Distributed Space-Time Code with Mutual Information Based Soft Information Relaying

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***Abstract*—In this letter, we investigate a distributed space-time coded soft information relaying. In particular, we focus on mutual information based soft forwarding (MIF) scheme with Alamouti space-time block code. In order to achieve a full diversity order, we propose a weighted amplifying factor based on average soft noise variance instead of instantaneous one. The exact analytical expression for average soft noise variance is first derived. A tight upper bound of the average soft noise variance is also derived in closed-form. It is shown that the proposed distributed MIF Alamouti scheme can achieve a full diversity order, and outperforms various distributed Alamouti schemes reported in the literature.**

***Index Terms*—Diversity order, Alamouti space-time block code, mutual information based soft forwarding.**

I. Introduction

RECENTLY, *cooperative relaying* as a means for extending cell coverage, increasing diversity order, and enhancing power efficiency, has drawn significant attention in both academia and industry. For instance, relaying has been adopted in Long Term Evolution Advanced (LTE-A) Release 10, as one of the cutting edge technologies for next generation commercial wireless communication systems. In order to facilitate the wireless communications assisted by relay, several relay protocols have been proposed, out of which, notables are amplify-forward (AF) [1], detect-forward (DF)[1], and mutual information based soft forwarding (MIF) [2], [3].

A soft forwarding scheme based on symbol-wise mutual information (SMI), referred to as MIF, has been proposed for parallel relay networks in [2]. MIF scheme was proposed to overcome the bottlenecks of AF and DF, where a soft decision of received signals is calculated and forwarded to the destination. MIF scheme has been shown to be very effective in reducing the error propagation to the destination, thereby, achieving an overall better bit error rate (BER) performance. It was shown in [2] that the MIF scheme achieves a superior BER performance in a parallel relay network compared with other memoryless forwarding schemes, such as AF, DF, and EF[4].

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In [5], it is shown that MIF scheme can achieve a full diversity order in orthogonal Rayleigh fading channels. To improve spectral efficiency, in this letter, we focus on MIF scheme in non-orthogonal fading channels. In specific, we propose a relaying scheme based on Alamouti space-time block code [6] in a distributed manner, which is referred to as distributed Alamouti MIF (DA-MIF) scheme. In order to achieve a full diversity order for DA-MIF scheme, we propose a modified amplifying factor at relay where average soft noise variance is employed instead of instantaneous soft noise variance. The exact analytical average soft noise variance is derived. An upper bound of average soft noise variance is also derived in closed form. It is shown that the proposed DA-MIF scheme can achieve a full diversity order, and it outperforms various distributed Alamouti schemes like Alamouti on-off AF scheme [7], and fixed gain Alamouti AF scheme[8].

II. System Model

We consider a parallel relay network in Rayleigh fading channels consisting of one source (S), one destination (D), and two parallel relays $(R_i, i = 1, 2)$, all of which have a single antenna. It is assumed that the direct link between the source and the destination is not available, which may result from heavy shadowing. The two relay nodes use the distributed Alamouti scheme to transmit signals to the destination, and the transmission consists of four time slots. At the first and second time slots, S transmits two binary phase-shift keying (BPSK) modulated symbols $x[1]$ and $x[2]$, respectively. The received signal at the i-th relay at the t-th time slot is given by

$$r_i[t] = \sqrt{P_S} h_{\mathrm{SR}_i} x[t] + n_i[t], \tag{1}$$

where $t = 1, 2$, P_S denotes the source symbol energy, h_{SR_i} is the fading coefficient between the source and i-th relay which is assumed to be fixed during the first two time slots with $h_{\mathrm{SR}_i} \sim \mathcal{CN}(0, 1)$, and $n_i[t]$ is an AWGN with zero mean and variance $\sigma_n^2 = N_0/2$ per dimension.

Assuming perfect knowledge of h_{SR_i} available to the two relays, the associated channel log-likelihood ratio (LLR) is computed at the i-th relay at time t as

$$\Lambda_i[t] = \log \frac{p(r_i[t]|x[t] = +1, h_{\mathrm{SR}_i})}{p(r_i[t]|x[t] = -1, h_{\mathrm{SR}_i})} = \frac{2\sqrt{P_S}}{\sigma_n^2} \mathrm{Re}\{h_{\mathrm{SR}_i}^* r_i[t]\} \tag{2}$$

with conditional probability density function (PDF) $p(r_i[t]|x[t], h_{\mathrm{SR}_i}) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{\left|r_i[t] - h_{\mathrm{SR}_i}\sqrt{P_S}x[t]\right|^2}{2\sigma_n^2}}$.

The soft information at the i-th relay of the MIF scheme in Rayleigh fading channels, denoted by $\tilde{x}_i[t]$, is given by [2]

$$\tilde{x}_i[t] = \text{sign}(\Lambda_i[t])\Theta(\Lambda_i[t]), \quad (3)$$

where $\Theta(\Lambda_i[t])$ denotes the SMI and it is calculated as [2]

$$\Theta(\Lambda_i[t])=\left(\frac{1}{1+e^{\Lambda_i[t]}}\log_2\frac{2}{1+e^{\Lambda_i[t]}}+\frac{1}{1+e^{-\Lambda_i[t]}}\log_2\frac{2}{1+e^{-\Lambda_i[t]}}\right).$$

The soft information $\tilde{x}_i[t]$ can be represented as

$$\tilde{x}_i[t] = \psi_i(x[t] + e_i[t]), \quad (4)$$

where $e_i[t]$ is the soft noise of the soft information $\tilde{x}_i[t]$ at the i-th relay, defined as $e_i[t] = \frac{\tilde{x}_i[t]}{\psi_i} - x[t]$ with variance $\sigma^2_{e,i}$, ψ_i is the scalar coefficient to take into account the correlation between $x[t]$ and the correlated soft noise, and it can be calculated as

$$\begin{aligned}\psi_i &= E[x[t]\tilde{x}_i[t]] = E[x[t]\hat{x}[t]\Theta(\Lambda_i[t])]\\ &= (1-P_e)E[x[t]x[t]\Theta(\Lambda_i[t])] + P_eE[x[t]{-}x[t]\Theta(\Lambda_i[t])]\\ &= (1-2P_e)E[\Theta(\Lambda_i[t])],\end{aligned} \quad (5)$$

where $\hat{x}[t]$ is the hard decision of $x[t]$, P_e is the average bit error probability at the relay, and $E[\Theta(\Lambda_i[t])]$ is the expectation of the symbol-wise mutual information, which is actually the mutual information of the i-th source-to-relay channel.

At the third time slot, the first relay transmits $\beta_1\tilde{x}_1[1]$, and the second relay transmits $-\beta_2\tilde{x}_2^*[2]$, where β_i is the amplifying coefficient at the i-th relay and it is given by

$$\beta_i = \sqrt{\frac{P_R}{\psi_i^2(1+\sigma^2_{e,i})}}. \quad (6)$$

Hence, the received signals at the destination is given by $y[1] = h_{\text{R}_1\text{D}}\beta_1\tilde{x}_1[1] - h_{\text{R}_2\text{D}}\beta_2\tilde{x}_2^*[2] + n[1]$, where $h_{\text{R}_i\text{D}}$ is the Rayleigh fading coefficient between the i-th relay and the destination with $h_{\text{R}_i\text{D}} \sim \mathcal{CN}(0,1)$, and $n[t]$ is an AWGN with zero mean and variance $\sigma_n^2 = N_0/2$ per dimension.

At the fourth time slot, the first relay transmits $\beta_1\tilde{x}_1[2]$, and the second relay transmits $\beta_2\tilde{x}_2^*[2]$. Hence, the received signals at the destination is given by $y[2] = h_{\text{R}_1\text{D}}\beta_1\tilde{x}_1[2] + h_{\text{R}_2\text{D}}\beta_2\tilde{x}_2^*[1] + n[2]$.

The received signals at the destination can be written in matrix form as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{e} + \mathbf{n}, \quad (7)$$

where $\mathbf{y} = \begin{bmatrix} y[1] \\ y[2]^* \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} \psi_1\beta_1h_{\text{R}_1\text{D}} & -\psi_2\beta_2h_{\text{R}_2\text{D}} \\ \psi_2\beta_2h^*_{\text{R}_2\text{D}} & \psi_1\beta_1h^*_{\text{R}_1\text{D}} \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x[1] \\ x[2]^* \end{bmatrix}$, $\mathbf{n} = \begin{bmatrix} n[1] \\ n[2]^* \end{bmatrix}$, and $\mathbf{e} = \begin{bmatrix} \psi_1\beta_1h_{\text{R}_1\text{D}}e_1[1] - \psi_2\beta_2h_{\text{R}_2\text{D}}e_2^*[2] \\ \psi_1\beta_1h^*_{\text{R}_1\text{D}}e_1^*[2] + \psi_2\beta_2h^*_{\text{R}_2\text{D}}e_2[1] \end{bmatrix}$.

Let $\hat{\mathbf{x}} := \begin{bmatrix} \hat{x}[1] \\ \hat{x}^*[2] \end{bmatrix}$, where $\hat{x}[1]$ and $\hat{x}[2]$ denote the estimate of $x[1]$ and $x[2]$, respectively, at the destination and it can be derived as [8]

$$\hat{\mathbf{x}} = \frac{1}{N}\mathbf{H}^H\mathbf{y} = \frac{1}{N}\mathbf{H}^H(\mathbf{Hx}+\mathbf{e}+\mathbf{n}), \quad (8)$$

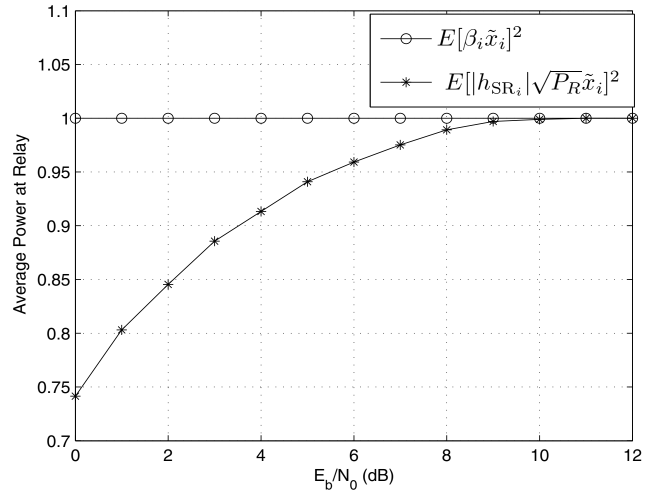

Fig. 1. Average transmit power at the relay.

where $N = \psi_1^2\beta_1^2h^2_{\text{R}_1\text{D}}\sigma^2_{e,1} + \psi_2^2\beta_2^2h^2_{\text{R}_2\text{D}}\sigma^2_{e,2} + \sigma_n^2$. Since $\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \psi_1^2\beta_1^2h^2_{\text{R}_1\text{D}} + \psi_2^2\beta_2^2h^2_{\text{R}_2\text{D}} & 0 \\ 0 & \psi_1^2\beta_1^2h^2_{\text{R}_1\text{D}} + \psi_2^2\beta_2^2h^2_{\text{R}_2\text{D}} \end{bmatrix}$, (8) can be written as

$$\begin{bmatrix} \hat{x}[1] \\ \hat{x}^*[2] \end{bmatrix} = diag\{\lambda,\lambda\}\begin{bmatrix} x[1] \\ x^*[2] \end{bmatrix} + \frac{1}{N}\mathbf{H}^H\mathbf{e} + \frac{1}{N}\mathbf{H}^H\mathbf{n}, \quad (9)$$

where $diag$ means the diagonal matrix with each of the diagonal terms indicated within parenthesis, and $\lambda = \frac{1}{N}(\psi_1^2\beta_1^2h^2_{\text{R}_1\text{D}} + \psi_2^2\beta_2^2h^2_{\text{R}_2\text{D}})$.

Interestingly, it can be seen later that with amplifying factor β_i, given in (6), DA-MIF scheme can not achieve a full diversity order. β_i is a function of instantaneous soft noise variance $\sigma^2_{e,i}$, and it was shown in [5] that $\sigma^2_{e,i}$ varies exponentially with h_{SR_i}. In [10], it has been shown that the variable gain Alamouti AF scheme, *i.e.*, AF scheme with amplifying coefficient, $\beta_{\text{Variable},i} = \sqrt{\frac{P_R}{|h_{\text{SR}_i}|^2+\sigma_n^2}}$, cannot achieve a full diversity order. With similar analogy given in [10], it can be shown that with β_i calculated from instantaneous soft noise variance, the received SNR of DA-MIF scheme goes to a very small value if either of source-relay channels experiences deep fading. It has been shown in [11] that Alamouti AF scheme can achieve a full diversity order with fixed gain amplifying coefficient, $\beta_{\text{AF}} = \sqrt{\frac{P_R}{1+\sigma_n^2}}$. Motivated by the use of fixed gain amplifying coefficient in [11], we propose a new amplifying coefficient ω_i, where the average soft noise variance $E[\sigma^2_{e,i}]$ is used instead of using instantaneous soft noise variance $\sigma^2_{e,i}$ as

$$w_i = |h_{\text{SR}_i}|\sqrt{\frac{P_R}{\psi_i^2(1+E[\sigma^2_{e,i}])}}. \quad (10)$$

In order to achieve a full diversity order, the statistics of both first-hop source-relay channels and second-hop relay-destination channels should be available at the destination. This is the reason why the proposed amplifying factor in (10) is weighted by the source-relay channel fading coefficient. From (5) we can see that at a very high SNR,

$\psi_i \to 1$ as $P_e \to 0$ and $E[\Theta(\Lambda_i[t])] \to 1$ at high SNRs. Moreover, in the next section, we will see that at a very high SNR, $E[\sigma_{e,i}^2] \to 0$. Hence, we can see that at high SNR, w_i converges to $|h_{\mathrm{SR}_i}|\sqrt{P_R}$. As $\tilde{x}_i \in [-1,1]$, for the effective relay signals $|h_{\mathrm{SR}_i}|\sqrt{P_R}\tilde{x}_i$, the average relay power $E[|h_{\mathrm{SR}_i}|\sqrt{P_R}\tilde{x}_i]^2 \le P_R$, so the power constraint at the relay P_R will not be violated, which is illustrated in Fig. 1, where $P_R = 1$ is assumed. We will see later that with the proposed amplifying coefficient the DA-MIF scheme can achieve a full diversity order.

III. Analytical Average Soft Noise Variance and Diversity Analysis of DA-MIF Scheme

In this section, we first derive exact and upper bound expressions for $E[\sigma_{e,i}^2]$ in integral and closed forms, respectively. Then, we prove that the DA-MIF scheme can achieve a full diversity order.

Theorem 1: The exact average soft noise variance at the i-th relay of the MIF scheme in Rayleigh fading channels can be derived as

$$E[\sigma_{e,i}^2]=\left(\frac{2\sqrt{1+2\sigma_n^2}\int_{-\infty}^{\infty}\tilde{x}_i^2|_{(x=1)}e^{\left[\frac{x\Lambda_i}{2}-\sqrt{1+2\sigma_n^2}\,\frac{|\Lambda_i|}{2}\right]}d\Lambda_i}{\sigma_n^2\left(\int_{-\infty}^{\infty}\tilde{x}_i|_{(x=1)}e^{\left[\frac{x\Lambda_i}{2}-\sqrt{1+2\sigma_n^2}\,\frac{|\Lambda_i|}{2}\right]}d\Lambda_i\right)^2}\right)-1, \tag{11}$$

and the asymptotic upper bound of the average soft noise variance at the i-th relay of the MIF scheme in Rayleigh fading channels can be derived as

$$\lim_{\gamma_{\mathrm{in},i}\to\infty} E[\sigma_{e,i}^2] \le \lim_{\gamma_{\mathrm{in},i}\to\infty}\frac{2}{1+\frac{P_S}{\sigma_n^2}}, \tag{12}$$

where $\gamma_{\mathrm{in},i} = h_{\mathrm{SR}_i}^2\frac{P_S}{\sigma_n^2}$.

Proof: The instantaneous exact soft noise variance is given by [2]

$$\sigma_{e,i}^2 = \left(\frac{\int_{-\infty}^{\infty}\tilde{x}_i^2|_{(x=1)}p(\Lambda_i|X=x,h_{\mathrm{SR}_i})d\Lambda_i}{\left(\int_{-\infty}^{\infty}\tilde{x}_i|_{(x=1)}p(\Lambda_i|X=x,h_{\mathrm{SR}_i})d\Lambda_i\right)^2}\right)-1.$$

We can calculate average soft noise variance at i-th relay, $E[\sigma_{e,i}^2]$ as follows

$$E[\sigma_{e,i}^2] = \left(\frac{\int_{-\infty}^{\infty}\tilde{x}_i^2|_{(x=1)}p(\Lambda_i|X=x)d\Lambda_i}{\left(\int_{-\infty}^{\infty}\tilde{x}_i|_{(x=1)}p(\Lambda_i|X=x)d\Lambda_i\right)^2}\right)-1, \tag{13}$$

where $p(\Lambda_i|X=x)$ is the PDF of Λ_i averaged over h_{SR_i} and is given by [9]

$$p(\Lambda_i|X=x) = \frac{\sigma_n^2}{2\sqrt{1+2\sigma_n^2}}\,e^{\left[\frac{x\Lambda_i}{2}-\sqrt{1+2\sigma_n^2}\,\frac{|\Lambda_i|}{2}\right]}. \tag{14}$$

Substituting (14) into (13), we get (11). Based on [6, (11)], an asymptotic upper bound of $\sigma_{e,i}^2$ can be derived as

$$\lim_{\gamma_{\mathrm{in},i}\to\infty}\sigma_{e,i}^2 \leqslant \lim_{\gamma_{\mathrm{in},i}\to\infty} 2\exp\left(-\gamma_{\mathrm{in},i}\right).$$

Hence, the upper bound of average soft noise variance can be calculated as

$$\lim_{\gamma_{\mathrm{in},i}\to\infty} E[\sigma_{e,i}^2] \leqslant \lim_{\gamma_{\mathrm{in},i}\to\infty}\int_0^{\infty}2\exp\left(-h_{\mathrm{SR}_i}^2\frac{P_S}{\sigma_n^2}\right)p(h_{\mathrm{SR}_i})dh_{\mathrm{SR}_i}$$
$$= \lim_{\gamma_{\mathrm{in},i}\to\infty}\int_0^{\infty}2\exp\left(-h_{\mathrm{SR}_i}^2\frac{P_S}{\sigma_n^2}\right)\frac{h_{\mathrm{SR}_i}}{\sigma_n^2}\exp\left(\frac{-h_{\mathrm{SR}_i}^2}{2\sigma_n^2}\right)dh_{\mathrm{SR}_i}$$
$$= \lim_{\gamma_{\mathrm{in},i}\to\infty}\frac{2}{1+\frac{P_S}{\sigma_n^2}}, \tag{15}$$

which completes the proof of Theorem 1. ■

The following theorem quantifies the diversity order of the distributed Alamouti MIF scheme in Rayleigh fading channels.

Theorem 2: The distributed Alamouti MIF scheme can achieve a full diversity order of 2 in Rayleigh fading channels.

Proof: With new amplifying coefficient, w_i, the instantaneous SNR γ_{MIF} of $\hat{\mathbf{x}}$ in (8) is given by

$$\gamma_{\mathrm{MIF}} = \frac{\psi_1^2\omega_1^2h_{\mathrm{R}_1\mathrm{D}}^2+\psi_2^2\omega_2^2h_{\mathrm{R}_2\mathrm{D}}^2}{\psi_1^2\omega_1^2h_{\mathrm{R}_1\mathrm{D}}^2\sigma_{e,1}^2+\psi_2^2\omega_2^2h_{\mathrm{R}_2\mathrm{D}}^2\sigma_{e,2}^2+\sigma_n^2}$$
$$= \gamma_{\mathrm{MIF},1}+\gamma_{\mathrm{MIF},2}, \tag{16}$$

where $\gamma_{\mathrm{MIF},1} = \frac{\tilde{\beta_i}^2|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2}{\tilde{\beta_i}^2|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2\sigma_{e,1}^2+\tilde{\beta_i}^2|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2\sigma_{e,2}^2+\sigma_n^2}$, and $\gamma_{\mathrm{MIF},2} = \frac{\tilde{\beta_i}^2|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2}{\tilde{\beta_i}^2|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2\sigma_{e,1}^2+\tilde{\beta_i}^2|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2\sigma_{e,2}^2+\sigma_n^2}$, with $\tilde{\beta}_i = \sqrt{\frac{P_R}{(1+E[\sigma_{e,i}^2])}}$.

$\gamma_{\mathrm{MIF},1}$ can be further expressed as $\gamma_{\mathrm{MIF},1} = \frac{1}{\frac{1}{\gamma_{\mathrm{out},1}}+\xi_1\frac{1}{\gamma_{\mathrm{out},2}}+\frac{\kappa_1}{\tilde{\beta_i}^2}}$, where $\gamma_{\mathrm{out},1} = \frac{1}{\sigma_{e,1}^2}$, $\gamma_{\mathrm{out},2} = \frac{1}{\sigma_{e,2}^2}$, $\xi_1 = \frac{|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2}{|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2}$, and $\kappa_1 = \frac{\sigma_n^2}{|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2}$. Similarly, $\gamma_{\mathrm{MIF},2}$ can be written as $\gamma_{\mathrm{MIF},2} = \frac{1}{\frac{1}{\gamma_{\mathrm{out},2}}+\xi_2\frac{1}{\gamma_{\mathrm{out},1}}+\frac{\kappa_2}{\tilde{\beta_i}^2}}$, where $\xi_2 = \frac{|h_{\mathrm{SR}_1}|^2h_{\mathrm{R}_1\mathrm{D}}^2}{|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2}$, and $\kappa_2 = \frac{\sigma_n^2}{|h_{\mathrm{SR}_2}|^2h_{\mathrm{R}_2\mathrm{D}}^2}$.

In comparison, the instantaneous SNR in fixed gain Alamouti AF scheme [8] γ_{AF} of $\hat{\mathbf{x}}$ can be derived as

$$\gamma_{\mathrm{AF}} = \frac{\beta_{\mathrm{AF}}^2h_{\mathrm{SR}_1}^2h_{\mathrm{R}_1\mathrm{D}}^2+\beta_{\mathrm{AF}}^2h_{\mathrm{SR}_2}^2h_{\mathrm{R}_2\mathrm{D}}^2}{\beta_{\mathrm{AF}}^2h_{\mathrm{R}_1\mathrm{D}}^2\sigma_n^2+\beta_{\mathrm{AF}}^2h_{\mathrm{R}_2\mathrm{D}}^2\sigma_n^2+\sigma_n^2}. \tag{17}$$

We write γ_{AF} as

$$\gamma_{\mathrm{AF}} = \gamma_{\mathrm{AF},1}+\gamma_{\mathrm{AF},2}, \tag{18}$$

where $\gamma_{\mathrm{AF},1} = \frac{1}{\frac{1}{\gamma_{\mathrm{in},1}}+\xi_1\frac{1}{\gamma_{\mathrm{in},2}}+\frac{\kappa_1}{\beta_{\mathrm{AF}}^2}}$, and $\gamma_{\mathrm{AF},2} = \frac{1}{\frac{1}{\gamma_{\mathrm{in},2}}+\xi_2\frac{1}{\gamma_{\mathrm{in},1}}+\frac{\kappa_2}{\beta_{\mathrm{AF}}^2}}$.

From (12) in Theorem 1, we can see that at a very high SNR, $E[\sigma_{e,i}^2] \to 0$. Hence, at a very high SNR, we can write $\tilde{\beta}_i = \beta_{\mathrm{AF}} \approx \sqrt{P_R}$.

Now, we can easily reach (19), given in top of the next page, where (a) is based on the fact that $\gamma_{\mathrm{out},i} \geqslant \gamma_{\mathrm{in},i}$[5]. Similarly, we can prove that

$$\lim_{\substack{\gamma_{\mathrm{in},1}\to\infty\\ \gamma_{\mathrm{in},2}\to\infty}}\frac{\gamma_{\mathrm{MIF},2}}{\gamma_{\mathrm{AF},2}} \ge 1, \tag{20}$$

which finally proves that $\gamma_{\mathrm{MIF}} \geqslant \gamma_{\mathrm{AF}}$ and gives the Theorem 2. ■

In Fig. 2, we plot the simulated average soft noise variance $E[\sigma_{e,i}^2]$, exact average analytical expression for the soft noise variance $E[\sigma_{e,i}^2]$ in (11), and the upper bound of the $E[\sigma_{e,i}^2]$

$$\lim_{\substack{\gamma_{\text{in},1}\to\infty \\ \gamma_{\text{in},2}\to\infty}} \frac{\gamma_{\text{MIF},1}}{\gamma_{\text{AF},1}} = \lim_{\substack{\gamma_{\text{in},1}\to\infty \\ \gamma_{\text{in},2}\to\infty}} \frac{\gamma_{\text{out},1} P_R \gamma_{\text{out},2}\gamma_{\text{in},2} + \gamma_{\text{out},2} P_R \xi_1 \gamma_{\text{in},1}\gamma_{\text{out},1} + \kappa_1 \gamma_{\text{in},1}\gamma_{\text{in},2}\gamma_{\text{out},1}\gamma_{\text{out},2}}{\gamma_{\text{in},1} P_R \gamma_{\text{out},2}\gamma_{\text{in},2} + \gamma_{\text{in},2} P_R \xi_1 \gamma_{\text{in},1}\gamma_{\text{out},1} + \kappa_1 \gamma_{\text{in},1}\gamma_{\text{in},2}\gamma_{\text{out},1}\gamma_{\text{out},2}} \overset{(a)}{\geq} 1. \quad (19)$$

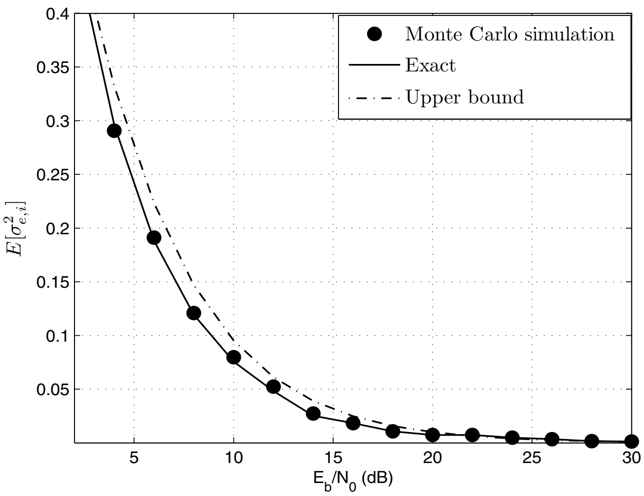

Fig. 2. Exact and upper bound of average soft noise variance of MIF scheme ($P_S = 1$).

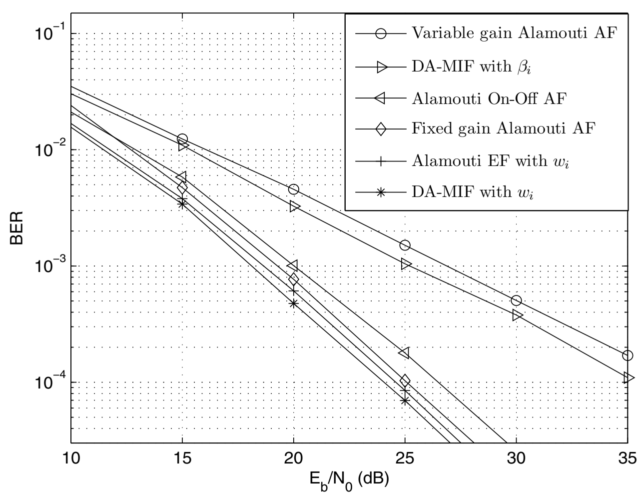

Fig. 3. BER performance comparison of various Alamouti schemes in Rayleigh fading channels.

in (12). The SNR is defined as $\frac{E_b}{N_0} = \frac{P_S}{2\sigma_n^2}$. It is clear that the exact analytical expression for $E[\sigma_{e,i}^2]$ closely matches the Monte Carlo simulation results. It can also be seen that the upper bound is fairly tight at high SNRs. This implies that the exact average soft noise variance can be calculated from channel SNR only, and no knowledge of exact or estimated information bits is required.

The BER performance of the proposed DA-MIF scheme in Rayleigh fading channels is presented in Fig. 3. In the simulation $P_S = P_R$ is assumed. In line with the theoretical prediction of diversity order in Theorem 2, the simulation results show that the proposed DA-MIF scheme with modified amplifying factor, w_i, can achieve a full diversity order of 2. It can be seen that DA-MIF scheme with w_i achieves around 0.6, 1.5 and 2.5 dB SNR gains, over Alamouti EF scheme, fixed gain Alamouti AF scheme[11] and Alamouti on-off AF scheme [7], respectively, at the BER=10^{-5}. The performance of variable gain Alamouti AF scheme with $\beta_{\text{Variable},i}$, and DA-MIF scheme with β_i are also included for reference purpose. Fig. 3 clearly verifies the performance superiority of the proposed DA-MIF scheme.

IV. Conclusion

A novel distributed Alamouti scheme with relaying based on mutual information forwarding (DA-MIF) was proposed. The proposed scheme features a source-relay channel weighted amplifying factor as a function of the average soft noise variance. The exact analytical expression and an upper bound of average soft noise variance in closed form were derived. All analytical results were confirmed by the Monte Carlo simulations. Moreover, it was shown that the proposed DA-MIF scheme can achieve a full diversity order in Rayleigh fading channels and can achieve about $0.6 - 2.5$ dB SNR gains over various distributed Alamouti relaying schemes.

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