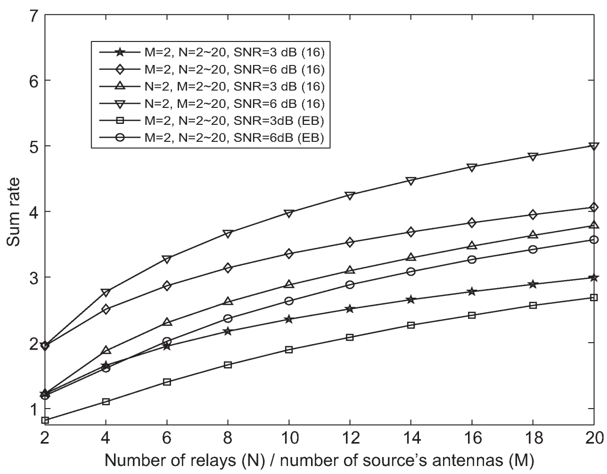

Fig. 6. Sum rate versus number of relays N and number of source antennas M with input SNR = 3, 6 dB using different relay amplifying matrices in (5) and (20) for two cases: case (1) $M = 2$ with N varying from 2 to 12 and case (2) $N = 2$ with M varying from 2 to 12.

matrix. The last difference is that the model in [1] uses one single relay of N multiple-antenna elements, whereas this paper employs N dispersed N relays, each with a single-antenna element. The same Rayleigh fading channel model is used for a fair comparison.

Fig. 5 shows the SER performance and the relay transmit power versus input SNR using (5) and the existing scheme in [1] with/without normalization of the relay transmit power for $M = 2$ and $N = 3$. Simulation results indicate that the transmit power p_r at the relay in [1] can be greater than 1, e.g., $p_r = 9 - 12$, when $M = 2$ and $N = 3$. This relay transmit power in [1] is significantly higher than $p_r = 1$ in the proposed scheme. This is why the proposed scheme shows a worse performance than the scheme in [1] at a low SNR, as shown in Fig. 4. When the relay transmit power in [1] is normalized as the proposed scheme in this paper for a fair comparison, then the proposed scheme shows much better performance (e.g., 4.1 dB at SER $= 10^{-2}$) than the scheme in [1] for both low and high SNRs, as shown in Fig. 5. The difference between the two results becomes larger as either the input SNR or N increases.

Fig. 6 shows the sum of the achievable rate $\mathcal{R}$ versus the number of relays N and the number of source antennas M with input SNR = 3, 6 dB using the optimum and the simplified relay amplifying matrices in (5) and (20) for the two cases: case (1) $M = 2$ with N varying from 2 to 12 and case (2) $N = 2$ with M varying from 2 to 12, respectively. The higher $\mathcal{R}$ is observed with the optimum relay amplifying matrix and the optimum transmit/receive beamforming vectors in (5)–(9) than with the simplified relay amplifying matrix and the EB vectors in (20).

V. Conclusion

This paper has presented an MMSE-based optimum relay amplifying matrix for the single-antenna multiple dispersed relays and optimum transmit/receive beamforming vectors for the multiple-antenna sources explicitly and iteratively in the two-way AF MIMO relay network under transmit power constraints at the two sources and relays. This paper also presented a simplified relay amplifying matrix with EB vectors. It was observed that both the proposed optimum and simplified schemes show significantly better performance than the existing schemes in [1]–[3] under the same environments. Finally, the MMSE cost function values are less than 2 at all times, regardless of the number of relays. The two-way AF wireless relay system proposed in this paper can be applicable in a future relay network with relays of CRAN to improve spectral efficiency.

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Compressive Soft Forwarding in Network-Coded Multiple-Access Relay Channels

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***Abstract*—In this paper, we design a novel estimate-and-forward (EF) protocol in multiple-access relay channels (MARCs) where two correlated sources transmit their information to the common destination with the help of a relay. With the application of network coding (NC) at the relay, the network-coded soft symbols contain redundant information due to the correlated sources, which can be compressed before being forwarded. Specifically, we first derive the value of the network-coded soft symbol by considering the correlation of the two sources. Due to this correlation, the majority of soft symbol values are close to one. Then, we convert network-coded soft symbol vector into a sparse vector, which is suitable for compression by using the compressive sensing (CS). Next, we analyze the performance of the proposed EF protocol by comparing the soft symbol value and received signal-to-noise ratio (SNR) with the conventional EF protocol. Simulations show that our protocol can achieve the same bit error rate (BER) as the conventional EF protocol with proper compression rates, and it consumes much less transmission time due to the compression.**

***Index Terms*—Compressive sensing, correlated sources, multiple access, network coding, soft forwarding.**

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I. Introduction

Two classical relay protocols, namely amplify-and-forward (AF) and decode-and-forward (DF) [1], are two protocols frequently considered for wireless relaying systems [2]. However, the AF suffers from noise amplification, and the DF propagates erroneous decisions to the destination. Consequently, soft information forwarding (SIF) has been proposed to achieve a better error performance by forwarding intermediate soft decisions at the relay [3]–[6]. The soft information can be in the form of log-likelihood ratio (LLR) [3], soft symbol [4], or soft mutual information [5]. Specifically, a soft-symbol-based SIF protocol in [4], also known as estimate-and-forward (EF), obtains soft symbols by calculating minimum mean square errors (MMSE) of the received symbols at the relay.

However, these SIF protocols are investigated in one-way or two-way relay channels, which are not directly applicable for general multisource relay channels, where multiple sources communicate with their destination with the help of the relay. Multiple-access relay channels (MARCs) are common in wireless networks, such as wireless sensor networks (WSNs) [7]. Wireless network coding (NC) has been considered to enhance the network performance in MARCs [8], [9]. Additionally, in WSNs where adjacent sensors observe the same phenomenon, be it the humidity, the temperature, or the pressure, the gathered information is correlated with each other [7], [10]. In this scenario, when NC is applied at the relay, the network-coded symbols of the correlated sources contain redundant information, which can be compressed before being forwarded to the destination. If we still adopt conventional relay protocols to the correlated sources, it will lead to a big amount of throughput loss.

In this paper, we investigate the SIF protocols in the MARC with NC and correlated sources. We will consider using compressive sensing (CS) [11]–[13] to compress and forward the network-coded soft information at the relay. Distributed CS design has mostly been considered for the application of WSNs. For instance, works in [14] and [15] both assumed the sensor networks where sensors gather highly correlated information. Specifically, we design a novel EF-based SIF protocol by compressing the soft network-coded symbols at the relay with CS, which is referred to as correlated EF with CS (CEF-CS). We first develop the expression of the network-coded soft symbol in the MARC with the correlated sources. Then, we investigate how to transform the network-coded soft symbol vector into a sparse vector, which is suitable for the CS. Next, we analyze the performance of the CEF-CS protocol by comparing the soft symbol value and the received signal-to-noise ratio (SNR) with the conventional EF protocol. Simulations show that our CEF-CS protocol can achieve the same bit error rate (BER) as the EF protocol with some properly selected compression rate, and it consumes much less transmission time due to the compression.

II. System Model

Consider a MARC with two correlated sources, one relay, and one destination, where the two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ broadcast their messages to the common destination $\mathcal{D}$, with the help of a half-duplex relay $\mathcal{R}$. Each transmission period consists of three phases. In the first phase, $\mathcal{S}_1$ broadcasts its message, and in the second phase, $\mathcal{S}_2$ broadcasts its message, to the relay and the destination. After the first two phases, the relay forms the network-coded message based on the signals from the two sources, which is then forwarded to the destination during the third phase. At the end of each transmission period, the destination decodes the messages of the two sources, based on the signals from the sources and the relay.

We denote by $h_{i\mathcal{R}}$, $i=1,2$, $h_{i\mathcal{D}}$, and $h_{\mathcal{R}\mathcal{D}}$, the channel coefficients between $\mathcal{S}_i$ and $\mathcal{R}$, between $\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively, and denote by $d_{i\mathcal{R}}$, $d_{i\mathcal{D}}$, and $d_{\mathcal{R}\mathcal{D}}$ the distances between $\mathcal{S}_i$ and $\mathcal{R}$, between $\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively. We assume that $h_{i\mathcal{R}}$, $h_{i\mathcal{D}}$, and $h_{\mathcal{R}\mathcal{D}}$ are independent and identically Rayleigh distributed with the channel gains denoted by $\lambda_{i\mathcal{R}}$, $\lambda_{i\mathcal{D}}$, and $\lambda_{\mathcal{R}\mathcal{D}}$, respectively. These channel gains are related to the corresponding distances with the attenuation exponent γ, i.e., $\lambda_{i\mathcal{R}} = 1/(d_{i\mathcal{R}})^\gamma$, $\lambda_{i\mathcal{D}} = 1/(d_{i\mathcal{D}})^\gamma$, and $\lambda_{\mathcal{R}\mathcal{D}} = 1/(d_{\mathcal{R}\mathcal{D}})^\gamma$. We consider quasi-static fading channels, i.e., the channel coefficients are constant during one transmission period and change independently from one period to another. We consider the binary phase-shift keying (BPSK) modulation due to low transmission power and simple hardware implementation of the sensor nodes, and we do not consider source and channel coding. In each transmission period, we assume that each source transmits l independent and identically distributed (i.i.d.) BPSK symbols. Thus, the symbol vector of $\mathcal{S}_i$ is denoted by $\boldsymbol{x}_i = (x_i^1,\ldots,x_i^l)^T$, $x_i^j \in \{\pm 1\}$ and $j \in \{1,\ldots,l\}$, with the power E_i. The received signals at the relay and at the destination from $\mathcal{S}_i$ can be expressed as

$$\boldsymbol{y}_{i\mathcal{R}} = h_{i\mathcal{R}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{R}}, \qquad \boldsymbol{y}_{i\mathcal{D}} = h_{i\mathcal{D}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{D}} \quad (1)$$

respectively, where the vectors $\boldsymbol{y}_{i\mathcal{R}}$ and $\boldsymbol{y}_{i\mathcal{D}}$ consist of l received signals, i.e., $\boldsymbol{y}_{i\mathcal{R}} = (y_{i\mathcal{R}}^1,\ldots,y_{i\mathcal{R}}^l)^T$ and $\boldsymbol{y}_{i\mathcal{D}} = (y_{i\mathcal{D}}^1,\ldots,y_{i\mathcal{D}}^l)^T$, the vector $\boldsymbol{n}_{i\mathcal{R}} = (n_{i\mathcal{R}}^1,\ldots,n_{i\mathcal{R}}^l)^T$ consists of l additive white Gaussian noise (AWGN) samples at the relay, and the vector $\boldsymbol{n}_{i\mathcal{D}} = (n_{i\mathcal{D}}^1,\ldots,n_{i\mathcal{D}}^l)^T$ consists of l AWGN samples at the destination. We assume that all the noise samples at the relay and the destination are i.i.d. Gaussian variables with a zero mean and variance σ^2, and the sources' power satisfies $E_1 = E_2 = E_\mathcal{S}$. We define the SNR as $\rho \triangleq E_\mathcal{S}/\sigma^2$.

After receiving $\boldsymbol{y}_{i\mathcal{R}}$, the relay detects $\boldsymbol{x}_i$ and forms the network-coded symbol vector denoted by $\boldsymbol{x}_\mathcal{R} = (x_\mathcal{R}^1,\ldots,x_\mathcal{R}^l)^T$, $x_\mathcal{R}^j \in \{\pm 1\}$. A network coded symbol $x_\mathcal{R}^j$ in $\boldsymbol{x}_\mathcal{R}$ can be calculated as $x_\mathcal{R}^j = x_1^j x_2^j$. However, in our CEF-CS protocol, the relay does not forward $\boldsymbol{x}_\mathcal{R}$ directly. Instead, it first obtains the soft symbol of $x_\mathcal{R}^j$ in $\boldsymbol{x}_\mathcal{R}$, denoted by $\tilde{x}_\mathcal{R}^j$. Note that the soft symbol of $x_\mathcal{R}^j$ is equivalent to the expectation of $x_\mathcal{R}^j$ given the received signals, i.e., $\tilde{x}_\mathcal{R}^j = \mathcal{E}[x_\mathcal{R}^j | y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j]$ [4]. Then, the relay compresses the soft symbol vector $\tilde{\boldsymbol{x}}_\mathcal{R} = (\tilde{x}_\mathcal{R}^1,\ldots,\tilde{x}_\mathcal{R}^l)^T$ by exploiting the correlation between the two sources, which will be discussed in detail in Section III. We denote by $\mathcal{C}(\tilde{\boldsymbol{x}}_\mathcal{R})$ the power-normalized vector after compression. The length of q, denoted by m, $(m < l)$, varies according to the compression rates. Then, the received signal vector at the destination in the third phase can be expressed as

$$\boldsymbol{y}_{\mathcal{R}\mathcal{D}} = h_{\mathcal{R}\mathcal{D}}\sqrt{E_\mathcal{R}}\mathcal{C}(\tilde{\boldsymbol{x}}_\mathcal{R}) + \boldsymbol{n}_{\mathcal{R}\mathcal{D}} \quad (2)$$

where $E_\mathcal{R}$ is the power at the relay, $\boldsymbol{y}_{\mathcal{R}\mathcal{D}} = (y_{\mathcal{R}\mathcal{D}}^1,\ldots,y_{\mathcal{R}\mathcal{D}}^m)^T$ is the received signal vector, and $\boldsymbol{n}_{\mathcal{R}\mathcal{D}} = (n_{\mathcal{R}\mathcal{D}}^1,\ldots,n_{\mathcal{R}\mathcal{D}}^m)^T$ is the AWGN vector at the destination.

At the end of the third phase, the destination first recovers $\tilde{\boldsymbol{x}}_\mathcal{R}$ from $\boldsymbol{y}_{\mathcal{R}\mathcal{D}}$, which are then combined with the signals from the source-to-destination channels to make hard decisions.

III. Correlated Estimate and Forward With Compressive Sensing

Here, we introduce our CEF-CS protocol. First, we derive the network-coded soft symbols based on the correlated messages from the two sources. We note that the correlated messages from the two sources lead to redundancy in the soft symbol vector $\tilde{\boldsymbol{x}}_\mathcal{R}$ at the relay. Then, we compress $\tilde{\boldsymbol{x}}_\mathcal{R}$ by using CS.

A. Network-Coded Soft Symbols With Correlated Sources

The correlation between the sources is modeled as follows [16]. Let $Z_j,\ j=1,\ldots,l$, be an i.i.d. binary random variable with $\Pr(Z_j=0)=\tau$, where $\tau\in[0,1]$. Without a loss of generality, we focus on the jth symbol x_i^j in $\boldsymbol{x}_i$. We define the correlation between x_1^j and x_2^j as follows. If $Z_j=0$, then x_1^j and x_2^j are independent, and if $Z_j=1$, then x_1^j and x_2^j are identical. To simplify the derivation of the network-coded soft symbol, we introduce a variable $\zeta(x_1^j,x_2^j)$ to equivalently represent the correlation of x_1^j and x_2^j, which can be expressed as

$$\zeta(x_1^j,x_2^j)=\begin{cases}\frac{2-\tau}{4}, & \text{if } x_1^j=1, x_2^j=1 \text{ or } x_1^j=-1, x_2^j=-1\\ \frac{\tau}{4}, & \text{if } x_1^j=1, x_2^j=-1 \text{ or } x_1^j=-1, x_2^j=1.\end{cases}\quad (3)$$

Based on $\zeta(x_1^j,x_2^j)$, we now derive the soft symbol $\tilde{x}_{\mathcal{R}}^j=\mathcal{E}[x_{\mathcal{R}}^j|y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j]$, and we have

$$\begin{aligned}\mathcal{E}&\left[x_{\mathcal{R}}^j|y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j\right]\\ &=\sum_{x_1^j=\pm1,x_2^j=\pm1} x_1^jx_2^j p\left(x_1^j,x_2^j|y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j\right)\\ &=\sum_{x_1^j=\pm1,x_2^j=\pm1} x_1^jx_2^j \frac{p\left(x_1^j,x_2^j,y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j\right)}{p\left(y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j\right)}\\ &=\sum_{x_1^j=\pm1,x_2^j=\pm1} x_1^jx_2^j \frac{p\left(y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j|x_1^j,x_2^j\right)\zeta\left(x_1^j,x_2^j\right)}{\sum\limits_{u=\pm1,v=\pm1} p\left(y_{1\mathcal{R}}^j,y_{2\mathcal{R}}^j|u,v\right)\zeta(u,v)}\\ &=\sum_{x_1^j=\pm1,x_2^j=\pm1} x_1^jx_2^j \frac{p\left(y_{1\mathcal{R}}^j|x_1^j\right)p\left(y_{2\mathcal{R}}^j|x_2^j\right)\zeta\left(x_1^j,x_2^j\right)}{\sum\limits_{u=\pm1,v=\pm1} p(y_{1\mathcal{R}}^j|u)p(y_{2\mathcal{R}}^j|v)\zeta(u,v)}\end{aligned}\quad (4)$$

where $p(\cdot)$ denotes the probability density function (pdf), and $p(y_{i\mathcal{R}}^j|x_i^j)$ is conditionally Gaussian distributed, which is expressed as

$$p\left(y_{i\mathcal{R}}^j|x_i^j\right)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(y_{i\mathcal{R}}^j-h_{i\mathcal{R}}x_i^j\right)^2}{2\sigma^2}\right).\quad (5)$$

B. Compressive Sensing of Soft Symbol Vector

The correlation between the two sources leads to redundancy in the soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$. By taking the advantage of this redundancy, we can convert $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ to a sparse vector. To show this sparsity, we first investigate the pdf of a network-coded soft symbol $\tilde{x}_{\mathcal{R}}^j$. Fig. 1 plots the histogram of a soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ with τ in (3) equal to 0.2, vector length $l=10\,000$, $h_{1\mathcal{R}}=0.5$, and $h_{2\mathcal{R}}=1$. We focus on the curves for the CEF-CS protocol and consider the SNR $\rho=0$, 10, and 25 dB (the curve "EF, SNR $=0$ dB" will be discussed later). We can see that the soft symbols are within the range from -1 to 1. When the SNR is large, the soft symbols are polarized around the areas close to -1 and 1. Furthermore, due to the correlation between the two sources, the majority of the soft symbols are close to 1.

Then, we obtain another length-l vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ by calculating $\tilde{\boldsymbol{x}}'_{\mathcal{R}}=\tilde{\boldsymbol{x}}_{\mathcal{R}}-1$. Obviously, the elements in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ are polarized around the areas close to -2 and 0, and the majority of the elements are close to 0. Therefore, the vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ can be viewed as an approximately sparse vector. Conventional CS [11], [12] is used for strictly sparse signals with the majority of zero entries and sparse nonzero entries. For the approximately sparse signals, they do not have strictly zero entries; instead, they have significant coefficients and small coefficients. If the signal meets the requirement that most of the information resides in a small number of significant coefficients, while a large number of the small coefficients can be neglected, CS under the Bayesian framework [13] can be utilized. We now connect the sparsity in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ with τ. We can show that, when ρ is large enough, there are $(1-(\tau/2))l$ elements in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ around zeros and $\tau l/2$ elements around -2. Therefore, the sparsity of the vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ is $\tau/2$.

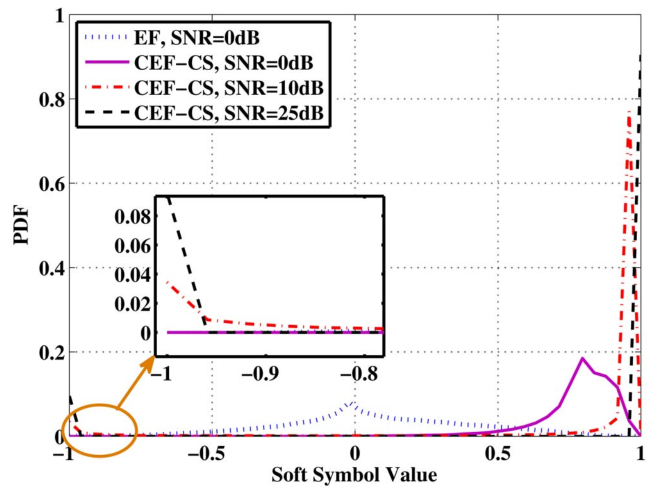

Fig. 1. PDF of a soft symbol vector with $\tau=0.2$, $l=10\,000$, $h_{1\mathcal{R}}=0.5$, and $h_{2\mathcal{R}}=1$.

Based on the sparsity of $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, we encode $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ with an $m\times l(m<l)$ sparse matrix $\boldsymbol{\Phi}$ composed only of the entries $\{1,-1,0\}$. Specifically, we select $\boldsymbol{\Phi}$ as a Rademacher encoding matrix according to [13]. For each element ϕ in $\boldsymbol{\Phi}$, we have

$$\phi=\begin{cases}+1, & \text{with probability } \frac{2}{\tau l}\\ 0, & \text{with probability } 1-\frac{4}{\tau l}\\ -1, & \text{with probability } \frac{2}{\tau l}.\end{cases}\quad (6)$$

The encoded vector $\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, called measurement vector, is of length m. Thus, by transmitting $\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ rather than $\tilde{\boldsymbol{x}}_{\mathcal{R}}$, the relay can save $l-m$ time slots. The vector $\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}})$ in (2) is written as

$$\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}})=\frac{\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}}{\sqrt{\mathcal{E}\left[|\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}|^2\right]}}\sqrt{\frac{l}{m}}.\quad (7)$$

Note that the term $\sqrt{l/m}$ in (7) is included to keep relay's transmission power constant during the third phase since the relay has less transmission time slots by compressing $\tilde{\boldsymbol{x}}_{\mathcal{R}}$.

From Fig. 1, we approximate the distribution of $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ as a mixture of two Gaussian variables with two different mean values and the same variance. This approximation can be equivalently treated as a two-state Gaussian mixture model described in [13], which requires $m=O((l\tau/2)\log(l))$ to guarantee the reconstruction. Thus, we need $O((l\tau/2)\log(l))$ measurements to recover $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$.

C. Reconstruction via Belief Propagation

Compared with the conventional CS schemes where the encoding matrices are dense, a sparse encoding matrix in this paper can be represented as a bipartite graph, and an efficient belief propagation (BP) algorithm can be applied for the fast decoding of compressed signals [13] at the destination. At the destination, BP is applied to recovering $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, which is of length l. We assume the pdfs of the soft symbol vector at different SNRs are known to the destination, which are used as the prior knowledge for BP [13]. We denote by $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$ the recovered $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ from the BP algorithm at the destination. It is easy to recover the network-coded soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$, denoted by $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$, from $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$ by

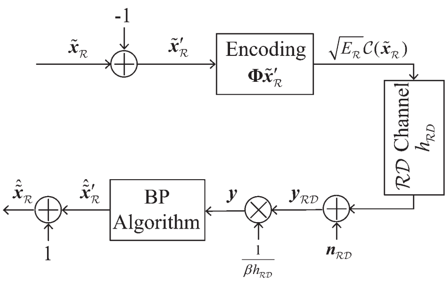

Fig. 2. Compression and reconstruction of $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ by using CS.

calculating $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}} = \hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}} + 1$. Then, the recovered soft symbol vector $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$ will be combined with the signals from the source-to-destination channels to make final decisions on the sources' messages. In the following, we focus on how to obtain $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$ from the received signal $\boldsymbol{y}_{\mathcal{RD}}$ by using BP.

To make the received signal $\boldsymbol{y}_{\mathcal{RD}}$ suitable for the BP algorithm, we first define

$$\beta = \sqrt{\frac{lE_{\mathcal{R}}}{m\mathcal{E}\left[|\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}|^2\right]}} \tag{8}$$

and we have $\boldsymbol{y}_{\mathcal{RD}} = \beta h_{\mathcal{RD}} \boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{RD}}$. Then, we obtain a new vector $\boldsymbol{y}$ by calculating $\boldsymbol{y} = \boldsymbol{y}_{\mathcal{RD}}/(\beta h_{\mathcal{RD}})$, i.e., $\boldsymbol{y} = \boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}} + \boldsymbol{n}$, where $\boldsymbol{n} = \boldsymbol{n}_{\mathcal{RD}}/(\beta h_{\mathcal{RD}})$. The vector $\boldsymbol{y}$ can be interpreted as the encoded vector $\boldsymbol{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ plus Gaussian noise. We use $\boldsymbol{y}$ as the input of the BP algorithm to recover $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ as in [13]. After we obtain $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$, we can thus obtain $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$. The compression and reconstruction of the network-coded soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ are shown in Fig. 2.

IV. Performance Analysis and Comparison

We analyze the soft symbol value and the received SNR at the destination in our CEF-CS protocol and compare them with the conventional EF protocol [4].

A. *Soft Symbol Value*

Recall that the soft symbol $x^j_{\mathcal{R}}$ of our CEF-CS protocol is derived in (4). However, in the conventional EF protocol, the soft symbol is calculated as [4]

$$\tilde{x}^j_{\mathcal{R}} = \tanh\left(\frac{\mathrm{LLR}_{x^j_1}}{2}\right)\tanh\left(\frac{\mathrm{LLR}_{x^j_2}}{2}\right) \tag{9}$$

where $\mathrm{LLR}_{x^j_i}$ represents the LLR of the symbol x^j_i at the relay, which is calculated as

$$\mathrm{LLR}_{x^j_i} = \ln\frac{p\left(y^j_{i\mathcal{R}}|x^j_i = 1\right)}{p\left(y^j_{i\mathcal{R}}|x^j_i = -1\right)} = \frac{2\sqrt{E_S}h_{i\mathcal{R}}y_{i\mathcal{R}j}}{\sigma^2}. \tag{10}$$

In fact, the calculation of $\tilde{x}^j_{\mathcal{R}}$ in (9) does not consider the correlation between the two sources.

By comparing the values of $\tilde{x}^j_{\mathcal{R}}$ calculated from (4) and (9), we have the following theorem.

Theorem 1: The soft symbol calculated by the CEF-CS in (4) has a higher reliability than that calculated by the conventional EF in (9).

Proof: See the Appendix.

From the proof of *Theorem 1*, we can see that the CEF-CS considers the correlation of the two sources when calculating $\tilde{x}^j_{\mathcal{R}}$, thus having a higher reliability. Therefore, the CEF-CS always has a better performance than the conventional EF.

In Fig. 1, we compare the probability distributions of the network-coded soft symbols in the CEF-CS protocol and the EF protocol when $\rho = 0$ dB. It can be seen that, with the correlation knowledge exploited by the CEF-CS protocol at the relay, more soft symbols than those in the EF protocol approach the value of 1. In fact, due to the correlation between the two sources, the majority of the network-coded symbols calculated in the CEF-CS protocol are 1. Therefore, the soft symbols in the CEF-CS have higher reliability compared with the conventional EF. In addition, since the soft symbols in the CEF-CS protocol are more polarized than those in the EF protocol (as shown in Fig. 1), it is more suitable to implement CS in our CEF-CS.

B. *Received SNR*

According to [4], the soft symbol of x^j_i is modeled as $\tilde{x}^j_i = \psi_i(x^j_i + e_i)$, where e_i is the soft noise, and ψ_i represents the scalar factor that makes the soft noise e_i uncorrelated to the information symbol x^j_i. We have $\mathcal{E}(e_i x^j_i) = 0$ and $\psi_i = \mathcal{E}(x^j_i \tilde{x}^j_i)/\mathcal{E}((x^j_i)^2)$. Similarly, the network-coded symbol is modeled as $\tilde{x}^j_{\mathcal{R}} = \psi_{\mathrm{EF}}(x^j_1 x^j_2 + e_{\mathrm{EF}})$, where e_{EF} is the soft noise, and ψ_{EF} represents the scalar factor that makes e_{EF} uncorrelated to the network-coded symbol $x^j_1 x^j_2$.

In the conventional EF, the relay forwards $\tilde{x}^j_{\mathcal{R}}$ in (9) without compression. At the destination, the receiver first estimates x^j_i from the received signal $y^j_{i\mathcal{D}}$ and obtains $\tilde{x}^j_i = \tanh(\mathrm{LLR}_{d,x^j_i}/2)$, where LLR_{d,x^j_i} is the LLR of x^j_i at the destination. Then, $\tilde{x}^j_i$ is multiplied with $y^j_{\mathcal{RD}}$ to cancel x^j_i, where $\bar{i} = 1$, 2 and $\bar{i} \neq i$. By defining $\kappa = \sqrt{E_{\mathcal{R}}/\mathcal{E}[|\tilde{\boldsymbol{x}}_{\mathcal{R}}|^2]}$, we have

$$\begin{aligned} y^j_{\mathcal{RD}}\tilde{x}^j_i &= \left(\kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\left(x^j_1 x^j_2 + e_{\mathrm{EF}}\right) + n^j_{\mathcal{RD}}\right)\left(\psi_i\left(x^j_i + e_i\right)\right) \\ &= \kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\psi_i x^j_{\bar{i}} + \kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\psi_i e_i x^j_1 x^j_2 \\ &\quad + \kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\psi_i e x^j_i + \kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\psi_i e_i e_{\mathrm{EF}} \\ &\quad + \psi_i n^j_{\mathcal{RD}} x^j_i + \psi_i e_i n^j_{\mathcal{RD}} \end{aligned} \tag{11}$$

where the noise term, denoted by $N_{i,\mathrm{EF}} = \kappa h_{\mathcal{RD}}\psi_{\mathrm{EF}}\psi_i(e_i x^j_1 x^j_2 + e x^j_i + e_i e_{\mathrm{EF}}) + \psi_i(n^j_{\mathcal{RD}} x^j_i + e^j_i n^j_{\mathcal{RD}})$, can be approximated as Gaussian distributed [4] with mean μ_i and variance σ^2_i. Since $\mathcal{E}[e_i x^j_i] = 0$, we have $\mu_i = 0$ and $\sigma^2_i = \kappa^2 h^2_{\mathcal{RD}}\psi^2_{\mathrm{EF}}\psi^2_i(\sigma^2_{e_i} + \sigma^2_e + \sigma^2_{e_i}\sigma^2_e) + \psi^2_i(\sigma^2 + \sigma^2_{e_i}\sigma^2)$, where $\sigma^2_{e_{\mathrm{EF}}}$ and $\sigma^2_{e_i}$ are the variances of e_{EF} and e_i, respectively.

Based on (11), we can obtain the LLR for $x^j_{\bar{i}}$, which is then combined with $y^j_{\bar{i}\mathcal{D}}$ to make the decision on $x^j_{\bar{i}}$. Here, we investigate the received SNR in (11), which can be expressed as

$$\rho_{i,\mathrm{EF}} = \frac{\kappa^2 h^2_{\mathcal{RD}}\psi^2_{\mathrm{EF}}\psi^2_i}{\kappa^2 h^2_{\mathcal{RD}}\psi^2_{\mathrm{EF}}\psi^2_i\left(\sigma^2_{e_i} + \sigma^2_{e_{\mathrm{EF}}} + \sigma^2_{e_i}\sigma^2_{e_{\mathrm{EF}}}\right) + \psi^2_i\left(\sigma^2 + \sigma^2_{e_i}\sigma^2\right)}. \tag{12}$$

For the CEF-CS, the network-coded soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ is compressed at the relay. When recovering $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ at the destination with BP, errors could be introduced [13], i.e., $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}} = \tilde{\boldsymbol{x}}_{\mathcal{R}} + \boldsymbol{n}_q$, where $\boldsymbol{n}_q$ is the quantization noise. Denote by $\hat{\tilde{x}}^j_{\mathcal{R}}$ and $\hat{n}^j_q$ the jth symbols in $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$ and $\boldsymbol{n}_q$, respectively. The recovered symbol $\hat{\tilde{x}}^j_{\mathcal{R}}$ is multiplied with $\tilde{x}^j_i$ to cancel $x^j_{\bar{i}}$. We have

$$\begin{aligned} \hat{\tilde{x}}^j_{\mathcal{R}}\tilde{x}^j_i &= \left(\psi_{\mathrm{CEF-CS}}\left(x^j_1 x^j_2 + e_{\mathrm{CEF-CS}}\right) + n^j_q\right)\left(\psi_i\left(x^j_i + e_i\right)\right) \\ &= \psi_{\mathrm{CEF-CS}}\psi_i x^j_{\bar{i}} + \psi_{\mathrm{CEF-CS}}\psi_i e_i x^j_1 x^j_2 \\ &\quad + \psi_{\mathrm{CEF-CS}}\psi_i e_{\mathrm{CEF-CS}} x^j_i + \psi\psi_i e_i e_{\mathrm{CEF-CS}} \\ &\quad + \psi_i n^j_q x^j_i + \psi_i e_i n^j_q \end{aligned} \tag{13}$$

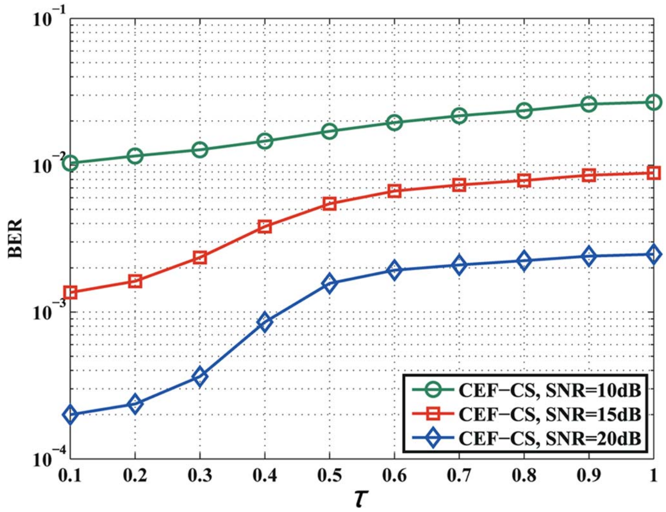

Fig. 3. BER performance of the CEF-CS protocol under different source correlation models at SNR = 10 dB, SNR = 15 dB, and SNR = 20 dB, and with $r = 1/2$.

where the noise term, denoted by $N_{i,\mathrm{CEF-CS}} = \psi_{\mathrm{CEF-CS}}\psi_i(e_i \times x_1^j x_2^j + e_{\mathrm{CEF-CS}} x_i^j + e_i e_{\mathrm{CEF-CS}}) + \psi_i(x_i^j n_q^j + e_i n_q^j)$, can be approximated as Gaussian distributed with mean μ_i and variance σ_i^2. Since $\mathcal{E}[e_i x_i^j] = 0$ and n_q^j is independent from the signal, we have $\mu_i = 0$ and $\sigma_i^2 = \psi^2\psi_i^2(\sigma_{e_i}^2 + \sigma_e^2 + \sigma_{e_i}^2\sigma_e^2) + \psi_i^2(\sigma_q^2 + \sigma_{e_i}^2\sigma_q^2)$, where σ_q^2 is the variance of n_q. The received SNR in (13) can be written as (14), shown at the bottom of the page.

V. Simulation Results

In the simulations, we consider quasi-static Rayleigh fading channels, assume that each source transmits $l = 200$ BPSK symbols in a transmission period, and run 1000 block errors to obtain simulation results. Moreover, we assume that the distances d_{S_iR} and d_{RD} are equal to 0.5, and d_{S_iD} is equal to 1. The attenuation exponent γ is set to be 2, and the sources and the relay have unit transmission power, namely $E_{\mathcal{S}} = E_{\mathcal{R}} = 1$. When applying the CS at the relay in the CEF-CS and EF-CS protocols, we denote by r the transmission rate at the relay, which means that the encoding matrix is of size $lr \times l$, and correspondingly the number of measurements transmitted from the relay is $m = lr$.

First, we investigate the BER performance of the CEF-CS protocol under different source correlation coefficients, as shown in Fig. 3. The system BER is defined as the average value of BERs of the two sources' signals at the destination. Note that the correlation coefficient of the two sources is defined in (3), which is derived from τ. Therefore, in Fig. 3, we simulate the system BERs for different values of τ. Moreover, in Fig. 3, we consider the SNRs of 10, 15, and 20 dB and a transmission rate $r = 1/2$. As shown in Fig. 3, with the increase in the value of τ, the BER performance becomes worse. This is because the increased τ, i.e., the reduced source correlation, leads to the less sparsity of the network-coded soft symbol vector. Furthermore, it is shown in Fig. 3 that, when τ exceeds 0.5, the system can only achieve diversity of 1; when τ falls below 0.3, the diversity gain is close to 2.

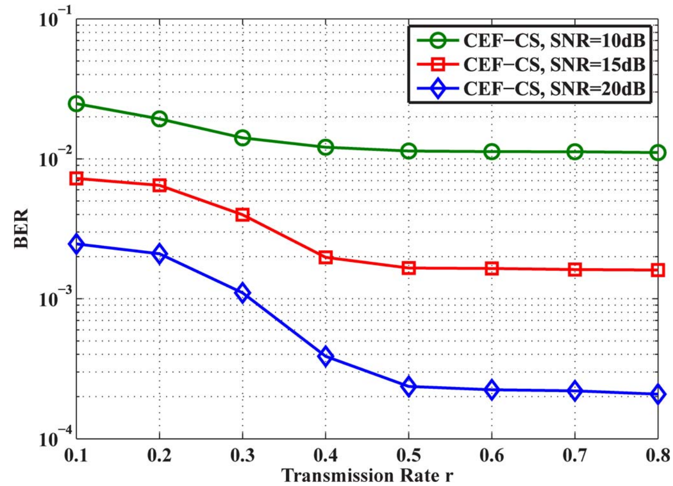

Fig. 4. BER as a function of the transmission rate r in the CEF-CS protocol at SNR = 10 dB, SNR = 15 dB, and SNR = 20 dB, as well as with $\tau = 0.2$.

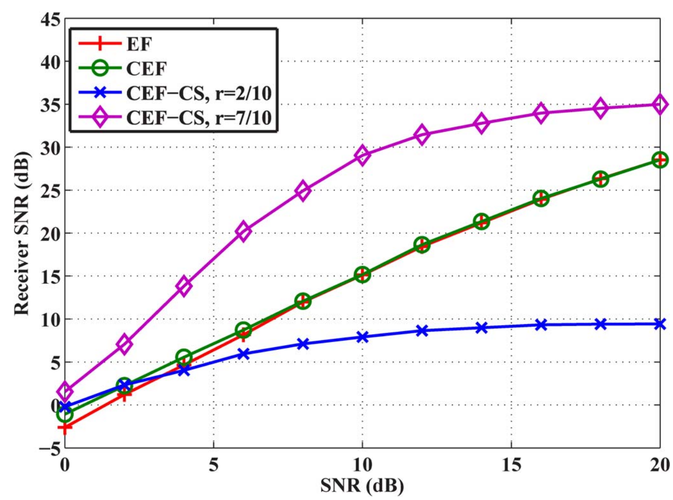

Fig. 5. Receiver SNR versus transmission SNR when $\tau = 0.2$.

Second, we study the impact of the transmission rate r on the BER performance for the CEF-CS protocol for a fixed τ. Here, we choose $\tau = 0.2$ since it offers a good error performance, as shown in Fig. 3. We investigate the system BERs with different transmission rates r under the SNRs of 10, 15, and 20 dB. As shown in Fig. 4, with the increase in r, the BER performance will gradually become better. Moreover, when r exceeds 0.5, the system diversity gain becomes closer to 2.

Third, we investigate the received SNRs discussed in Section IV-B. We compare our CEF-CS protocol with two other protocols, namely the conventional EF protocol and the CEF protocol, with $\tau = 0.2$. Specifically, in the CEF, the relay directly forwards the network-coded soft symbol vector calculated by (4) without any compression. The received SNRs in Fig. 5 for the EF and the CEF-CS protocols are calculated from (12) and (14), respectively. The received SNR for the CEF protocol can be obtained by following the similar method as in

$$\rho_{i,\mathrm{CEF-CS}} = \frac{\psi_{\mathrm{CEF-CS}}^2\psi_i^2}{\psi_{\mathrm{CEF-CS}}^2\psi_i^2\left(\sigma_{e_i}^2 + \sigma_{e_{\mathrm{CEF-CS}}}^2 + \sigma_{e_i}^2\sigma_{e_{\mathrm{CEF-CS}}}^2\right) + \psi_i^2\left(\sigma_q^2 + \sigma_{e_i}^2\sigma_q^2\right)} \quad (14)$$

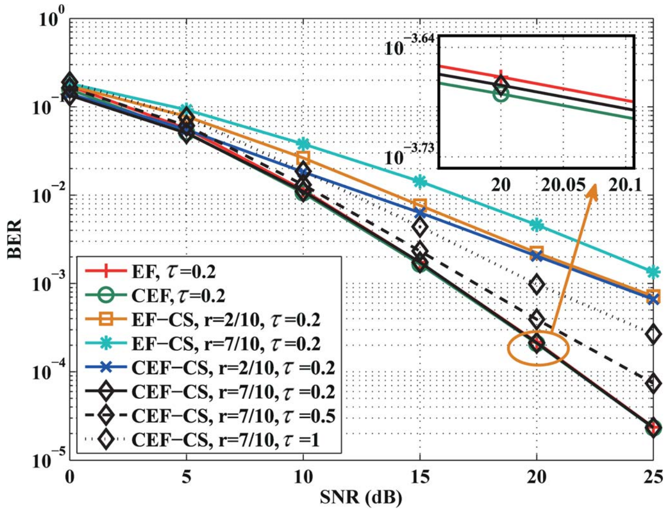

Fig. 6. BER performance of different schemes over fading channels.

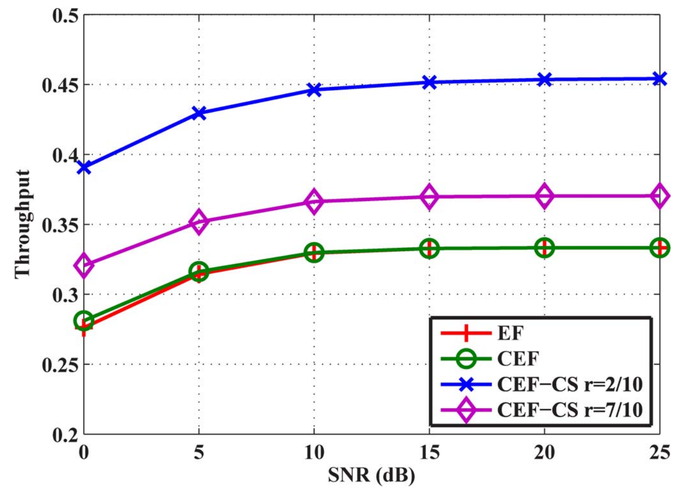

Fig. 7. Throughput performance of different schemes over fading channels when $\tau = 0.2$.

the EF protocol. It is shown in Fig. 5 that the received SNR of the CEF protocol always outperforms that of the EF protocol. However, the gap between the received SNRs of the CEF and EF goes to zero in the high SNR region, which means that the error performance of these two protocols tend to be equal in the high SNR region. For the CEF-CS protocol, it is shown that the CEF-CS with $r = 2/10$ obtains a lower receiver SNR than that in the EF and CEF protocols. For the transmission rate $r = 7/10$, the CEF-CS has the best receiver SNR compared with the other protocols. This is due to the fact that the quantization noise n_q^j in (13) is strictly constrained in the range of (0, 2), whereas the value range of $n_{\mathcal{RD}}$ in (11) is infinite. In particular, when the SNR is large enough, the majority of n_q^j gathers around 0.

Fourth, we investigate the BER performance of the CEF-CS, the CEF, and the EF protocols. We also consider another benchmark protocol, namely the EF-CS, where the relay applies CS to the network-coded soft symbol vector obtained from the EF protocol. As shown in Fig. 6, when $\tau = 0.2$, the CEF-CS protocol with a rate of 2/10 and the EF-CS protocol with rates of 2/10 and 7/10 can only achieve a diversity gain of one, whereas the CEF-CS with a rate of 7/10, the CEF, and the EF can achieve a diversity gain of two. The worse performance of the EF-CS implies that the CS is not suitable for the EF. This is because the soft symbol vector in the EF does not ensure the sparsity for CS. Moreover, the CEF outperforms the CEF-CS with a rate of 7/10 and the CEF-CS with a rate of 7/10 outperforms the EF. However, the performance gap between these three protocols goes to zero in the high SNR region. We also compare the BER performance of the CEF-CS protocol under other correlation values, namely $\tau = 0.5$ and $\tau = 1$. As can be seen, with the decrease in the correlation, the BER performance becomes worse.

Furthermore, since the relay node uses fewer time slots to forward information in the compressed protocols, the throughput performance of the compressed protocols will be much improved. Specifically, we define the throughput as the number of the correctly received symbols divided by the number of all transmitted symbols per transmission period, which can be calculated by $(2l - e)/(2l + m)$, where e denotes the number of symbol errors for both sources. For the EF and CEF protocols without CS, m is a constant equal to l. For the CEF-CS protocol, we have $m < l$ due to CS. As shown in Fig. 6, the EF, the CEF, and the CEF-CS at $r = 7/10$ have equivalent BER performance. However, in terms of the throughput performance, the CEF-CS at $r = 7/10$ will improve by 10% compared with the EF and the CEF protocols, as shown in Fig. 7. Moreover, although the BER performance of the CEF-CS at $r = 2/10$ cannot achieve full diversity, the throughput performance improves by about 36%, compared with the uncompressed protocols.

VI. Conclusion

This paper presents an EF technique for correlated sources in a MARC and exploits the sources' correlation at the relay by implementing CS on the soft symbols. We first develop the expression of the soft symbol of the CEF-CS protocol and prove that the CEF-CS protocol outperforms the conventional EF protocol with more reliable soft symbols. Then, we convert the soft symbol vector to a sparse vector, which is suitable for CS. A BP algorithm is utilized to recover the compressed soft symbol vector. Furthermore, we analyze the performance of the CEF-CS by comparing its soft symbol value and received SNR with the conventional EF. Simulations show that our protocol can achieve the same BER performance as the conventional EF protocol with proper compression rates, and it consumes much less transmission time due to the compression.

Appendix
Proof of Theorem I

We further develop the soft symbol $\tilde{x}_{\mathcal{R}}^j$ in (4) as

$$\tilde{x}_{\mathcal{R}}^j = \frac{\alpha(2-\tau) - \theta\tau}{\alpha(2-\tau) + \theta\tau} \tag{15}$$

where

$$\alpha = \sum_{x_1^j x_2^j = 1} p\left(y_{1\mathcal{R}}^j | x_1^j\right) p\left(y_{2\mathcal{R}}^j | x_2^j\right) \tag{16}$$

$$\theta = \sum_{x_1^j x_2^j = -1} p\left(y_{1\mathcal{R}}^j | x_1^j\right) p\left(y_{2\mathcal{R}}^j | x_2^j\right). \tag{17}$$

Obviously, we have $\alpha > 0$ and $\theta > 0$.

The conventional EF derives $\tilde{x}_{\mathcal{R}}^j$ in (9) by assuming that the two sources are independent, i.e., (9) can be obtained by considering $\tau = 1$ in (15). In the CEF-CS protocol, we derive $\tilde{x}_{\mathcal{R}}^j$ by considering the real τ. In correlated sources, we always have $\tau < 1$. Given α and θ, $\tilde{x}_{\mathcal{R}}^j$ can be regarded as a function of τ, which is denoted by $\delta(\tau)$. To investigate the relationship between $\delta(\tau)$ and τ, we derive the first-order derivative of $\delta(p)$ as follows:

$$\frac{d\delta(\tau)}{d\tau} = \frac{-8\alpha\theta}{(\alpha(2-\tau)+\theta\tau)^2}. \tag{18}$$

Since $\alpha > 0$ and $\theta > 0$, we always have $d\delta(\tau)/dp < 0$, which means that $\delta(\tau)$ is a decreasing function in terms of τ. When correlation exists between the sources, we have $\tau < 1$. Therefore, the soft symbol values in the CEF protocol are always larger than that in the conventional EF protocol. As the majority of the soft symbols $\tilde{x}^j_{\mathcal{R}}$ approach the value of 1, the soft symbols in the CEF protocol are more reliable than those of the EF protocol.

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Achieving Maximum Degrees of Freedom of Two-Hop MIMO Alternate Half-Duplex Relaying System For Linear Transceivers: A Unified Transmission Framework for DF and AF Protocols

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***Abstract*—In this paper, a novel unified transmission framework in the context of both decode-and-forward (DF) and amplify-and-forward (AF) protocols was proposed for a two-hop multiple-input–multiple-output (MIMO) alternate half-duplex (HD) relaying system with M antennas for each node, which can achieve the maximum M degrees of freedom (DoFs) of the system, compared with the existing interference alignment (IA) schemes that can only get $3M/4$ DoF. Furthermore, the proposed schemes require only two relays against three relays invoked in the existing IA schemes. Moreover, the proposed schemes are valid for the cases where the number of antennas M can be either even or odd, whereas the existing IA schemes can only apply for an even number of antennas M.**

***Index Terms*—Alternate relay, degree of freedom (DoF), interference alignment (IA), multiple input multiple output (MIMO).**

I. Introduction

It is well known that multiple-input–multiple-output (MIMO) system can substantially improve spectral efficiency [1]. Furthermore, relay-aided transmission has the potential ability to extend the coverage area and provide the spatial diversity [2]. Hence, the composite system consisting of MIMO and relay has attracted intensive research effort.

If the relay can simultaneously transmit and receive, it is called a full-duplex (FD) relay. On the contrary, if the relay can only transmit or receive at a time, it is referred to as a half-duplex (HD) relay. As an FD relay is difficult to implement, an HD relay is more popular. However, when an HD relay is adopted, the prelog loss factor of 1/2 of the capacity loss is induced, particularly for a high-signal-to-noise-ratio region [3].

Different schemes have been proposed to recover this prelog loss factor of one half performance loss [3]. Among them, the alternate relaying scheme, where two relays successively forward information from a source to a destination, has attracted much interest [3]–[5]. However, for alternate relaying scheme, the interrelay interference (IRI) may significantly degrade the system performance [3].

For a single-input–single-output alternate relaying system, the IRI could be canceled at the destination for both the decode-and-forward (DF) and amplify-and-forward (AF) relaying protocols [3]. Alternatively, the IRI imposed by self-interference could be canceled at the

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