Design of Physical Layer Network Coded LDPC Code for a Multiple-Access Relaying System

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***Abstract*—In this letter we propose a novel physical layer network coded LDPC code structure, named PNC-LDPC code, for a non-orthogonal multiple access relay channel. An iterative detection-and-decoding receiver is designed to tackle the multi-user interference at the destination. Based on the code structure and the iterative receiver, we optimize the PNC-LDPC code profile to approach the system achievable rate by utilizing the extrinsic mutual information transfer (EXIT) chart. Simulations show that the performance of our PNC-LDPC code outperforms other conventional network coding based LDPC codes.**

***Index Terms*—LDPC code, network coding, EXIT chart, multiple access relay channel.**

I. Introduction

In wireless multiple access relay channels (MARC) with multiple sources, one relay, and one destination, the sources transmit signals to the destination with the help of the relay. Conventional decode-and-forward (DF) protocols of the classic triangle cooperative channels [1] can be readily extended to the MARC. The capacity outer bound and the achievable rate region of the MARC with the DF protocol have been investigated in [2].

Wireless network coding [3] combined with a powerful channel code, e.g., low-density parity check codes [4, 5], is an effective method to approach the achievable rate of the DF based MARC system [6]. In these combined network-channel coding schemes, the relay explicitly/fully decodes the messages of each source and combines these messages based on the channel code to obtain network coded digits. However, these schemes assume that all the sources' signals are transmitted in orthogonal channels, e.g., time/frequency division multiple access (T/FDMA). In the non-orthogonal MARC where the sources' signals are transmitted in the same time and frequency domains, fully decoding of each source's messages at the relay will degrade the error performance due to the multi-user interference.

Physical-layer network coding (PNC) [7] is proposed in two-way relay channels (TWRC) to enhance the error performance of the source-to-relay multiple access channels (MAC). The main advantage of the PNC compared to the full-decoding based network coding (FNC) is that it applies partial decoding at the relay by viewing the source-to-relay MAC as a single link channel. This partial decoding of the PNC enables the collaboration of the two sources' signals rather than treating one source's signals as interferences to the other. Simulations in [7] show that the PNC achieves a better error performance relative to the FNC at the relay. This inspiring result motivates us to focus on the applications of the PNC in the MARC.

In this letter we focus on a non-orthogonal MARC network with two sources, one half-duplexing relay and one destination. We consider a strong interference scenario at the relay, i.e., the received signal-to-noise ratios (SNR) of the two sources at the relay are the same. The contributions in this letter are as follows. we first propose a novel PNC-LDPC code based on the multi-edge type LDPC code structure [5]. Then an iterative detection-and-decoding receiver is designed to deal with the multi-user interference at the destination. Based on the code structure and the iterative receiver, we optimize the degree distribution of the PNC-LDPC code to approach the system achievable rate by utilizing EXIT chart [8]. Simulations show that the bit error rate (BER) performance of our PNC-LDPC code outperforms the FNC based LDPC code by 0.8 dB.

II. System Model

We consider a non-orthogonal MARC network with two sources, one relay and one destination. The two sources $\mathcal{S}_1$, $\mathcal{S}_2$ transmit their information to the destination $\mathcal{D}$ with the help of a half-duplex relay $\mathcal{R}$. We assume that the two sources have the same distance to the relay, and have different ones to the destination. Although not practical in general, this scenario could happen when two sources are around the relay and far from the destination. The distance between $\mathcal{S}_i$ $(i = 1, 2)$ and the relay $\mathcal{R}$ is denoted by $d_{\mathcal{R}}$, the distance between $\mathcal{R}$ and the destination $\mathcal{D}$ is $d_{\mathcal{RD}}$, and the distance between $\mathcal{S}_i$ and $\mathcal{D}$ is $d_{i\mathcal{D}}$, where $d_{1\mathcal{D}} \neq d_{2\mathcal{D}}$. The path losses of all the channels are related to their distances with the same attenuation exponent γ. Therefore, the channel coefficients between $\mathcal{S}_i$ and $\mathcal{R}$, $\mathcal{S}_i$ and $\mathcal{D}$, $\mathcal{R}$ and $\mathcal{D}$ are calculated as $h_{\mathcal{R}} = 1/\sqrt{(d_{\mathcal{R}})^\gamma}$, $h_{i\mathcal{D}} = 1/\sqrt{(d_{i\mathcal{D}})^\gamma}$, and $h_{\mathcal{RD}} = 1/\sqrt{(d_{\mathcal{RD}})^\gamma}$, respectively.

We split one transmission period (n time slots) into two phases. The first phase has tn time slots ($0 < t < 1$, and tn is an integer), in which the two sources simultaneously broadcast their channel encoded codewords C_i, $i = 1, 2$, to both the destination and the relay. The second phase has $(1 - t)n$ time slots, in which the two sources keep silent, while the relay decodes the two source's information, generates the network-

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coded bit vector $\bar{C}_{\mathcal{R}}$ based on the sources' information, encodes $\bar{C}_{\mathcal{R}}$ by using a channel code, and forwards the codeword $C_{\mathcal{R}}$ to the destination. In the first phase, the signal vectors received by the destination and the relay are denoted by Y_1 and $Y_{\mathcal{R}}$, respectively, while in the second phase, the signal vector received at the destination is denoted by Y_2.

We assume that the codewords C_i and $C_{\mathcal{R}}$ are modulated to X_i and $X_{\mathcal{R}}$ by BPSK modulations, respectively. We have $X_i = [x_i^1, \cdots, x_i^{tn}]^T$, where x_i^j, $j = 1, \cdots, tn$ is a BPSK symbol, and $X_{\mathcal{R}} = [x_{\mathcal{R}}^1, \cdots, x_{\mathcal{R}}^{(1-t)n}]^T$, where $x_{\mathcal{R}}^{j'}$, $j' = 1, \cdots, (1-t)n$ is a BPSK symbol. The information part of X_i is denoted by $\bar{X}_i = [x_i^1, \cdots, x_i^{tnR_i}]^T$, where R_i is $\mathcal{S}_i$'s code rate. The information part of $X_{\mathcal{R}}$ is denoted by $\bar{X}_{\mathcal{R}} = [x_{\mathcal{R}}^1, \cdots, x_{\mathcal{R}}^{(1-t)nR_{\mathcal{R}}}]^T$, where $R_{\mathcal{R}}$ is the relay's code rate, and $\bar{X}_{\mathcal{R}}$ is the BPSK modulation for $\bar{C}_{\mathcal{R}}$. All the noises are modeled as AWGN distributed with a zero mean and variance σ^2. To make the total power constant, we assume that each source has the same power one and the relay has the transmission power of two, i.e., $x_i^j \in \{\pm 1\}$ and $x_{\mathcal{R}}^{j'} \in \{\pm\sqrt{2}\}$.

Similar to the assumptions made in [2, 7], we assume the channel phases have been synchronized before transmissions, which makes the PNC applicable to our MARC. Therefore, the received signal vectors are expressed as $Y_{\mathcal{R}} = h_{\mathcal{R}}(X_1+X_2)+N_{\mathcal{R}}$, $Y_1 = h_{1\mathcal{D}}X_1 + h_{2\mathcal{D}}X_2 + N_1$, and $Y_2 = h_{\mathcal{R}\mathcal{D}}X_{\mathcal{R}} + N_2$, where $N_{\mathcal{R}}$ is the noise vector observed by the relay, N_1 and N_2 are the noise vectors observed by the destination in the first and second phases, respectively.

III. Structure of PNC-LDPC Codes

The relay first decodes $C_1 + C_2$ (here "+" is the addition operator in the real domain) from $Y_{\mathcal{R}}$ by viewing it as a ternary code, and maps it to the XOR of C_1 and C_2, i.e., $C_1 \oplus C_2$ [7]. Specifically, 0 and 2 in $C_1 + C_2$ are mapped to 0 in $C_1 \oplus C_2$, and 1 in $C_1 + C_2$ is mapped to 1 in $C_1 \oplus C_2$. Then the relay generates the network coded bit vector $\bar{C}_{\mathcal{R}}$ based on $C_1 \oplus C_2$, which are then encoded into the codeword $C_{\mathcal{R}}$. The destination firstly decodes $\bar{C}_{\mathcal{R}}$ from Y_2 and then decodes the two sources's information based on Y_1 and $\bar{C}_{\mathcal{R}}$. We assume $\bar{C}_{\mathcal{R}}$ can be perfectly decoded at the destination.

The source $\mathcal{S}_1$ is encoded by a single link LDPC code with code rate R_1, which is denoted by $\mathbb{C}_1$. The source $\mathcal{S}_2$ is encoded by another LDPC code $\mathbb{C}_2$ with code rate R_2. We assume $R_2 < R_1$ and design the code $\mathbb{C}_2$ such that its codewords form a subset of $\mathbb{C}_1$'s codewords. $\mathbb{C}_2$ is designed as a concatenated code $(\mathbb{C}_{out}, \mathbb{C}_{in})$. We set $\mathbb{C}_{in} = \mathbb{C}_1$. The information of $\mathcal{S}_2$ is firstly encoded by the outer code $\mathbb{C}_{out}$ with rate $\frac{R_2}{R_1}$, and then the output of $\mathbb{C}_{out}$ is encoded by the inner code $\mathbb{C}_1$ (with rate R_1). We denote the parity check matrix of $\mathbb{C}_{out}$ and $\mathbb{C}_1$ as $\mathbf{H}_{out}$ and $\mathbf{H}_1$, respectively. Then the parity check matrix $\mathbf{H}_2$ of $\mathbb{C}_2$ can be expressed as

$$\mathbf{H}_2 = \begin{bmatrix} \mathbf{H}_{out} & \mathbf{O} \\ ------ & -- \\ \leftarrow & \mathbf{H}_1 \ \rightarrow \end{bmatrix}, \tag{1}$$

where $\mathbf{O}$ is a zero matrix.

At the relay, $C_1 + C_2$ is viewed as a codeword of a non-binary code which is derived from $\mathbb{C}_1$, and $\mathcal{S}_2$'s outer code $\mathbb{C}_{out}$ is transparent to the relay. The relay generates the network coded bit vector $\bar{C}_{\mathcal{R}}$ from $C_1 \oplus C_2$ based on a matrix $\mathbf{H}_{\mathcal{R}}$, i.e., $\bar{C}_{\mathcal{R}} = \mathbf{H}_{\mathcal{R}}(C_1 \oplus C_2)$, which are then encoded by a desired channel code to the codeword $C_{\mathcal{R}}$. At the destination, the decoder is designed by viewing $[C_1^T \ C_2^T \ \bar{C}_{\mathcal{R}}^T]^T$ as its codeword, with its parity check matrix denoted by $\mathbf{H}_{\mathcal{D}}$.

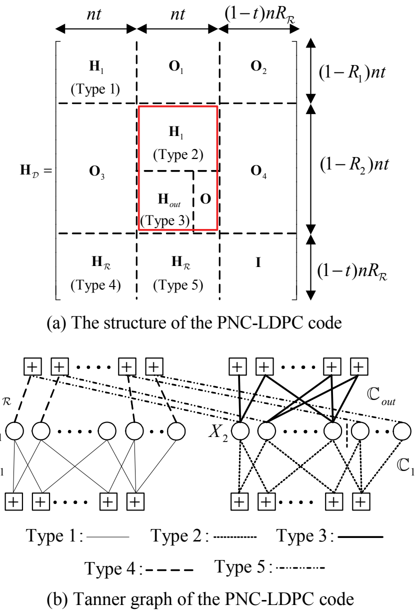

Fig. 1. Multi-edge type structure and Tanner Graph of the PNC-LDPC code.

Fig. 1(a) shows the structure of the parity check matrix $\mathbf{H}_{\mathcal{D}}$. In Fig. 1(a), the sub-matrices $\mathbf{O}_1$, $\mathbf{O}_2$, $\mathbf{O}_3$ and $\mathbf{O}_4$ are four zero matrices, $\mathbf{I}$ is an identity matrix. The sub-matrix $[\mathbf{H}_{\mathcal{R}} \ \mathbf{H}_{\mathcal{R}}]$ of $\mathbf{H}_{\mathcal{D}}$ corresponds to the network coding at the relay. We denote the code that corresponds to the sub-matrix $[\mathbf{H}_{\mathcal{R}} \ \mathbf{H}_{\mathcal{R}}]$ of $\mathbf{H}_{\mathcal{D}}$ as $\mathbb{C}_{\mathcal{R}}$, denote the code that corresponds to the left matrix $\mathbf{H}_{\mathcal{R}}$ of $[\mathbf{H}_{\mathcal{R}} \ \mathbf{H}_{\mathcal{R}}]$ as $\mathbb{C}_{\mathcal{R},1}$, and denote the code that corresponds to the right matrix $\mathbf{H}_{\mathcal{R}}$ of $[\mathbf{H}_{\mathcal{R}} \ \mathbf{H}_{\mathcal{R}}]$ as $\mathbb{C}_{\mathcal{R},2}$.

We utilize multi-edge type structure [5] to represent the code structure at the destination. The multi-edge type ensemble can be specified through two polynomials, one is associated with variable nodes, and the other is associated with check nodes. The polynomials are given by $v(\mathbf{r}, \mathbf{w}) = \sum v_{\mathbf{b},\mathbf{d}} \mathbf{r}^{\mathbf{b}} \mathbf{w}^{\mathbf{d}}$, and $\mu(\mathbf{w}) = \sum \mu_{\mathbf{d}} \mathbf{w}^{\mathbf{d}}$, where $\mathbf{d} = [d_1, d_2, \cdots, d_{n_\varsigma}]$ is the *edge degree vector* (EDV) of length n_ς and $\mathbf{b} = [b_0, b_1, \cdots, b_{n_\tau}]$ is the *received degree vector* (RDV) of length $n_\tau + 1$.

Fig. 1 (b) shows the Tanner Graph of the PNC-LDPC code represented by a multi-edge type code structure. The bits of $\bar{C}_{\mathcal{R}}$ are neglected since $\bar{C}_{\mathcal{R}}$ has been successfully decoded at the destination. We denote the five edge types by Type m, $m = 1, 2, 3, 4, 5$, corresponding to the codes $\mathbb{C}_1$, $\mathbb{C}_1$ (the inner code of $\mathbb{C}_2$), $\mathbb{C}_{out}$, $\mathbb{C}_{\mathcal{R},1}$, and $\mathbb{C}_{\mathcal{R},2}$, respectively. Next, we assign the structure of the PNC-LDPC code with two types of the received degree (i.e., $n_\tau = 2$), since X_1 and X_2 experience two different signal-to-interference-noise ratio. With five different edge-types and two types of received degree in $\mathbf{H}_{\mathcal{R}}$, the polynomials for $\mathbf{H}_{\mathcal{D}}$ can be written as $v(\mathbf{r}, \mathbf{w}) = r_1 \sum_{a=1}^{d_{v,1}} \sum_{d=0}^{d_{v,4}} v_{[0,1,0],[a,0,0,d,0]} w_1^a w_4^d + r_2 \sum_{b=1}^{d_{v,2}} \sum_{c=0}^{d_{v,3}} \sum_{e=0}^{d_{v,5}} v_{[0,0,1],[0,b,c,0,e]} w_2^b w_3^c w_5^d$, and $\mu(\mathbf{w}) =$

$$l_{\text{dec}}^{\text{mud}}(x_1^j) = \ln\left(\frac{p(y_1^j|x_1^j=1,x_2^j=1)P(x_1^j=1)P(x_2^j=1)+p(y_1^j|x_1^j=1,x_2^j=-1)P(x_1^j=1)P(x_2^j=-1)}{p(y_1^j|x_1^j=-1,x_2^j=1)P(x_1^j=-1)P(x_2^j=1)+p(y_1^j|x_1^j=-1,x_2^j=-1)P(x_1^j=-1)P(x_2^j=-1)}\right) \tag{2}$$

$$\begin{aligned} I_{Ev,1}^{(k,q)} &= \sum_{a=1}^{d_{v,1}}\sum_{d=1}^{d_{v,4}} J\left(\sqrt{(a-1)\left(J^{-1}\left(I_{Av,1}^{(k,q)}\right)\right)^2 + d\left(J^{-1}\left(I_{Av,4}^{(k,q)}\right)\right)^2 + \left(\sigma_1^{(k)}\right)^2}\right)\lambda_{[a,0,0,d,0]}^{(1)}, \\ I_{Ev,2}^{(k,q)} &= \sum_{b=1}^{d_{v,2}}\sum_{c=1}^{d_{v,3}}\sum_{e=1}^{d_{v,5}} J\left(\sqrt{(b-1)\left(J^{-1}\left(I_{Av,2}^{(k,q)}\right)\right)^2 + c\left(J^{-1}\left(I_{Av,3}^{(k,q)}\right)\right) + e\left(J^{-1}\left(I_{Av,5}^{(k,q)}\right)\right)^2 + \left(\sigma_2^{(k)}\right)^2}\right)\lambda_{[0,b,c,0,e]}^{(2)}. \end{aligned} \tag{3}$$

$\sum_{a=1}^{d_{c,1}} \mu_{[a,0,0,0,0]} w_1^a + \sum_{b=1}^{d_{c,2}} \mu_{[0,b,0,0,0]} w_2^b + \sum_{c=1}^{d_{c,3}} \mu_{[0,0,c,0,0]} w_3^c + \sum_{d=1}^{d_{c,4}} \mu_{[0,0,0,d,0]} w_4^d + \sum_{e=1}^{d_{c,5}} \mu_{[0,0,0,0,e]} w_5^d$, where r_1 and r_2 denote the variable nodes associated with X_1 and X_2, respectively. More specifically, r_1 and r_2 are associated with the $\mathcal{S}_1$-to-destination channel and the $\mathcal{S}_2$-to-destination channel, respectively. The variable nodes transmitted in the $\mathcal{S}_1$-to-destination channel (i.e., the symbols in X_1) are connected to the edges of Type 1 and Type 4, while the variable nodes transmitted in the $\mathcal{S}_2$-to-destination channel (i.e., the symbols in X_2) are connected to the edges of Type 2, Type 3 and Type 5. The EDV $\mathbf{d} = [a, b, c, d, e]$ represents five types of edge degree with a, b, c, d and e denoting the variable or check node' degrees of Type m, $m = 1, 2, 3, 4, 5$, respectively.

IV. Receiver Structure and Code Optimization

A. Iterative Receiver

The iterative receiver structure is shown in Fig. 2. There is a soft-in-soft-out (SISO) multiuser detector (denoted by MUD), and a SISO belief propagation (BP) decoder (denoted by DEC). The extrinsic log-likelihood ratios (LLR), i.e., $l_{\text{dec}}^{\text{mud}}(x_i^j)$ and $l_{\text{mud}}^{\text{dec}}(x_i^j)$ are exchanged between the MUD and the DEC in each iteration. In the MUD, the probability $P(x_i^j)$ is updated as $P(x_i^j = 1) = \frac{\exp(l_{\text{mud}}^{\text{dec}}(x_i^j))}{1+\exp(l_{\text{mud}}^{\text{dec}}(x_i^j))}$, and $P(x_i^j = -1) = \frac{1}{1+\exp(l_{\text{mud}}^{\text{dec}}(x_i^j))}$. Eq. (2) gives the expression of $l_{\text{dec}}^{\text{mud}}(x_1^j)$ based on $P(x_i^j)$.

In the DEC, $l_{\text{dec}}^{\text{mud}}(x_i^j)$ is utilized as the extrinsic channel LLRs. BP decoding is applied to the parity check matrix $\mathbf{H}_{\mathcal{D}}$. The DEC structure is shown in Fig. 2. We can see that joint decoding of the two sources' codewords is enabled by the sub-matrix $[\mathbf{H}_{\mathcal{R}}\ \mathbf{H}_{\mathcal{R}}]$ in the parity check matrix $\mathbf{H}_{\mathcal{D}}$. We denote the output LLR of the code $\mathbb{C}_i$ by $l_i(x_i^j)$, and denote the output LLR from $\mathbb{C}_{\mathcal{R}}$ to $\mathbb{C}_i$ by $l_{\mathcal{R}}^i(x_i^j)$. Then the output LLR of the DEC is calculated as $l_{\text{mud}}^{\text{dec}}(x_i^j) = l_i(x_i^j) - l_{\text{dec}}^{\text{mud}}(x_i^j) - l_{\mathcal{R}}^i(x_i^j)$.

B. Code Optimization

We first optimize the code $\mathbb{C}_1$ as a single link ternary LDPC code, which guarantees a good decoding performance of $C_1 + C_2$ at the relay. The optimization can follow a method similar to the one in [7]. Then based on $\mathbb{C}_1$ and the iterative receiver, we jointly optimize $\mathbb{C}_{out}$ and $\mathbb{C}_{\mathcal{R}}$.

The EXIT chart [8] is utilized to optimize $\mathbb{C}_{out}$ and $\mathbb{C}_{\mathcal{R}}$. We denote by $I_{Ev,m}^{(k,q)}$ the average extrinsic mutual information sent on the edges of Type m from the variable nodes to the check nodes in the q-th iteration of the BP decoding and the k-th iteration between the MUD and the DEC. We denote by $I_{Ec,m}^{(k,q)}$ the average extrinsic mutual information

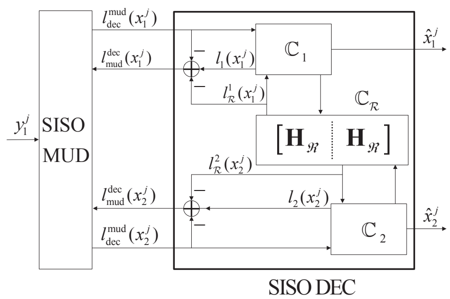

Fig. 2. Iterative receiver structure at the destination.

sent on the edges of Type m from the check nodes to the variable nodes in the q-th iteration of the BP decoding and the k-th iteration between the MUD and the DEC. Also note that the extrinsic mutual information on an edge connecting the variable nodes to the check nodes, at the output of the variable nodes, is the *a-priori* mutual information for the check nodes in the current iteration of BP decoding, i.e., $I_{Ac,m}^{(k,q)} = I_{Ev,m}^{(k,q)}$. Similarly, we have $I_{Av,m}^{(k,q+1)} = I_{Ec,m}^{(k,q)}$. We use the $J(\cdot)$ function to represent the mutual information of a single link BIAWGN channel, which is derived in [8]. We denote the variance of the LLR $l_{\text{dec}}^{\text{mud}}(x_i^j)$ as $\left(\sigma_i^{(k)}\right)^2$ in the k-th iteration between the MUD and the DEC. According to [8], we have $I_{Ev,1}^{(k,q)}$ and $I_{Ev,2}^{(k,q)}$ shown as Eq. (3). In (3), we have $\lambda_{[a,0,0,d,0]}^{(1)} = \frac{v_{[0,1,0],[a,0,0,d,0]}a}{\sum_{a'=1}^{d_{v,1}}\sum_{d'=1}^{d_{v,4}} v_{[0,1,0],[a',0,0,d',0]}a'}$, and $\lambda_{[0,b,c,0,e]}^{(2)} = \frac{v_{[0,0,1],[0,b,c,0,e]}b}{\sum_{b'=1}^{d_{v,2}}\sum_{c'=1}^{d_{v,3}}\sum_{e'=1}^{d_{v,5}} v_{[0,0,1],[0,b',c',0,e']}b'}$. Note that the calculations for $I_{Ev,2}^{(k,q)}$, $I_{Ev,3}^{(k,q)}$, and $I_{Ev,5}^{(k,q)}$ can be obtained by following the same method.

At the end of Q iterations of the BP decoding, the extrinsic mutual information from the DEC to the MUD, at output of the DEC is given by

$$\begin{aligned} I_{1,DEC\to MUD}^{(k)} &= \sum_{a=1}^{d_{v,1}}\sum_{d=1}^{d_{v,4}} J\left(\sqrt{a\left(J^{-1}\left(I_{Av,1}^{(k,Q)}\right)\right)^2}\right)\lambda_{[a,0,0,d]}^{(1)}, \\ I_{2,DEC\to MUD}^{(k)} &= \sum_{b=1}^{d_{v,2}}\sum_{c=1}^{d_{v,3}}\sum_{d=1}^{d_{v,4}} \\ & J\left(\sqrt{b\left(J^{-1}\left(I_{Av,2}^{(k,Q)}\right)\right)^2 + c\left(J^{-1}\left(I_{Av,3}^{(k,Q)}\right)\right)}\right)\lambda_{[0,b,c,d]}^{(2)}. \end{aligned} \tag{4}$$

We assume that the receiver at the destination conducts total K iterations between the MUD and the DEC. At the end of the K iterations, we ensure that $I_{i,DEC\to MUD}^{(K)} \to 1$. Given

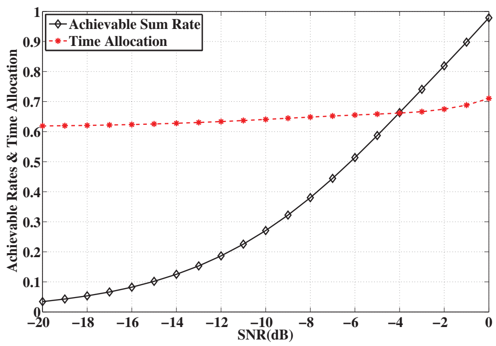

Fig. 3. Optimal time allocations and achievable rates of the MARC with the FNC and the PNC schemes.

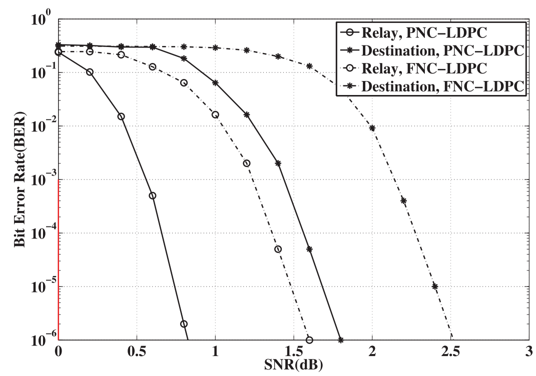

Fig. 4. BER curves for PNC-LDPC and FNC-LDPC.

these requirements, we optimize the code by searching a code profile with the maximum threshold σ_{th}.

V. Simulations and Numerical Results

In the simulations, we assume that $d_{\mathcal{R}} = 0.5$, $d_{\mathcal{RD}} = 0.5$, $d_{1\mathcal{D}} = 0.7$, $d_{2\mathcal{D}} = 1.0$, and $\gamma = 2$. The SNR is defined as the transmission SNR of each source, i.e., $\frac{1}{\sigma^2}$. According to [2], we can obtain the system achievable sum rate as $R_{sum} = \frac{1}{n}\min\{I(X_1, X_2; Y_{\mathcal{R}}), I(X_1, X_2; Y_1) + I(X_{\mathcal{R}}; Y_2)\}$. Fig. 3 shows the optimized achievable rates of the MARC based on time allocations t at different SNRs, where t is optimized by letting $I(X_1, X_2; Y_{\mathcal{R}}) = I(X_1, X_2; Y_1) + I(X_{\mathcal{R}}; Y_2)$.

Without loss of generality, we design the codes at the SNR of 0 dB, where we have $I(X_1, X_2; Y_{\mathcal{R}}) = 1.39nt$, $I(X_1, X_2; Y_1) = 0.99nt$, $I(X_{\mathcal{R}}; Y_2) = 0.99n(1-t)$, and the optimal time allocation $t = 0.71$. Hence we have the system achievable sum rate at 0 dB as $1.39nt = 0.99nt + 0.99n(1-t) = 0.99$. In the code design, the length of the codeword X_i is nt, and thus the information length is ntR_i. Also, the relay forwards the network coded digits to the destination. The information part of these network coded digits, i.e., $X_{\mathcal{R}}$, has a length of $n(1-t)R_{\mathcal{R}}$. We determine the code rates as follows. First, we choose the code rates $R_1 = 0.8$ and $R_2 = 0.59$ for X_1 and X_2, respectively. Second, in the relay-to-destination channel, we choose relay's code rate $R_{\mathcal{R}}$ as $R_{\mathcal{R}} = 0.99$. The extra rate provided by the relay (relative to the codeword length) is calculated as $\frac{1-t}{t}R_{\mathcal{R}} = 0.4$. At the destination, the equivalent sum code rate of the two sources is $1.39 - 0.4 = 0.99$, which equals the achievable sum rate of the MARC network at 0 dB (See Fig. 3).

Next, we optimize the code profiles, and design a LDPC code for the FNC as a benchmark, which is denoted by FNC-LDPC code. In the design of FNC-LDPC code, we optimize the codes for $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{R}$, respectively. Each code is designed to be a good code in single link channels. The PNC-LDPC is optimized by following the method in Section IV.

We set $n = 10000$ in the BER simulations. Fig. 4 shows the BER curves of two coding schemes. First, we investigate the decoding performance at the relay. From Fig. 4 we can see that, in the FNC, because the relay has to fully decode two sources' information in a strongly interfered source-to-relay MAC, its decoding performance is much worse than that of the PNC at the relay. The PNC-LDPC has a very good performance at the relay because the PNC makes the source-to-relay MAC as a single link channel for the ternary codeword $C_1 \oplus C_2$. Then we focus on the decoding performance at the destination. We can see that the performance of the proposed PNC-LDPC is about 1.5 dB away from the capacity, i.e., 0 dB, at BER 10^{-4}. Also, our PNC-LDPC code is 0.8 dB better compared with the FNC-LDPC code. The superiority of the PNC-LDPC is due to (a) the good performance at the relay, which reduces the errors propagated to the destination, (b) and our optimized code profile with the iterative receiver.

VI. Conclusion

In this letter we propose a PNC-LDPC code structure based on the multi-edge type LDPC codes for a non-orthogonal MARC network. An iterative detection-and-decoding receiver is designed to deal with the multi-user interference at the destination. We optimize the PNC-LDPC code to approach the achievable rates by utilizing the EXIT chart of the iterative detection-and-decoding receiver. Simulations show that the BER performance of our PNC-LDPC code is only 1.5 dB away from the capacity. Also, relative to the LDPC code optimized for the FNC, the PNC-LDPC code has a gain of 0.8 dB gain.

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