and bit error rate (BER) performance of MIMO with imperfect CSI is considered in [14]–[16]. In [14], the authors investigated the lower and upper bounds of mutual information under CSI error. In [15], the authors studied the BER performance of a MIMO system under combined beamforming and maximal ratio combining with imperfect CSI. In [16], bit error probability is analyzed based on Taylor approximation. Some optimization problem has been investigated with imperfect CSI in [17]–[21]. In [17], the authors maximized a lower bound of capacity by optimally configuring the number of antennas with imperfect

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CSI. In [19], assuming only imperfect CSI at the relay, the optimization problem of maximizing the upper bound of mutual information is presented and solved. In [21], the authors studied the tradeoff between accuracy of channel estimation and data transmission, and showed that the optimal number of training symbols is equal to the number of transmit antennas. In [22], the authors investigated the effects of channel estimation error on the receiver of MIMO AF two-way relaying.

Recently, two efficient relay beamformers for the dual-hop MIMO multirelay networks have been presented in [23], which is based on the matched filter (MF) and regularized ZF (RZF), and utilize QR decomposition (QRD) of the effective system channel matrix at the destination node [24]. The beamformers at the relay nodes can exploit the distributed array gain by diagonalizing both the $\mathcal{S}$-$\mathcal{R}$ and $\mathcal{R}$-$\mathcal{D}$ channels. The QRD can exploit the intranode array gain by SIC detection. These two beamforming schemes not only have advantageous performance than that of conventional schemes such as QR-P-QR or QR-P-ZF in [12] but also have lower complexity because they only need one QRD at destination. However, such advantageous performances are based on perfect CSI, and the imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ is not considered. It is worth to know the capacity performances of these efficient beamforming schemes and the validity of the scaling law in [9], when the imperfect $\mathcal{R}$-$\mathcal{D}$ CSI is present at the relays. In addition, the asymptotic capacities for these beamforming schemes and the optimal regularizing factor in the MF-RZF beamforming are not derived in [23].

Inspired by the works on imperfect CSI and [23], in this paper, we investigate the performance of three efficient beamforming schemes for dual-hop MIMO relay networks under the condition of imperfect $\mathcal{R}$-$\mathcal{D}$ CSI at relays. The three beamforming schemes are based on MF, ZF precoding, and RZF precoding techniques. We first derive the ergodic capacities in terms of the instantaneous CSI of $\mathcal{S}$-$\mathcal{R}$ and $\mathcal{R}$-$\mathcal{D}$. Using the Law of Large Number, we obtain the asymtotic capacities for the three beamformers. Based on the asymptotic capacity of MF-RZF, we derive the optimal regularizing factor. Simulation results show that the asymptotic capacities match with the ergodic capacities very well. Analysis and simulations demonstrate that the capacity of MF-ZF drops fast when the $\mathcal{R}$-$\mathcal{D}$ CSI error increases. We observe that MF-RZF always outperforms MF as in [23] when perfect CSI is available at relays, whereas MF-RZF underperforms MF when $\mathcal{R}$-$\mathcal{D}$ CSI error is present. However, the optimized MF-RZF always outperforms MF for any power of the CSI error. The ceiling effect of capacity is also discussed in this paper.

The remainder of this paper is organized as follows: In Section II, the system model of a dual-hop MIMO multirelay network is introduced. In Section III, we briefly explain the three beamforming schemes and QRD. In Section IV, we derive the instantaneous SNR on each antenna link at the destination with $\mathcal{R}$-$\mathcal{D}$ CSI error at each relay. Using the Law of Large Number, we obtain the asymptotic capacities in Section V. Section VI devotes to simulation results, followed by conclusion in Section VII.

In this paper, boldface lowercase letter and boldface uppercase letter represent vectors and matrices, respectively. Notations $(\mathbf{A})_i$ and $(\mathbf{A})_{i,j}$ denote the ith row and (i,j)th entry of matrix $\mathbf{A}$. Notations $\mathrm{tr}(\cdot)$ and $(\cdot)^H$ denote trace and conjugate transpose operation of a matrix, respectively. Term $\mathbf{I}_N$ is an $N \times N$ identity matrix. $\|\mathbf{a}\|$ stands for the Euclidean norm of a vector $\mathbf{a}$, and $\xrightarrow{w.p.}$ represents convergence with probability one. Finally, we denote the expectation operation by $\mathrm{E}[\cdot]$.

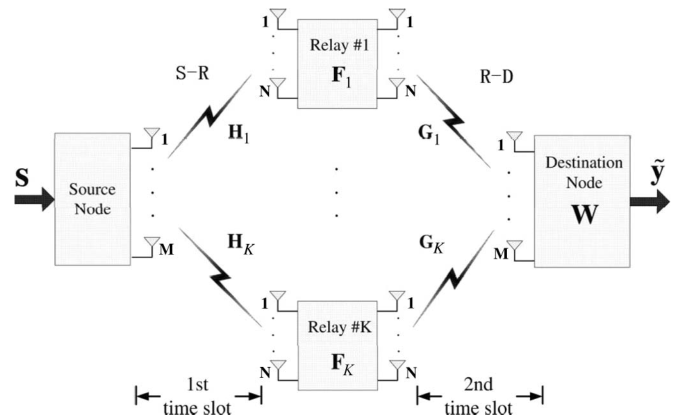

Fig. 1. System model of a dual-hop MIMO multirelay network with relay beamforming and SIC at the destination.

II. System Model

The considered MIMO multirelay network consists of a single source and destination node, both equipped with M antennas, and K N-antenna relay nodes distributed between the source–destination pair, as shown in Fig. 1. When the source node implements spatial multiplexing, the requirement $N \geq M$ must be satisfied if each relay node is supposed to support all the M independent data streams. We consider half-duplex nonregenerative relaying throughout this paper, where it takes two non-overlapping time slots for the data to be transmitted from the source to the destination node via the source-to-relay ($\mathcal{S}$-$\mathcal{R}$) and relay-to-destination ($\mathcal{R}$-$\mathcal{D}$) channels. Due to deep large-scale fading effects produced by the long distance, we assume that there is no direct link between the source and the destination. In this paper, perfect CSI of $\mathcal{S}$-$\mathcal{R}$ and imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ is assumed to be available at relay nodes. In a practical system, each relay needs to transmit training sequences or pilots to acquire the CSI of all the forward channels. Here, we assume that the destination node can estimate the CSI of all the forward channels during the pilot phase. However, due to the large number of relay nodes, a feedback delay and training overhead are expected at each relay. Thus, perfect CSI of $\mathcal{R}$-$\mathcal{D}$ is hard to be obtained at relays. We assume that the CSI of $\mathcal{R}$-$\mathcal{D}$ is imperfect with a Gaussian distributed error at relay nodes as in [25], and the destination node only knows the statistical distribution of these $\mathcal{R}$-$\mathcal{D}$ CSI errors.

In the first time slot, the source node broadcasts the signal to all the relay nodes through $\mathcal{S}$-$\mathcal{R}$ channels. Let the $M \times 1$ vector $\mathbf{s}$ be the transmit signal vector satisfying the power constraint $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = (P/M)\mathbf{I}_M$, where P is defined as the transmit power at the source node. Let $\mathbf{H}_k \in \mathbb{C}^{N\times M}$, $(k = 1, \ldots, K)$ stand for the $\mathcal{S}$-$\mathcal{R}$ MIMO channel matrix from the source node to the kth relay node. All the relay nodes are supposed to be located in a cluster. Then, all the $\mathcal{S}$-$\mathcal{R}$ channels $\mathbf{H}_1, \ldots, \mathbf{H}_K$ can be supposed to be independently and identically distributed (i.i.d.) and experience the same Rayleigh flat fading. Assume that the entries of $\mathbf{H}_k$ is zero-mean complex Gaussian random variables with variance one. Then, the corresponding received signal at the kth relay can be written as

$$\mathbf{r}_k = \mathbf{H}_k\mathbf{s} + \mathbf{n}_k \tag{1}$$

where the term $\mathbf{n}_k$ is the spatio-temporally white zero-mean complex additive Gaussian noise vector, independent across k, with the covariance matrix $\mathrm{E}\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_1^2\mathbf{I}_N$. Therefore, noise variance σ_1^2 represents the noise power at each relay node.

In the second time slot, first, each relay node performs linear processing by multiplying $\mathbf{r}_k$ with an $N \times N$ beamforming matrix $\mathbf{F}_k$. This $\mathbf{F}_k$ is based on its perfect $\mathcal{S}$-$\mathcal{R}$ CSI $\mathbf{H}_k$ and imperfect $\mathcal{R}$-$\mathcal{D}$ CSI $\widehat{\mathbf{G}}_k$. Consequently, the signal vector sent from the kth relay node is

$$\mathbf{t}_k = \mathbf{F}_k\mathbf{r}_k. \tag{2}$$

From more practical consideration, we assume that each relay node has its own power constraint satisfying $\mathrm{E}\{\mathbf{t}_k^H\mathbf{t}_k\} \le Q$, which is independent of power P. Hence, a power constraint condition of $\mathbf{t}_k$ can be derived as

$$p(\mathbf{t}_k) = \mathrm{tr}\left\{\mathbf{F}_k\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{F}_k^H\right\} \le Q. \tag{3}$$

After linear relay beamforming processing, all the relay nodes simultaneously forward their data to the destination. Thus, the signal vector received by the destination can be expressed as

$$\mathbf{y} = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{t}_k + \mathbf{n}_d = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{F}_k\mathbf{H}_k\mathbf{s} + \sum_{k=1}^{K}\mathbf{G}_k\mathbf{F}_k\mathbf{n}_k + \mathbf{n}_d \tag{4}$$

where $\mathbf{G}_k$, under the same assumption as $\mathbf{H}_k$, is the $M \times N$ $\mathcal{R}$-$\mathcal{D}$ channel matrix between the kth relay node and the destination. $\mathbf{n}_d \in \mathbb{C}^M$, satisfying $\mathrm{E}\{\mathbf{n}_d\mathbf{n}_d^H\} = \sigma_2^2\mathbf{I}_M$, denotes the zero-mean white circularly symmetric complex additive Gaussian noise vector at the destination node with the noise power σ_2^2.

III. Relay Beamforming and QR Detection

In this section we will consider MF, ZF, and RZF beamforming at relays. The destination applied QRD detection to successively cancel the interference from other antennas.

A. Beamforming at Relay Nodes

Denote $\widehat{\mathbf{G}}_k$ as the imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ at the kth relay. When MF is chosen, beamforming at the kth relay is

$$\mathbf{F}_k^{\mathrm{MF}} = \widehat{\mathbf{G}}_k^H\mathbf{H}_k^H \tag{5}$$

where we set MF as both the receiver of the $\mathcal{S}$-$\mathcal{R}$ channel and the precoder of the $\mathcal{R}$-$\mathcal{D}$ channel.

When MF-ZF is chosen, beamforming at the kth relay is

$$\mathbf{F}_k^{\mathrm{MF-ZF}} = \widehat{\mathbf{G}}_k^H\left(\widehat{\mathbf{G}}_k\widehat{\mathbf{G}}_k^H\right)^{-1}\mathbf{H}_k^H \tag{6}$$

where we set ZF as the precoder of the $\mathcal{R}$-$\mathcal{D}$ channel. Here, the requirement that $N \ge M$ is also indispensable for the inversion of $(\widehat{\mathbf{G}}_k\widehat{\mathbf{G}}_k^H)$.

When MF-RZF is chosen, beamforming at the kth relay is

$$\mathbf{F}_k^{\mathrm{MF-RZF}} = \widehat{\mathbf{G}}_k^H\left(\widehat{\mathbf{G}}_k\widehat{\mathbf{G}}_k^H + \alpha_k\mathbf{I}_M\right)^{-1}\mathbf{H}_k^H \tag{7}$$

where we set RZF as the precoder of the $\mathcal{R}$-$\mathcal{D}$ channel. Note that MF-ZF is a special case of MF-RZF when $\alpha_k = 0$.

B. QRD and SIC Detection

A QRD detector is utilized as the destination receiver $\mathbf{W}$ in this paper, which is proved to be asymptotically equivalent to that of the maximum-likelihood detector [24]. Let $\sum_{k=1}^{K}\mathbf{G}_k\mathbf{F}_k\mathbf{H}_k = \mathbf{H}_{\mathcal{SD}}$. Then, (4) can be rewritten as

$$\mathbf{y} = \mathbf{H}_{\mathcal{SD}}\mathbf{s} + \widehat{\mathbf{n}} \tag{8}$$

where $\mathbf{H}_{\mathcal{SD}}$ represents the effective channel between the source and destination node, and $\widehat{\mathbf{n}} = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{F}_k\mathbf{n}_k + \mathbf{n}_d$ is the effective noise vector cumulated from the noise $\mathbf{n}_k$ at the kth relay node and the noise vector $\mathbf{n}_d$ at the destination. Finally, in order to cancel the interference from other antennas, QRD of the effective channel is implemented as

$$\mathbf{H}_{\mathcal{SD}} = \mathbf{Q}_{\mathcal{SD}}\mathbf{R}_{\mathcal{SD}} \tag{9}$$

where $\mathbf{Q}_{\mathcal{SD}}$ is an $M \times M$ unitary matrix, and $\mathbf{R}_{\mathcal{SD}}$ is an $M \times M$ right upper triangular matrix. Therefore, the QRD detector at the destination node is chosen as $\mathbf{W} = \mathbf{Q}_{\mathcal{SD}}^H$, and the signal vector after QRD detection becomes

$$\tilde{\mathbf{y}} = \mathbf{Q}_{\mathcal{SD}}^H\mathbf{y} = \mathbf{R}_{\mathcal{SD}}\mathbf{s} + \mathbf{Q}_{\mathcal{SD}}^H\mathbf{n}. \tag{10}$$

A power control factor $\widehat{\rho_k}$ is set with $\mathbf{F}_k$ in (2) to guarantee that the kth relay transmit power is equal to Q. The transmit signal from each relay node after linear beamforming and power control becomes

$$\mathbf{t}_k = \widehat{\rho_k}\mathbf{F}_k\mathbf{r}_k \tag{11}$$

where the power control factor $\widehat{\rho_k}$ can be derived from (3) as

$$\widehat{\rho_k} = \left(Q/\mathrm{tr}\left\{\mathbf{F}_k\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{F}_k^H\right\}\right)^{\frac{1}{2}}. \tag{12}$$

From the cut-set theorem in network information theory [4], the upper bound capacity of the dual-hop MIMO relay networks is

$$C_{\mathrm{upper}} = \mathrm{E}_{\{\mathbf{H}_k\}_{k=1}^K}\left\{\frac{1}{2}\log\det\left(\mathbf{I}_M + \frac{P}{M\sigma_1^2}\sum_{k=1}^{K}\mathbf{H}_k^H\mathbf{H}_k\right)\right\}. \tag{13}$$

IV. Ergodic Capacities

In this section, we will derive the ergodic capacities of the three relay beamformers under the condition of imperfect $\mathcal{R}$-$\mathcal{D}$ CSI at relays. These ergodic capacities are based on the instantaneous SNR of the source-to-destination channel. For the kth relay, we denote the accurate $\mathcal{R}$-$\mathcal{D}$ channel matrix as $\mathbf{G}_k$. We assume the $\mathcal{R}$-$\mathcal{D}$ CSI error caused by large delay and reciprocity mismatch to be complex Gaussian distributed and model the imperfect $\mathcal{R}$-$\mathcal{D}$ CSI received at the kth relay as [25]

$$\widehat{\mathbf{G}}_k = \mathbf{G}_k + e\boldsymbol{\Omega}_k \tag{14}$$

where $\boldsymbol{\Omega}_k$ is a matrix independent of $\mathbf{G}_k$, whose entries is i.i.d. zero-mean complex Gaussian, with unity variance, and e is the gain of channel loss. Therefore, the power of CSI error is e^2 [14]. In this paper, we consider $e \ll 1$.

A. Preliminaries

In this section, we give some lemmas associated with $\boldsymbol{\Omega}_k$, which will be used to derive the instantaneous SNR of the source-to-destination channel.

Lemma 1: $\mathrm{E}[\mathrm{tr}(\mathbf{A}\boldsymbol{\Omega}_k^H)\boldsymbol{\Omega}_k] = \mathbf{A}$ for any complex matrix $\mathbf{A} \in \mathbb{C}^{M\times N}$.

Proof: Note that $\mathrm{E}[(\boldsymbol{\Omega}_k)_{i,j}] = 0$ and $\mathrm{E}[(\boldsymbol{\Omega}_k)_{i,j}(\boldsymbol{\Omega}_k)_{i,j}^*] = 1$. Since the entries in $\boldsymbol{\Omega}_k$ are independent, we have

$$\begin{aligned}&\mathrm{E}\left[\left(\mathrm{tr}\left(\mathbf{A}\boldsymbol{\Omega}_k^H\right)\boldsymbol{\Omega}_k\right)_{i,j}\right]\\ &\quad= \mathrm{E}\left[\Sigma_{m=1}^{M}\Sigma_{l=1}^{N}(\mathbf{A})_{m,l}(\boldsymbol{\Omega}_k)_{m,l}^*(\boldsymbol{\Omega}_k)_{i,j}\right] = (\mathbf{A})_{i,j}.\end{aligned} \tag{15}$$

■

Lemma 2: $\mathrm{E}[\mathrm{tr}(\mathbf{A}\boldsymbol{\Omega}_k^H)\mathrm{tr}(\mathbf{B}\boldsymbol{\Omega}_k)] = \mathrm{tr}(\mathbf{AB})$, for any $\mathbf{A} \in \mathbb{C}^{M\times N}$ and $\mathbf{B} \in \mathbb{C}^{N\times M}$.

Proof:

$$\begin{aligned}&\mathrm{E}\left[\mathrm{tr}\left(\mathbf{A}\boldsymbol{\Omega}_k^H\right)\mathrm{tr}(\mathbf{B}\boldsymbol{\Omega}_k)\right]\\ &\quad= \mathrm{E}\left[\left(\Sigma_{m=1}^{M}\Sigma_{n=1}^{N}(\mathbf{A})_{m,n}(\boldsymbol{\Omega}_k)_{m,n}^*\right)\right.\\ &\qquad\times\left.\left(\Sigma_{n=1}^{N}\Sigma_{m=1}^{M}(\mathbf{B})_{n,m}(\boldsymbol{\Omega}_k)_{m,n}\right)\right]\\ &\quad= \Sigma_{m=1}^{M}\Sigma_{n=1}^{N}(\mathbf{A})_{m,n}(\mathbf{B})_{n,m} = \mathrm{tr}(\mathbf{AB}).\end{aligned} \tag{16}$$

■

Lemma 3: $\mathrm{E}[\boldsymbol{\Omega}_k\mathbf{A}\boldsymbol{\Omega}_k^H] = \mathrm{tr}(\mathbf{A})\mathbf{I}_M$ for any $\mathbf{A} \in \mathbb{C}^{M\times M}$.

Proof:

$$\begin{aligned}\mathrm{E}\left[(\boldsymbol{\Omega}_k\mathbf{A}\boldsymbol{\Omega}_k^H)_{i,j}\right] &= \sum_{m=1}^{M}\sum_{n=1}^{M}\mathrm{E}\left[(\boldsymbol{\Omega}_k)_{i,m}(\mathbf{A})_{m,n}\left(\boldsymbol{\Omega}_k^H\right)_{n,j}\right]\\ &= \sum_{m=1}^{M}\sum_{n=1}^{M}(\mathbf{A})_{m,n}\mathrm{E}\left[(\boldsymbol{\Omega}_k)_{i,m}(\boldsymbol{\Omega}_k)_{j,n}^*\right]\\ &= \mathrm{tr}(\mathbf{A})\delta[m-n]\end{aligned} \tag{17}$$

where $\delta[x] = 1$ for $x = 0$ and 0 otherwise. ■

B. Instantaneous SNR of MF Beamforming

In the presence of imperfect $\mathcal{R}$-$\mathcal{D}$ CSI at relays, the MF beamforming at the kth relay is

$$\mathbf{F}_k = \widehat{\mathbf{G}}_k^H\mathbf{H}_k^H. \tag{18}$$

Thus, the signal vector received by the destination is

$$\mathbf{y} = \sum_{k=1}^{K}\widehat{\rho}_k\mathbf{G}_k\widehat{\mathbf{G}}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{s} + \mathbf{n}_k\right) + \mathbf{n}_d. \tag{19}$$

Since $e \ll 1$ and the kth relay node only knows an imperfect $\mathcal{R}$-$\mathcal{D}$ CSI $\widehat{\mathbf{G}}_k$, the power control factor becomes

$$\begin{aligned}\widehat{\rho_k} &= \left(Q/\mathrm{tr}\left\{\widehat{\mathbf{G}}_k^H\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\widehat{\mathbf{G}}_k\right\}\right)^{\frac{1}{2}}\\ &\cong \rho_k^{\frac{1}{2}}\left(1 - \frac{ev_k}{2u_k}\right)\end{aligned} \tag{20}$$

where

$$u_k = \mathrm{tr}\left(\mathbf{G}_k^H\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\mathbf{G}_k\right) \tag{21}$$

$$v_k = \mathrm{tr}\left(\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\left(\mathbf{G}_k\boldsymbol{\Omega}_k^H + \boldsymbol{\Omega}_k\mathbf{G}_k^H\right)\right) \tag{22}$$

$$\rho_k = \left(\frac{Q}{u_k}\right)^{\frac{1}{2}}. \tag{23}$$

Using Lemma 1 and Lemma 2, through some manipulations and omitting some relatively small terms, (19) becomes

$$\begin{aligned}\mathbf{y} \cong \sum_{k=1}^{K}\rho_k\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{s} + e\sum_{k=1}^{K}\rho_k\mathbf{G}_k\boldsymbol{\Omega}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{s}\\ + \sum_{k=1}^{K}\rho_k\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{n}_k + \mathbf{n}_d.\end{aligned} \tag{24}$$

We observe that the second term in the right-hand side of (24) is the additional noises introduced by the $\mathcal{R}$-$\mathcal{D}$ CSI error. The third term is the noise introduced by the noises at each relays. We denote the last three terms in the right-hand side of (24) as

$$\widehat{\mathbf{n}} = e\sum_{k=1}^{K}\rho_k\mathbf{G}_k\boldsymbol{\Omega}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{s} + \sum_{k=1}^{K}\rho_k\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{n}_k + \mathbf{n}_d \tag{25}$$

and refer to it as the effective postprocessing noise. Denote

$$\mathbf{H}_{\mathcal{SD},MF} = \sum_{k=1}^{K}\rho_k\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{H}_k \tag{26}$$

as the effective transmitting matrix of the whole network. Then, (24) becomes

$$\mathbf{y} = \mathbf{H}_{\mathcal{SD},MF}\mathbf{s} + \widehat{\mathbf{n}}. \tag{27}$$

Using QRD $\mathbf{H}_{\mathcal{SD},MF} = \mathbf{Q}_{\mathrm{MF}}\mathbf{R}_{\mathrm{MF}}$ at the destination, we have

$$\mathbf{Q}_{\mathrm{MF}}^H\mathbf{y} = \mathbf{R}_{\mathrm{MF}}\mathbf{s} + \mathbf{Q}_{\mathrm{MF}}^H\widehat{\mathbf{n}}. \tag{28}$$

Thus, the power of the mth transmitted signal stream becomes $P/M(\mathbf{R}_{\mathrm{MF}})_{m,m}^2$. We now calculate the covariance matrix of the effective postprocessing noise $\widehat{\mathbf{n}}$ as

$$\begin{aligned}\mathrm{E}\left[\mathbf{Q}_{\mathrm{MF}}^H\widehat{\mathbf{n}}\left(\mathbf{Q}_{\mathrm{MF}}^H\widehat{\mathbf{n}}\right)^H\right] &= e^2\frac{P}{M}\sum_{k=1}^{K}\rho_k^2\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\right)\mathbf{G}_k^H\mathbf{Q}_{\mathrm{MF}} \\ &\quad + \sigma_1^2\sum_{k=1}^{K}\rho_k^2\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{G}_k\mathbf{G}_k^H\mathbf{Q}_{\mathrm{MF}} + \sigma_2^2\mathbf{I}_M\end{aligned} \tag{29}$$

where we used Lemma 3. Thus, the effective noise power of the mth data stream is

$$\begin{aligned}\mathrm{E}\left[(\widehat{\mathbf{n}}\widehat{\mathbf{n}}^H)_{m,m}\right] &= e^2\frac{P}{M}\sum_{k=1}^{K}\rho_k^2\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\right)\left\|\left(\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\right)_m\right\|^2 \\ &\quad + \sigma_1^2\sum_{k=1}^{K}\rho_k^2\left\|\left(\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\right)_m\right\|^2 + \sigma_2^2.\end{aligned} \tag{30}$$

Thus, the postprocessing SNR per symbol of the mth stream can be expressed as (31), shown at the bottom of the page. Compared to the covariance of the effective noise under the condition of perfect CSI ($e = 0$), we see that the covariance of the effective postprocessing noise under the condition of imperfect $\mathcal{R}$-$\mathcal{D}$ CSI consists of an additional term, which is related to transmit power P, $\mathcal{R}$-$\mathcal{D}$ CSI error gain e, and the CSI of $\mathcal{S}-\mathcal{R}$ and $\mathcal{R}-\mathcal{D}$. We call this term as channel-error-generated noise power (CEG-noise power).

C. Instantaneous SNR of MF-ZF Beamforming

In the presence of imperfect CSI of $\mathcal{R}$-$\mathcal{D}$, the ZF beamforming at the kth relay is

$$\widehat{\mathbf{F}}_k = \widehat{\mathbf{G}}_k^{\dagger}\mathbf{H}_k^H \tag{32}$$

where $\widehat{\mathbf{G}}_k^{\dagger} = \widehat{\mathbf{G}}_k^H(\widehat{\mathbf{G}}_k\widehat{\mathbf{G}}_k^H)^{-1}$ is the pseudoinverse of matrix $\widehat{\mathbf{G}}_k$. The signal vector received by the destination is

$$\mathbf{y} = \sum_{k=1}^{K}\widehat{\rho_k}\mathbf{G}_k\widehat{\mathbf{G}}_k^{\dagger}\mathbf{H}_k^H(\mathbf{H}\mathbf{s} + \mathbf{n}_k) + \mathbf{n}_d. \tag{33}$$

Since $e \ll 1$, the pseudoinverse of matrix $\mathbf{G}_k$ can be approximated using the Taylor expansion as

$$\widehat{\mathbf{G}}_k^{\dagger} \cong \left(\mathbf{I}_N - e\mathbf{G}_k^{\dagger}\mathbf{\Omega}_k\right)\mathbf{G}_k^{\dagger} \tag{34}$$

where $\mathbf{G}_k^{\dagger} = \mathbf{G}_k^H(\mathbf{G}_k\mathbf{G}_k^H)^{-1}$ is the pseudoinverse of $\mathbf{G}_k$. The relay power control factor can be approximated as

$$\begin{aligned}\widehat{\rho_k} &= \left(Q/\mathrm{tr}\left\{\widehat{\mathbf{G}}_k^{\dagger}\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\left(\widehat{\mathbf{G}}_k^{\dagger}\right)^H\right\}\right)^{\frac{1}{2}} \\ &\cong \rho_k^{\frac{1}{2}}\left(1 + \frac{ev_k}{2u_k}\right)\end{aligned} \tag{35}$$

where

$$u_k = \mathrm{tr}\left(\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\left(\mathbf{G}_k\mathbf{G}_k^H\right)^{-1}\right) \tag{36}$$

$$\begin{aligned}v_k = \mathrm{tr}\Bigg(&\mathbf{H}_k^H\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H + \sigma_1^2\mathbf{I}_N\right)\mathbf{H}_k\left(\mathbf{G}_k\mathbf{G}_k^H\right)^{-1} \\ &\times\left(\mathbf{G}_k\mathbf{\Omega}_k^H + \mathbf{\Omega}_k\mathbf{G}_k^H\right)\left(\mathbf{G}_k\mathbf{G}_k^H\right)^{-1}\Bigg)\end{aligned} \tag{37}$$

$$\rho_k = \left(\frac{Q}{u_k}\right)^{\frac{1}{2}}. \tag{38}$$

By substituting (34) and (35) into (33) and omitting some relatively small terms, the received signal at the destination can be further written as

$$\begin{aligned}\mathbf{y} \cong \sum_{k=1}^{K}\rho_k\mathbf{H}_k^H\mathbf{H}_k\mathbf{s} + \sum_{k=1}^{K}\rho_k\mathbf{H}_k^H\mathbf{n}_k \\ - e\sum_{k=1}^{K}\rho_k\mathbf{\Omega}_k\mathbf{G}_k^{\dagger}\mathbf{H}_k^H\mathbf{H}_k\mathbf{s} + \mathbf{n}_d,\end{aligned} \tag{39}$$

where we used Lemma 1 and Lemma 2. Just as the case of MF beamforming, the third term in the right-hand side of (39) is caused by the $\mathcal{R}$-$\mathcal{D}$ CSI error. The effective postprocessing noise is the last three terms in the right-hand side of (39), i.e.,

$$\widehat{\mathbf{n}} = \sum_{k=1}^{K}\rho_k\mathbf{H}_k^H\mathbf{n}_k - e\sum_{k=1}^{K}\rho_k\mathbf{\Omega}_k\mathbf{G}_k^{\dagger}\mathbf{H}_k^H\mathbf{H}_k\mathbf{s} + \mathbf{n}_d. \tag{40}$$

Denote

$$\mathbf{H}_{\mathcal{SD},MF-ZF} = \sum_{k=1}^{K}\rho_k\mathbf{H}_k^H\mathbf{H}_k. \tag{41}$$

as the effective transmitting matrix. Using QRD as $\mathbf{H}_{\mathcal{SD},MF-ZF} = \mathbf{Q}_{\mathrm{MF-ZF}}\mathbf{R}_{\mathrm{MF-ZF}}$, we have

$$\mathbf{y} = \mathbf{H}_{\mathcal{SD},MF-ZF}\mathbf{x} + \widehat{\mathbf{n}} = \mathbf{Q}_{\mathrm{MF-ZF}}\mathbf{R}_{\mathrm{MF-ZF}}\mathbf{x} + \widehat{\mathbf{n}}. \tag{42}$$

$$\gamma_m^{\mathrm{MF}} = \frac{\frac{P}{M}(\mathbf{R}_{\mathrm{MF}})_{m,m}^2}{\underbrace{e^2\frac{P}{M}\sum_{k=1}^{K}\rho_k^2\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\right)\|\left(\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\right)_m\|^2}_{\text{channel-error-generated noise power}} + \sigma_1^2\sum_{k=1}^{K}\rho_k^2\left\|\left(\mathbf{Q}_{\mathrm{MF}}^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\right)_m\right\|^2 + \sigma_2^2} \tag{31}$$

The covariance matrix of the effective postprocessing noise is

$$\begin{aligned}\mathrm{E}\left[\mathbf{Q}_{\mathrm{MF-ZF}}^{H}\widehat{\mathbf{n}}\left(\mathbf{Q}_{\mathrm{MF-ZF}}^{H}\widehat{\mathbf{n}}\right)^{H}\right] &= e^{2}\sigma_{1}^{2}\sum_{k=1}^{K}\rho_{k}^{2}\mathrm{tr}\left(\left(\mathbf{H}_{k}^{H}\mathbf{H}_{k}\right)^{2}\left(\mathbf{G}_{k}\mathbf{G}_{k}^{H}\right)^{-1}\right)\mathbf{I}_{M}\\ &+\sigma_{1}^{2}\sum_{k=1}^{K}\rho_{k}^{2}\mathbf{Q}_{\mathrm{MF-ZF}}^{H}\mathbf{H}_{k}^{H}\mathbf{H}_{k}\mathbf{Q}_{\mathrm{MF-ZF}}+\sigma_{2}^{2}\mathbf{I}_{M}\end{aligned} \tag{43}$$

where we used Lemma 3 in the derivation. The preceding formulas result in the postprocessing SNR per symbol of the mth stream as (44), shown at the bottom of the page. Once again, the imperfect $\mathcal{R}$-$\mathcal{D}$ CSI generates the CEG-noise power term in the postprocessing SNR.

D. Instantaneous SNR of MF-RZF Beamforming

To simplify the analysis, consider $\alpha_k = \alpha$. In the presence of imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ at each relay, the MF-RZF beamforming at the kth relay is

$$\widehat{\mathbf{F}}_{k}=\widehat{\mathbf{G}}_{k}^{H}\left(\widehat{\mathbf{G}}_{k}\widehat{\mathbf{G}}_{k}^{H}+\alpha\mathbf{I}_{M}\right)^{-1}\mathbf{H}_{k}^{H}. \tag{45}$$

When $e \ll 1$, using Taylor expansion, we have

$$\begin{aligned}&\widehat{\mathbf{G}}_{k}^{H}\left(\widehat{\mathbf{G}}_{k}\widehat{\mathbf{G}}_{k}^{H}+\alpha\mathbf{I}_{M}\right)^{-1}\\ &\cong\left(\mathbf{G}_{k}^{\dagger}-e\mathbf{G}_{k}^{\dagger}\boldsymbol{\Omega}_{k}\mathbf{G}_{k}^{\dagger}\right)\left(\mathbf{I}_{M}-\alpha\mathbf{G}_{k}^{\alpha}+e\alpha\mathbf{G}_{k}^{\alpha}\mathbf{G}_{k}^{e}\mathbf{G}_{k}^{\alpha}\right)\\ &=\mathbf{G}_{k}^{H}\left(\mathbf{G}_{k}\mathbf{G}_{k}^{H}+\alpha\mathbf{I}_{M}\right)^{-1}-e\mathbf{G}_{k}^{E}\end{aligned} \tag{46}$$

where

$$\mathbf{G}_{k}^{\alpha}=\left(\mathbf{G}_{k}\mathbf{G}_{k}^{H}+\alpha\mathbf{I}_{M}\right)^{-1}\quad\mathbf{G}_{k}^{E}=\mathbf{G}_{k}^{\dagger}\boldsymbol{\Omega}_{k}\mathbf{G}_{k}^{H}\mathbf{G}_{k}^{\alpha}. \tag{47}$$

If α increases, the norm of entries of $\mathbf{G}_k^E$ will decrease. Thus, the impact of imperfect CSI will be reduced. In addition, the power control factor $\widehat{\rho_k}$ will increase, resulting in a reduced unnormalized transmit power at relays. However, if α becomes too large, interference from other antennas at the destination will be considerable. In this paper, we try to obtain the optimal α to maximize the SINR on each data stream.

Let $\mathbf{G}_k^e = \mathbf{G}_k\boldsymbol{\Omega}_k^H + \boldsymbol{\Omega}_k\mathbf{G}_k^H$. The relay power control factor can be approximated as

$$\widehat{\rho_{k}}\cong\rho_{k}^{\frac{1}{2}}\left(1+\frac{ev_{k}}{2u_{k}}\right) \tag{48}$$

where

$$u_{k}=\mathrm{tr}\left(\mathbf{H}_{k}^{H}\left(\frac{P}{M}\mathbf{H}_{k}\mathbf{H}_{k}^{H}+\sigma_{1}^{2}\mathbf{I}_{N}\right)\mathbf{H}_{k}\left(\mathbf{G}_{k}^{\alpha}-\alpha\left(\mathbf{G}_{k}^{\alpha}\right)^{2}\right)\right) \tag{49}$$

$$\begin{aligned}v_{k}=\mathrm{tr}\Bigg(&\mathbf{H}_{k}^{H}\left(\frac{P}{M}\mathbf{H}_{k}\mathbf{H}_{k}^{H}+\sigma_{1}^{2}\mathbf{I}_{N}\right)\mathbf{H}_{k}\mathbf{G}_{k}^{\alpha}\\ &\times\left(\mathbf{G}_{k}^{e}\mathbf{G}_{k}^{\alpha}\mathbf{G}_{k}\mathbf{G}_{k}^{H}+\mathbf{G}_{k}\mathbf{G}_{k}^{H}\mathbf{G}_{k}^{\alpha}\mathbf{G}_{k}^{e}-\mathbf{G}_{k}^{e}\right)\mathbf{G}_{k}^{\alpha}\Bigg)\end{aligned} \tag{50}$$

$$\rho_{k}=\left(\frac{Q}{u_{k}}\right)^{\frac{1}{2}}. \tag{51}$$

Using Lemma 1 and Lemma 2, substituting (45)–(48) into (4) and omitting some relatively small terms, the received signal at destination can be expanded as

$$\begin{aligned}\mathbf{y}=&\sum_{k=1}^{K}\rho_{k}\left(\mathbf{I}_{M}-\alpha\mathbf{G}_{k}^{\alpha}\right)\mathbf{H}_{k}^{H}\mathbf{H}_{k}\mathbf{s}+\sum_{k=1}^{K}\rho_{k}\left(\mathbf{I}_{M}-\alpha\mathbf{G}_{k}^{\alpha}\right)\mathbf{H}_{k}^{H}\mathbf{n}_{k}\\ &+e\sum_{k=1}^{K}\rho_{k}\boldsymbol{\Omega}_{k}\mathbf{G}_{k}^{H}\mathbf{G}_{k}^{\alpha}\mathbf{H}_{k}^{H}\mathbf{H}_{k}\mathbf{s}+\mathbf{n}_{d}.\end{aligned} \tag{52}$$

Denote

$$\mathbf{H}_{\mathcal{SD},MF-RZF}=\sum_{k=1}^{K}\rho_{k}\left(\mathbf{I}_{M}-\alpha\mathbf{G}_{k}^{\alpha}\right)\mathbf{H}_{k}^{H}\mathbf{H}_{k}. \tag{53}$$

Then, (52) becomes

$$\begin{aligned}\mathbf{y}&=\mathbf{H}_{\mathcal{SD},MF-RZF}\mathbf{x}+\widehat{\mathbf{n}}\\ &\triangleq\mathbf{Q}_{\mathcal{SD},MF-RZF}\mathbf{R}_{\mathcal{SD},MF-RZF}\mathbf{x}+\widehat{\mathbf{n}}\end{aligned} \tag{54}$$

where the effective postprocessing noise

$$\begin{aligned}\widehat{\mathbf{n}}=&\,e\sum_{k=1}^{K}\rho_{k}\boldsymbol{\Omega}_{k}\mathbf{G}_{k}^{H}\mathbf{G}_{k}^{\alpha}\mathbf{H}_{k}^{H}\mathbf{H}_{k}\mathbf{s}\\ &+\sum_{k=1}^{K}\rho_{k}\left(\mathbf{I}_{M}-\alpha\mathbf{G}_{k}^{\alpha}\right)\mathbf{H}_{k}^{H}\mathbf{n}_{k}+\mathbf{n}_{d}.\end{aligned} \tag{55}$$

$$\gamma_{m}^{\mathrm{MF-ZF}}=\frac{\frac{P}{M}\left(\mathbf{R}_{\mathrm{MF-ZF}}\right)_{m,m}^{2}}{\underbrace{e^{2}\frac{P}{M}\sum_{k=1}^{K}\rho_{k}^{2}\mathrm{tr}\left(\left(\mathbf{H}_{k}^{H}\mathbf{H}_{k}\right)^{2}\left(\mathbf{G}_{k}\mathbf{G}_{k}^{H}\right)^{-1}\right)}_{\text{channel−error−generated noise power}}+\sigma_{1}^{2}\sum_{k=1}^{K}\rho_{k}^{2}\left\|\left(\mathbf{Q}_{\mathrm{MF-ZF}}^{H}\mathbf{H}_{k}^{H}\right)_{m}\right\|^{2}+\sigma_{2}^{2}} \tag{44}$$

The covariance of effective postprocessing noise is

$$\begin{aligned}\mathrm{E}&\left[\mathbf{Q}_{\mathrm{MF-RZF}}^{H}\widehat{\mathbf{n}}\left(\mathbf{Q}_{\mathrm{MF-RZF}}^{H}\widehat{\mathbf{n}}\right)^{H}\right]\\&=e^{2}\frac{P}{M}\sum_{k=1}^{K}\rho_k^2\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\mathbf{G}_k^{\alpha}\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\right)\\&\quad+\sigma_1^2\sum_{k=1}^{K}\rho_k^2\mathbf{Q}_{\mathrm{MF-RZF}}^{H}\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\mathbf{H}_k^H\\&\quad\mathbf{H}_k\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)^H\mathbf{Q}_{\mathrm{MF-RZF}}+\sigma_2^2\mathbf{I}_M\end{aligned}\tag{56}$$

where we used Lemma 3. Then, the postprocessing SNR per symbol of the mth stream is calculated as (57), shown at the bottom of the page.

E. Ergodic Capacity

The ergodic capacity is derived by summing up all the data rates on each antenna link, i.e.,

$$C=\mathrm{E}_{\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^{K}}\left\{\frac{1}{2}\sum_{m=1}^{M}\log_2\left(1+\gamma_m\right)\right\}.\tag{58}$$

We can see, from the instantaneous SNRs in (31), (44), and (57) that a ceiling effect [25] can be expected when the transmit power-to-noise ratio (PNR) (P/σ_1^2) and the relay power-to-noise ratio (QNR) $(Q/\sigma_2^2)\to\infty$. This is because γ_m cannot tend to infinity due to the CEG-noise power term in the denominator of γ_m, which will also increase as the effective power does. Simulations will confirm the ceiling effect.

V. Asymptotic Capacities and the Optimized MF-RZF

To further investigate the capacity performance under imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ channel at relays, we derive asymptotic capacities for large K. To simplify the analysis, we assume a fixed power control factor for all relays in this section. Simulation results will validate this assumption. The power control factor is chosen as the average one, i.e.,

$$\rho_k=\left(Q/\mathrm{E}\left[\mathrm{tr}\left(\mathbf{F}_k\left(\frac{P}{M}\mathbf{H}_k\mathbf{H}_k^H+\sigma_1^2\mathbf{I}_N\right)\mathbf{F}_k^H\right)\right]\right)^{\frac{1}{2}}.\tag{59}$$

Then

$$\rho_{\mathrm{MF}}=\left(Q/\left(\left(P(M+N)+M\right)N^2\right)\right)^{\frac{1}{2}}\tag{60}$$

$$\rho_{\mathrm{MF-ZF}}=\left(Q(N-M)/\left(\left(P(M+N)+M\right)\right)\right)^{\frac{1}{2}}\tag{61}$$

$$\rho_{\mathrm{MF-RZF}}=\left(Q/\left(\left(P(M+N)N+MN\right)\right)\mathrm{E}\left[\frac{\lambda}{(\lambda+\alpha)^2}\right]\right)^{\frac{1}{2}}.\tag{62}$$

In (62), we used the decomposition $\mathbf{G}\mathbf{G}^H=\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{\Lambda}=\mathrm{diag}\{\lambda_1,\ldots,\lambda_M\}$ and $\mathbf{U}$ is independent to each other [27]. For the case of large K, using the Law of Large Number, we have

$$\mathbf{H}_{\mathcal{SD},MF}\xrightarrow{w.p.}K\left(\mathrm{E}\left[\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_k^H\mathbf{H}_k\right]\right)=KN^2\mathbf{I}_M\tag{63}$$

$$\mathbf{H}_{\mathcal{SD},MF-ZF}\xrightarrow{w.p.}K\left(\mathrm{E}\left[\mathbf{H}_k^H\mathbf{H}_k\right]\right)=KN\mathbf{I}_M\tag{64}$$

$$\begin{aligned}\mathbf{H}_{\mathcal{SD},MF-RZF}&\xrightarrow{w.p.}K\left(\mathrm{E}\left[\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\mathbf{H}_k^H\mathbf{H}_k\right]\right)\\&=KN\mathrm{E}\left[\mathbf{U}\mathrm{diag}\left\{\frac{\lambda_1}{\lambda_1+\alpha},\ldots,\frac{\lambda_M}{\lambda_M+\alpha}\right\}\mathbf{U}^H\right].\end{aligned}\tag{65}$$

Note that

$$\begin{aligned}\mathrm{E}&\left[\left(\mathbf{U}\mathrm{diag}\left\{\frac{\lambda_1}{\lambda_1+\alpha},\ldots,\frac{\lambda_M}{\lambda_M+\alpha}\right\}\mathbf{U}^H\right)_{m,n}\right]\\&=\mathrm{E}\left[\Sigma_{j=1}^{M}(\mathbf{U})_{m,j}\frac{\lambda_j}{\lambda_j+\alpha}(\mathbf{U})_{n,j}^{*}\right]=\mathrm{E}\left[\frac{\lambda}{\lambda+\alpha}\right]\delta[m-n].\end{aligned}\tag{66}$$

Since the asymptotic effective channel matrices are all diagonal, we have $\mathbf{Q}\xrightarrow{w.p.}\mathbf{I}_M$ for large K. Since $(\mathbf{G}_k\mathbf{G}_k^H)^{-1}$ is a complex inverse Wishart distribution with N degrees of freedom [26]. We have $\mathrm{E}[(\mathbf{G}_k\mathbf{G}_k^H)^{-1}]=M/N-M$ [28] and $\mathrm{E}[(\mathbf{H}_k^H\mathbf{H}_k)^2]=(MN+N^2)\mathbf{I}_M$ [29]. Using the Law of Large Number, for a large K, we have the asymptotic capacities in (67)–(69), shown at the bottom of the next page. From the asymptotic capacities, we see that they satisfy the scaling law in [9], i.e., $C=(M/2)\log(K)+O(1)$ for large K. Obviously, the capacities will increase as K increases and decrease when e increases. In addition, the CEG-noise power in $C_{\mathrm{MF-ZF}}$ is the largest among those in the three asymptotic capacities, resulting in worse capacity performance of MF-ZF, which will be confirmed by simulations.

From (67)–(69), it is observed that, when QNR (Q/σ_2^2) grows to infinite for a fixed PNR (P/σ_1^2), the capacities of the three beamformers will reach a limit, which demonstrates the "ceiling effect" that will be confirmed by simulations. When PNR $(=\mathrm{QNR})$ grows to infinite, the capacities will linearly

$$\gamma_m^{\mathrm{MF-RZF}}=\frac{\frac{P}{M}\left(\mathbf{R}_{\mathrm{MF-RZF}}\right)_{m,m}^2}{\underbrace{e^2\frac{P}{M}\sum_{k=1}^{K}\rho_k^2\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\mathbf{G}_k^{\alpha}\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\right)}_{\text{channel-error-generated noise power}}+\sigma_1^2\sum_{k=1}^{K}\rho_k^2\left\|\left(\mathbf{Q}_{\mathrm{MF-RZF}}^H\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\mathbf{H}_k^H\right)_m\right\|^2+\sigma_2^2}\tag{57}$$

grow with PNR (decibels) for perfect $\mathcal{R}$-$\mathcal{D}$ CSI or reach a limit for imperfect $\mathcal{R}$-$\mathcal{D}$ CSI, which also demonstrates the "ceiling effect" that will be confirmed by simulations.

Consider the $\mathcal{R}$-$\mathcal{D}$ CSI error varying with the number of relays (K). Let $e = \sigma_q + K\sigma_d$, where σ_q denotes the quantization error due to limited bits of feedback, and σ_d denotes the error weight caused by feedback delay in each relay. Substituting such e into the asymptotic capacities, it will generate terms $O(1/K) + O(K)$ in the denominators of the asymptotic SNR, which implies that the denominator will reach a minimum value at some K. Therefore, there exists an optimal number of relays to maximize the asymptotic capacities, which will be confirmed by simulations.

Note that (68) holds when $N > M$ because $\mathrm{E}[\mathrm{tr}(\mathbf{G}\mathbf{G}^H)^{-1}] = \infty$ and $\mathrm{E}[\rho_{\mathrm{ZF}}^{-2}] = \infty$ for $M = N$ [27]. A bad capacity performance of MF-ZF can be expected due to the infinite expectation of CEG-noise power when $M = N$, which will be confirmed by simulations. To optimize the regularizing factor α, we shall use the following approximations:

$$\mathrm{E}\left[\frac{\lambda}{\lambda+\alpha}\right] = \frac{1}{KM}\Sigma_{k=1}^{K}\Sigma_{m=1}^{M}\frac{\lambda_{m,k}}{\lambda_{m,k}+\alpha}$$

$$\mathrm{E}\left[\frac{\lambda}{(\lambda+\alpha)^2}\right] = \frac{1}{KM}\Sigma_{k=1}^{K}\Sigma_{m=1}^{M}\frac{\lambda_{m,k}}{(\lambda_{m,k}+\alpha)^2}$$

$$\mathrm{E}\left[\frac{\lambda^2}{(\lambda+\alpha)^2}\right] = \frac{1}{KM}\Sigma_{k=1}^{K}\Sigma_{m=1}^{M}\frac{\lambda_{m,k}^2}{(\lambda_{m,k}+\alpha)^2}$$

where $\lambda_{m,k}$ denotes the mth eigenvalue of $\mathbf{G}_k\mathbf{G}_k^H$. Take the derivative of (69) with respect to α, and manipulate as [27]. Then, we get the optimal regularizing factor as

$$\alpha_{\mathrm{opt}} = \frac{\frac{P(M+N)+M}{Q}\sigma_2^2 + e^2PK(M+N)}{K\sigma_1^2}. \qquad (70)$$

VI. Simulation Results

In this section, numerical results are carried out to validate what we draw from the analysis in the previous sections for the three relay beamforming schemes. The advantage of the optimized MF-RZF beamformer is also demonstrated.

A. Capacity Versus Number of Relays

In Fig. 2, we compare the ergodic/asymptotic capacities of the three beamforming schemes at the relays. We consider perfect CSI of the $\mathcal{S}$-$\mathcal{R}$ channel at relays and imperfect CSI of $\mathcal{R}$-$\mathcal{D}$ at the relays. The solid curves are ergodic capacities, and the dashed curves are the asymptotic capacities. Capacities versus K is demonstrated when $M = 4$, $N = 6$, and PNR = QNR = 10 dB. When the CSI of $\mathcal{R}$-$\mathcal{D}$ is perfect at relays $(e = 0)$, MF-RZF is the best choice. When the CSI of $\mathcal{R}$-$\mathcal{D}$ is imperfect at relays, MF-ZF has apparently the worst performance. MF and MF-RZF have almost the same performance. The poor performance of MF-ZF under

$$C_{\mathrm{MF}} \xrightarrow{w.p.} \frac{M}{2}\log_2\left(1+\frac{\frac{P}{M}(KN^2)^2}{e^2\frac{PK}{M}\mathrm{E}\left[\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\right)\left(\mathbf{G}_k\mathbf{G}_k^H\right)_{m,m}\right]+\sigma_1^2K\mathrm{E}\left[\left(\mathbf{H}_k^H\mathbf{H}_k\left(\mathbf{G}_k\mathbf{G}_k^H\right)^2\right)_{m,m}\right]+\sigma_2^2\rho_{\mathrm{MF}}^{-2}}\right)$$

$$= \frac{M}{2}\log_2\left(1+\frac{PK^2N}{\left(e^2P+\sigma_1^2\right)KM\frac{M+N}{N}+\frac{PM(M+N)+M^2}{QN}\sigma_2^2}\right) \qquad (67)$$

$$C_{\mathrm{MF-ZF}} \xrightarrow{w.p.} \frac{M}{2}\log_2\left(1+\frac{\frac{P}{M}(KN)^2}{e^2\frac{PK}{M}\mathrm{E}\left[\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\left(\mathbf{G}_k\mathbf{G}_k^H\right)^{-1}\right)\right]+\sigma_1^2K\mathrm{E}\left[\left(\mathbf{H}_k^H\mathbf{H}_k\right)_{m,m}\right]+\sigma_2^2\rho_{\mathrm{MF-ZF}}^{-2}}\right)$$

$$= \frac{M}{2}\log_2\left(1+\frac{PK^2N}{e^2PK\frac{M(M+N)}{N-M}+KM\sigma_1^2+\frac{PM(M+N)+M^2}{Q(N-M)}\sigma_2^2}\right) \qquad (68)$$

$$C_{\mathrm{MF-RZF}} \xrightarrow{w.p.} \frac{M}{2}$$

$$\times\log_2\left(1+\frac{\frac{P}{M}\left(KN\mathrm{E}\left[\frac{\lambda}{\lambda+\alpha}\right]\right)^2}{e^2\frac{PK}{M}\mathrm{E}\left[\mathrm{tr}\left(\left(\mathbf{H}_k^H\mathbf{H}_k\right)^2\mathbf{G}_k^{\alpha}\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\right)\right]+\sigma_1^2K\mathrm{E}\left[\left(\mathbf{H}_k^H\mathbf{H}_k\left(\left(\mathbf{I}_M-\alpha\mathbf{G}_k^{\alpha}\right)\right)^2\right)_{m,m}\right]+\sigma_2^2\rho_{\mathrm{MF-RZF}}^{-2}}\right)$$

$$= \frac{M}{2}\log_2\left(1+\frac{PK^2N\left(\mathrm{E}\left[\frac{\lambda}{\lambda+\alpha}\right]\right)^2}{e^2PKM(M+N)\mathrm{E}\left[\frac{\lambda}{(\lambda+\alpha)^2}\right]+KM\mathrm{E}\left[\frac{\lambda^2}{(\lambda+\alpha)^2}\right]\sigma_1^2+\frac{PM(M+N)+M^2}{Q}\mathrm{E}\left[\frac{\lambda}{(\lambda+\alpha)^2}\right]\sigma_2^2}\right) \qquad (69)$$

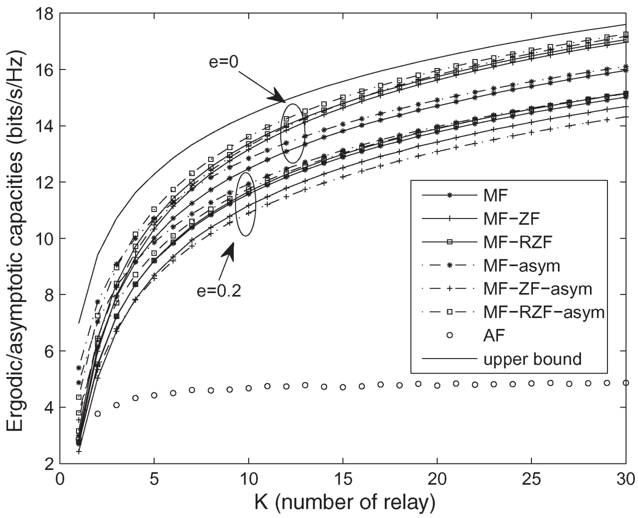

Fig. 2. Ergodic/asymptotic capacity versus K (number of relays) ($M = 4, N = 6, \mathrm{PNR} = \mathrm{QNR} = 10$ dB). In this figure, MF-RZF is fixed with $\alpha = 0.5$.

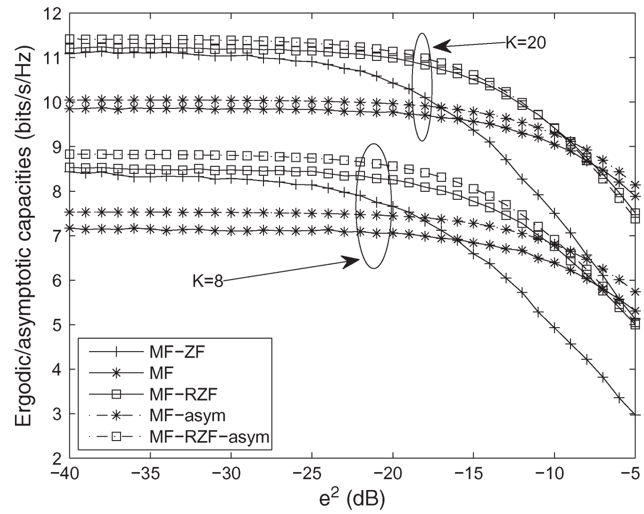

Fig. 3. Capacity versus the power of $\mathcal{R}$-$\mathcal{D}$ CSI error ($N = M = 4, \mathrm{PNR} = 5$ dB, $\mathrm{QNR} = 20$ dB). Asymptotic capacities match with ergodic capacities very well for $K = 20$. Since $\mathrm{E}[\mathrm{tr}(\mathbf{GG}^H)^{-1}] = \infty$ for $M = N$, asymptotic capacity for MF-ZF does not exist. MF-RZF is fixed with $\alpha = 0.5$.

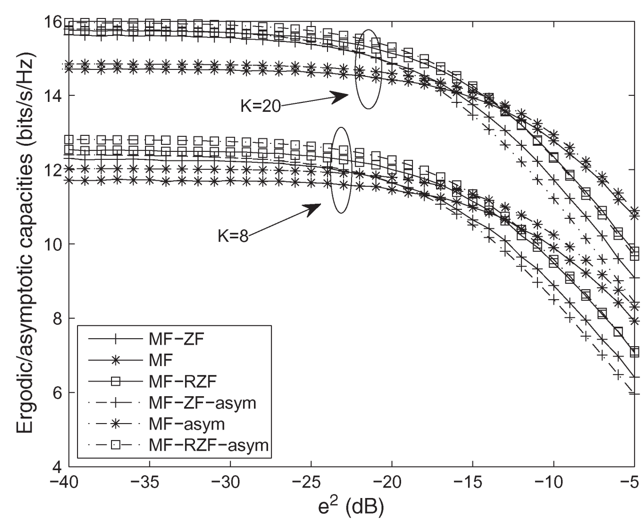

Fig. 4. Capacity versus the power of $\mathcal{R}$-$\mathcal{D}$ CSI error ($M = 4$, $N = 6, \mathrm{PNR} = 10$ dB, $\mathrm{QNR} = 10$ dB). MF-RZF is fixed with $\alpha = 0.5$.

imperfect CSI comes from the inverse Wishart distribution term in its CEG-noise power, which can be clearly seen from the asymptotic capacity (68). Note that the asymptotic capacity of MF-ZF is not tight enough to its ergodic capacity since its power control factor has an inverse Wishart distribution in the denominator. Thus, the dynamic power control factor of MF-ZF has a much larger variance than those of MF and MF-RZF. Since we use an average power control factor, instead of a dynamic power control factor, to derive the asymptotic capacity, it results in a gap between the two types of capacities. We find that the ergodic capacities still satisfy the scaling law in [9], i.e., $C = (M/2)\log(K) + O(1)$ for large K in the presence of $\mathcal{R}$-$\mathcal{D}$ CSI error. This is also consistent with the asymptotic capacities for the three beamformers. Note that AF keeps as the worst relaying strategy, which cannot utilize the distributed array gain.

B. Capacity Versus Power of CSI Error

In Fig. 3, we show the ergodic/asymptotic capacities versus power of $\mathcal{R}$-$\mathcal{D}$ CSI error for $M = N = 4$. The ergodic capacity of MF-ZF quickly drops when CSI error occurs, which validates what we observed in (68). We see that the asymptotic capacities of MF and MF-RZF match well with their ergodic capacities for $K = 20$, which also shows that static power allocation has almost the same performance as dynamic power allocation for large K. Similarly, Fig. 4 is the case for $M = 4$ and $N = 6$. It is observed that, when $N > M$, MF-ZF and MF-RZF obviously outperform MF with perfect $\mathcal{R}$-$\mathcal{D}$ CSI at relays, whereas their performance will be upside down in the presence of $\mathcal{R}$-$\mathcal{D}$ CSI error. Since ergodic capacities match well with the asymptotic capacities, in the rest of the figures, we only plot the asymptotic capacities to demonstrate the advantage of optimized MF-RZF. Fig. 5 shows that the optimized MF-RZF consistently has the best performance for any powers of CSI error.

C. Capacity Versus PNR and QNR

A more apparent superiority of optimized MF-RZF can be observed in Fig. 6, where we fix the SNR of the $\mathcal{S}$-$\mathcal{R}$ channel (PNR) and increase the SNR of the $\mathcal{R}$-$\mathcal{D}$ channel (QNR). Note that the capacity in this scenario is limited by the $\mathcal{S}$-$\mathcal{R}$ channels [9], which is called "ceiling phenomenon" in [25]. We also include the conventional optimized RZF ($\alpha = M\sigma_2^2/Q = M/QNR$) for perfect CSI [27] in Fig. 6 and refer to it as the conventional MF-RZF. We find that the optimized MF-RZF outperforms MF, MF-ZF, the conventional MF-RZF, and MF-RZF with fixed α and has the highest ceiling for QNR >1 dB. When QNR increases, the α in the conventional MF-RZF approaches zero. Thus, MF-RZF will converge to the MF-ZF beamformer. In Fig. 7, we simultaneously increase PNR and QNR. When $\mathcal{R}$-$\mathcal{D}$ CSI is perfect at relays, the capacities of all the three beamformings linearly grow with the PNR ($=$ QNR) in decibels. When $\mathcal{R}$-$\mathcal{D}$ CSI error occurs, we see

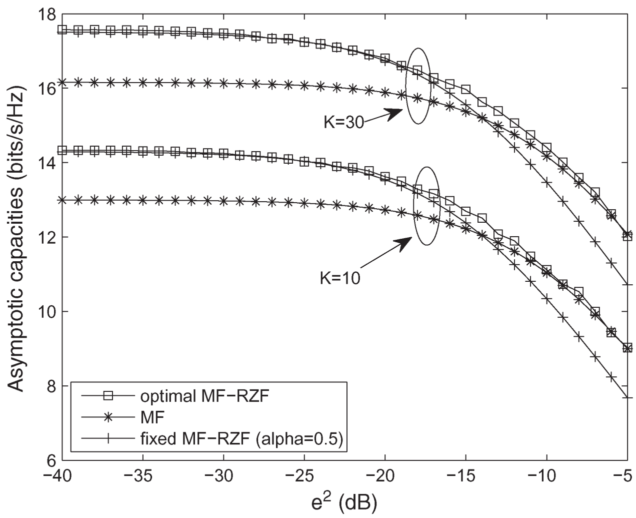

Fig. 5. Capacity versus the power of $\mathcal{R}$-$\mathcal{D}$ CSI error for MF and different MF-RZFs $(M = 4, N = 6, \mathrm{PNR} = 10\ \mathrm{dB}, \mathrm{QNR} = 20\ \mathrm{dB})$. The optimized MF-RZF outperforms MF for any power of $\mathcal{R}$-$\mathcal{D}$ CSI error.

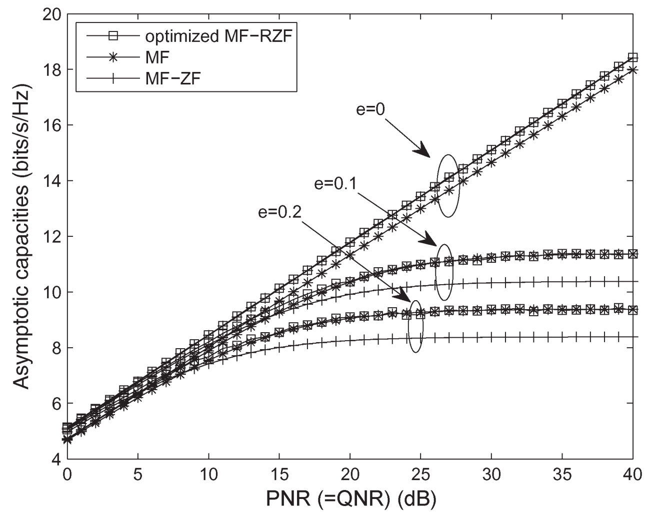

Fig. 7. Capacity versus PNR $(= \mathrm{QNR})$ for $e = 0, 0.1, 0.2$. All schemes experience the ceiling effect in the presence of $\mathcal{R}$-$\mathcal{D}$ CSI error.

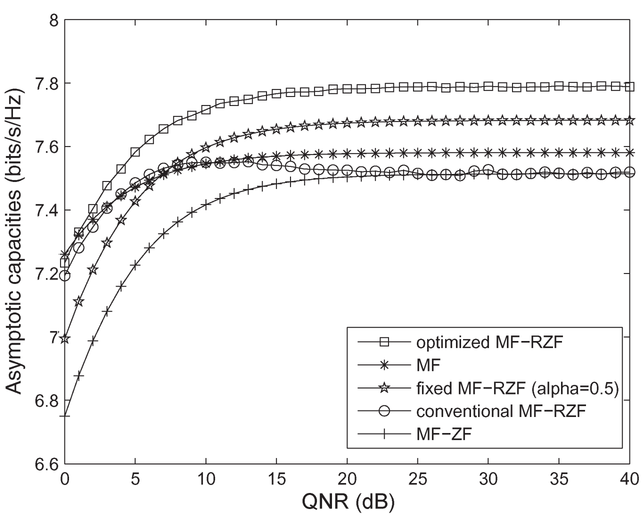

Fig. 6. Capacity versus QNR $(M = 2, N = 4, \mathrm{PNR} = 10\ \mathrm{dB}, e = 0.1)$. Optimized MF-RZF outperforms the MF-RZF with fixed α, MF, the conventional MF-RZF, and MF-ZF in the presence of $\mathcal{R}$-$\mathcal{D}$ CSI error. All schemes experience the ceiling effect.

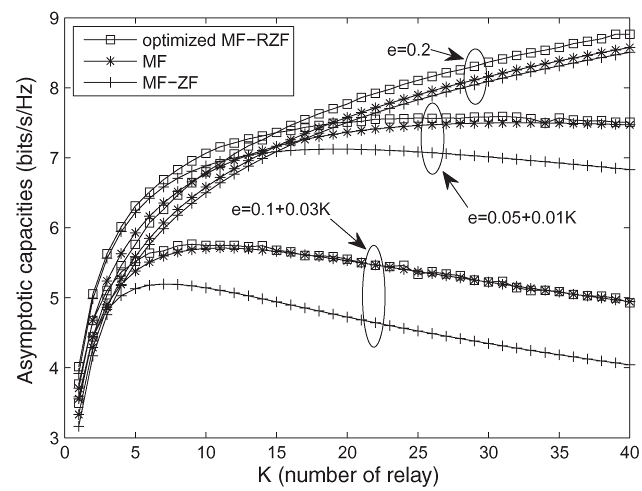

Fig. 8. Capacity versus K for $e = 0.2$ and $e = \sigma_q + K\sigma_d$. Each scheme has an optimal number of relays to maximize the capacity in the presence of CSI error. The optimal point of K is obvious if σ_d is big.

different capacity limits for different CSI error powers. This is the "ceiling effect" discussed in Section V.

D. Capacity Versus Relay Number for Dynamic CSI Error

Fig. 8 shows the capacities versus the relay number K when CSI error $e = \sigma_q + K\sigma_d$. We also include the capacities versus relay number K with constant CSI error for comparison in Fig. 8. It is observed that the capacity achieves maximum at some optimal relay number in the presence of CSI error. When σ_d is bigger, the optimal K is smaller.

VII. Conclusion

In this paper, considering three efficient relay beamforming schemes based on the MF, MF-ZF, and MF-RZF techniques, we have investigated the effect of imperfect $\mathcal{R}$-$\mathcal{D}$ CSI at relays to the capacity in a dual-hop MIMO multirelay network with AF relaying protocol. Supposing Gaussian distributed $\mathcal{R}$-$\mathcal{D}$ CSI error and perfect $\mathcal{S}$-$\mathcal{R}$ CSI at relays, we give the ergodic capacities of the three beamformers in terms of instantaneous SNR. Using the Law of Large Number, we have derived the asymptotic capacities of the three beamformers for a large number of relays, upon which the optimized MF-RZF is derived. Simulation results show that the asymptotic capacities match with the respective ergodic capacities very well. Analysis and simulations demonstrate that the MF-ZF beamformer has the worst performance in the presence of $\mathcal{R}$-$\mathcal{D}$ CSI error. The capacity of MF-RZF drops faster than that of MF as the $\mathcal{R}$-$\mathcal{D}$ CSI error increases and has a small performance loss, compared to that of MF, whereas the optimized MF-RZF consistently has the best performance for any power of $\mathcal{R}$-$\mathcal{D}$ CSI error. Although we consider imperfect $\mathcal{R}$-$\mathcal{D}$ CSI caused by limited feedback and large delay, imperfect $\mathcal{S}$-$\mathcal{R}$ CSI is still a practical consideration when estimation error presents, and we will consider this case as our future work.

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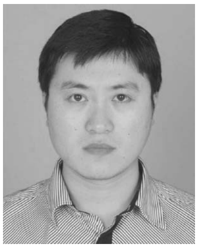

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