LDPC Coded Soft Forwarding with Network Coding for the Two-Way Relay Channel

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Abstract—**This paper investigates a low-density parity-check (LDPC) coded soft-decode-and-forward (SDF) relaying protocol for a two way relay channel (TWRC). In this SDF protocol, the relay evaluates the reliabilities, expressed as log-likelihood ratios (LLRs), of the received signals from the two sources. The relay then forwards a network-coded combination of the parity symbols of both sources, but in the "soft" domain to avoid error propagation from the relay. We introduce a model for the effective noise experienced by the soft network coded symbols, constituting the new parameters of *soft scalar* and *soft error*; this model is then used to compute the log-likelihood ratios (LLRs) at the destination. To facilitate this latter computation, we have derived an analytical expression for soft error variance, as well as a simplified expression based on a common assumption on the statistical behavior of the LLRs. This enables low-complexity computation and tracking of the soft error variance on-the-fly. The proposed system outperforms standard competing schemes reported in the literature in terms of error rate performance over Rayleigh fading links.**

I. INTRODUCTION

Cooperative communication via relays has been widely studied as a potential means to combat the multipath fading effects inherent in wireless communication channels [1]. Two principal relay protocols are amplify-and-forward (AF) and decode-and-forward (DF). The AF protocol has a lower implementation complexity, but the relay amplifies the noise signal. In the DF protocol, the relay completely removes the effects of noise when the received signal is decoded correctly, but in a poor channel condition it may forward an erroneously decoded signal to the destination, thus introducing severe error propagation.

The technique of soft information relaying (SIR) has been proposed [2]-[8] for contemporary wireless systems to reap the benefits of AF and DF while avoiding their respective drawbacks. The idea of transmitting soft symbols was proposed in [2] for an uncoded system. In [3], the implementation of SIR in conjunction with distributed turbo coding (DTC) was studied. The idea of "soft network coding" has been investigated for message exchange in a bi-directional relay system in [5]. A model to incorporate the effective noise associated with SIR has been proposed (for uncoded transmission) in [8] using hard, and Gaussian-distributed soft, errors. Recently, a soft decode-compress-forward scheme was proposed in [9]; this work featured a new model, referred to as the *soft scalar model*, to facilitate the LLR computation at the destination.

In this paper, we consider the problem of information exchange between two users by means of a network-coded relay transmission. We consider a "soft" form of network coding at the relay, and we introduce a model for the effective noise experienced by the soft network coded symbols. This model is similar to that of [9] but in the former case, the model is used for soft bits generated via the expected value of the BPSK symbol ("tanh" function) at the relay; in this paper, the model is used for the network-coded *a posteriori* LLRs produced by the LDPC decoder. This model is then used to compute the log-likelihood ratios (LLRs) at the destination. For this purpose, an analytical expression is derived for the soft error variance, as well as a simplification which is very easy to compute and adapt on-the-fly. This makes the destination's LLR computation precise as well as practical.

In this paper, we investigate a network coded TWRC scheme over fading channels, and consider both the LDPC coded and uncoded systems. Simulation results demonstrate that the proposed scheme is robust even under poor source-relay link conditions, and that the proposed scheme achieves improved error rate performance as compared to competing techniques.

II. SYSTEM MODEL

Two-way communication is a common scenario where two parties transmit information to each other via a common relay. We consider a two way relay channel involving three nodes as shown in Fig. 1. User[1] A broadcasts its message $\boldsymbol{x}_A$ to user B and the relay in the first time slot. In the second time slot, user B transmits a message $\boldsymbol{x}_B$ to the relay and user A. In the third time slot, the relay broadcasts a "soft network coded" signal $\check{\boldsymbol{x}}_R$ to users A and B.

More specifically, in the first time slot, user A encodes a bit vector $\boldsymbol{u}_A$ of length K using an LDPC encoder of rate $R = K/N$ to produce the codeword $\boldsymbol{c}_A$ of length N; this LDPC code is defined by a $(N-K) \times N$ parity check matrix $\boldsymbol{H}$, i.e., we have $\boldsymbol{H}\boldsymbol{c}_A^T = \boldsymbol{0}$. The bit vector $\boldsymbol{c}_A$ is mapped to a

[1]In this paper, the letters A and B denote the users (sources), and the letter R denotes the relay. The subscript $i \in \{A, B\}$ stands for a source, and $\bar{i}$ is the opposite source of i. Vectors are denoted by bold letters, and the jth element of vector $\boldsymbol{a}$ is represented as a^j. We use regular letters to denote scalars (including random variables). For a random variable x, we use $\mathbb{E}(x)$ to denote the expected value of x. The "soft symbol" corresponding to symbol a is represented by $\tilde{a}$.

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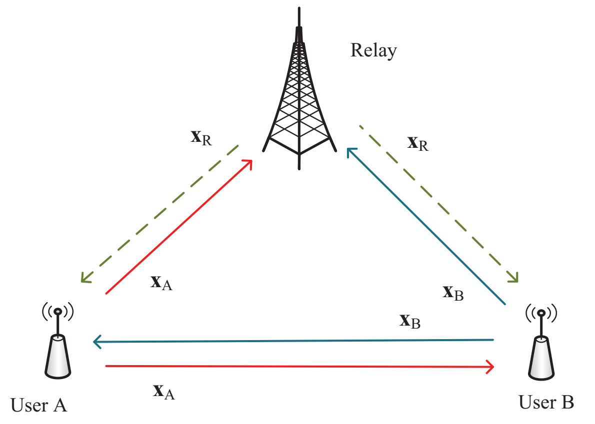

Figure 1. The proposed two way relay system in half-duplex mode.

BPSK symbol vector $\boldsymbol{x}_A$, via the mapping $0 \mapsto +1$, $1 \mapsto -1$, before transmission. In the second time slot, the process at user B is similar to that at user A in the first time slot. The received signals at each of the nodes in the first and second time slots are

$$\boldsymbol{y}_{iR} = \sqrt{P_i} h_{iR} \mathbf{x}_i + \boldsymbol{n}_{iR}, \quad (1)$$

and

$$\boldsymbol{y}_{i\bar{i}} = \sqrt{P_i} h_{i\bar{i}} \mathbf{x}_i + \boldsymbol{n}_{i\bar{i}}, \quad (2)$$

where $\boldsymbol{n}_{iR}$ and $\boldsymbol{n}_{i\bar{i}}$ are vectors having i.i.d. real Gaussian (noise) entries with zero mean and variance σ^2_{iR} and $\sigma^2_{i\bar{i}}$ respectively (both of which are here assumed to be equal to $N_0/2$, where N_0 denotes the channel noise power spectral density). Here $i, \bar{i} \in \{A, B\}$ with $i \neq \bar{i}$. Also, P_i is the transmit power constraint at node i, and h_{ik} stands for the Rayleigh fading coefficient between nodes i and k where $k \in \{\bar{i}, R\}$.

In the third time slot, the relay employs an LDPC decoder for decoding (using the parity-check matrix $\boldsymbol{H}$) the noisy codewords received via the user-relay links. As an attempt to achieve diversity for the third time slot transmission, the relay seeks to broadcast a network-coded parity symbol vector[2] to the users, i.e., $\boldsymbol{c}_R = \boldsymbol{c}_A \oplus \boldsymbol{c}_B$ where $\oplus$ denotes the XOR operation. This is equivalent to the multiplication of the corresponding BPSK symbols, i.e., $x^j_R = \hat{x}^j_A \hat{x}^j_B$, where $\hat{x}^j_i$ is the (BPSK symbol) hard decision corresponding to x^j_i at the relay.

In the low signal-to-noise (SNR) regime, errors may occur in the decoding process at the relay and error propagation may be introduced. Therefore, in the proposed scheme, the relay transmits a "soft" version of these network-coded parity symbols. This process will be elucidated in the next section.

III. Soft Information Relaying Scheme

This section will explain in detail how the relay performs the function of soft information relaying (SIR). The first step required for SIR is the calculation of the *a posteriori* LLRs $\lambda_{iR}(x^j_i|\boldsymbol{y}_{iR}) = \log\left[\frac{P(x^j_i=+1|\boldsymbol{y}_{iR})}{P(x^j_i=-1|\boldsymbol{y}_{iR})}\right]$ for each user i. This computation can be easily performed using an LDPC decoder based on the received signal frame $\boldsymbol{y}_{iR}$ from each user i. Then, the relay computes the corresponding soft network coded symbols. The network coding operation can be approximately implemented in the soft domain using the computed *a posteriori* LLR values as (see, e.g., [5])

$$\tilde{x}^j_R \approx \text{sign}(\tilde{x}^j_A \tilde{x}^j_B) \min\left(|\tilde{x}^j_A|, |\tilde{x}^j_B|\right), \quad (3)$$

where $\tilde{x}^j_A = \lambda_{AR}(x^j_A|\boldsymbol{y}_{AR})$ and $\tilde{x}^j_B = \lambda_{BR}(x^j_B|\boldsymbol{y}_{BR})$. An intuition for this formula is that since $\tilde{x}^j_R$ is an estimate of $x^j_A x^j_B$ from $\tilde{x}^j_A$ and $\tilde{x}^j_B$, its sign should equal the sign of $\tilde{x}^j_A \tilde{x}^j_B$, and its magnitude should not be greater than the minimum of $|\tilde{x}^j_A|$ and $|\tilde{x}^j_B|$. The signal transmitted from the relay can be viewed as the hard decision of the network coded BPSK symbol multiplied by a reliability measurement based on the *a posteriori* LLRs of the underlying source symbols. Therefore, if both $\tilde{x}^j_A$ and $\tilde{x}^j_B$ are reliable, then $\tilde{x}^j_R$ is reliable; on the other hand, if any one of $\tilde{x}^j_A$ and $\tilde{x}^j_B$ is not reliable, then $\tilde{x}^j_R$ is not reliable. Finally, the signal transmitted from the relay can be written as

$$\breve{\boldsymbol{x}}_R = \beta \tilde{\boldsymbol{x}}_R, \quad (4)$$

where the factor β is chosen to satisfy the transmit power constraint at the relay, i.e., $\mathbb{E}((\breve{x}^j_R)^2) = 1$. Thus, the received signal at source i in the third time slot can be written as

$$\boldsymbol{y}_{Ri} = \sqrt{P_R} h_{Ri} \beta \tilde{\boldsymbol{x}}_R + \boldsymbol{n}_{Ri}, \quad (5)$$

where $\boldsymbol{n}_{Ri}$ is a vector having i.i.d. real Gaussian entries each having zero mean and variance $\sigma^2_{Ri} = N_0/2$. In fact, in the proposed scheme, the relay forwards *only the parity portion* of the soft network-coded LLRs; this will help to conserve transmission power at the relay.

A. Calculation of LLR at the Destination

The source A receives two different signals via two independent fading routes in the second and third time slots, i.e., $\boldsymbol{y}_{BA}$ and $\boldsymbol{y}_{RA}$. Similarly, the source B receives $\boldsymbol{y}_{AB}$ and $\boldsymbol{y}_{RB}$ in the first and third time slots respectively. In the following, we describe the computation of LLRs at the destination for the proposed cooperative communication system.

We adopt the following model for the relationship between the correct symbols $x^j_R = x^j_A x^j_B$ and the soft symbols (LLRs) $\tilde{x}^j_R$:

$$\tilde{x}^j_R = \eta x^j_R + \tilde{n}^j, \quad (6)$$

where $\tilde{n}^j$ is called the *soft error* variable, and the constant η is called the *soft scalar* (its effect is somewhat like that of a fading coefficient). Note that this model is quite similar to one proposed in [9], except that here the model is applied to the LLRs and not to the "soft modulated" symbols (an advantage of the current approach is that the LLRs may be modeled as having a Gaussian distribution). For this model, we choose the

[2]Actually, in this paper, the relay is assumed to transmit only the parity portion of this vector. In principle, any subset could be transmitted.

value of η which minimizes the mean-square value of the soft error, i.e., $\eta = \mathbb{E}[x_R \tilde{x}_R]$ (c.f. [9]).

Assuming the model of (5), the received signal at each source i in the third time slot can be written as $\boldsymbol{y}_{Ri} = \sqrt{P}_R h_{Ri} \beta \eta \mathbf{x}_R + \hat{\boldsymbol{n}}_{Ri}$, where $\hat{\boldsymbol{n}}_{Ri} = \boldsymbol{n}_{Ri} + \sqrt{P_R} h_{Ri} \beta \tilde{\boldsymbol{n}}$. We model this equivalent noise $\hat{\boldsymbol{n}}_{Ri}$ as having a Gaussian distribution (which is equivalent to assuming that $\tilde{\boldsymbol{n}}$ is Gaussian). The equivalent zero-mean noise at the destination has variance

$$\hat{\sigma}^2_{Ri} = \sigma^2_{Ri} + P_R h^2_{Ri} \beta^2 \sigma^2_{\tilde{n}}. \tag{7}$$

The analysis provided in Section III-B is focused on the soft error variance $\sigma^2_{\tilde{n}}$, as this needs to be estimated in order to compute (7) at each node i. The LLR corresponding to the third time slot transmission is given by

$$\lambda_{Ri}(x^j_R | \boldsymbol{y}_{Ri}) = \log \left[\frac{P(x^j_R = +1 | \boldsymbol{y}_{Ri})}{P(x^j_R = -1 | \boldsymbol{y}_{Ri})} \right] = \frac{2\sqrt{P}_R h_{Ri} \beta \eta}{\hat{\sigma}^2_{Ri}} y^j_{Ri}. \tag{8}$$

Note that the *a priori* LLR at source i corresponding to the source $\bar{i}$ is easily calculated as

$$\lambda_{\bar{i}i}(x^j_{\bar{i}} | \boldsymbol{y}_{\bar{i}i}) = \frac{2\sqrt{P_{\bar{i}}} h_{\bar{i}i}}{\sigma^2_{\bar{i}i}} y^j_{\bar{i}i}. \tag{9}$$

Next, the network decoded soft symbols at source i are computed via

$$\bar{\lambda}_{Ri}(x^j_{\bar{i}} | \boldsymbol{y}_{Ri}) = \lambda_{Ri}(x^j_R | \boldsymbol{y}_{Ri}) \cdot x^j_i. \tag{10}$$

At each source i, the parity bit LLRs derived from the relay transmission (as given by (10)) will be combined with the parity bit LLRs derived from the transmission from source $\bar{i}$ (as given by (9)) as follows,

$$\boldsymbol{\lambda}^{(p)}_i(\boldsymbol{x}_{\bar{i}}) = \boldsymbol{\lambda}^{(p)}_{\bar{i}i}(\boldsymbol{x}_{\bar{i}} | \boldsymbol{y}_{\bar{i}i}) + \boldsymbol{\lambda}^{(p)}_{Ri}(\boldsymbol{x}_{\bar{i}} | \boldsymbol{y}_{Ri}) \,. \tag{11}$$

Here we adopt the convention that bracketed superscripts s and p designate the systematic (information) and parity sections of the LLR vectors.

Each source i is now in possession of two different LLR vectors, derived from different time slots. The overall LLR vector, for use by the decoder of user i, is computed by concatenating the relevant LLR vectors, i.e.,

$$\boldsymbol{\lambda}^{\text{total}}_i(\boldsymbol{x}_{\bar{i}}) = [\boldsymbol{\lambda}^{(s)}_{\bar{i}i}(\boldsymbol{x}_{\bar{i}} | y_{\bar{i}i}) \; \boldsymbol{\lambda}^{(p)}_i(\boldsymbol{x}_{\bar{i}})], \tag{12}$$

where $\boldsymbol{\lambda}^{(s)}_{\bar{i}i}(\boldsymbol{x}_{\bar{i}} | \boldsymbol{y}_{\bar{i}i})$ are the systematic bit LLRs derived from the transmission from source $\bar{i}$.

B. Soft Error Variance Analysis

In this section, we analyze the variance of the soft error variable as defined by the model of (6). Throughout the section, we will remove the superscript j for ease of presentation, i.e.,

$$\tilde{x}_R = \eta x_R + \tilde{n} \,. \tag{13}$$

In the following, we derive an analytical expression for the soft error variance. We also show that under a commonly used assumption on the decoder output LLRs, a very computationally efficient expression can be derived. First note that since the symbols x_R are equidistributed in $\{-1, +1\}$, we have $\mathbb{E}(x_R) = 0$, and invoking symmetry of the channel, BPSK modulation and LDPC decoding process we also have $\mathbb{E}(\tilde{x}_R) = 0$; it follows that $\mathbb{E}(\tilde{n}) = 0$.

Our basic assumption, which is motivated by the symmetry of BPSK modulation as well as that of LDPC decoding, is that

$$p_{\tilde{x}_R}(\Lambda | x_R = 1) = p_{\tilde{x}_R}(-\Lambda | x_R = -1) \tag{14}$$

for all $\Lambda \in \mathbb{R}$ (see [10]).

First, we have the following lemma.

Lemma 1. *The PDF of the soft error variable conditioned on the network-coded relay symbol satisfies* $p_{\tilde{n}}(\Lambda | x_R = 1) = p_{\tilde{n}}(-\Lambda | x_R = -1)$ *for all* $\Lambda \in \mathbb{R}$.

Proof: From (13), $x_R = 1$ is equivalent to $\tilde{x}_R = \eta + \tilde{n}$, and $x_R = -1$ is equivalent to $\tilde{x}_R = -\eta + \tilde{n}$. Therefore, for an arbitrary value Λ, we have

$$\begin{aligned} p_{\tilde{n}}(\Lambda | x_R = 1) &= p_{\tilde{x}_R}(\Lambda + \eta | x_R = 1) \\ &= p_{\tilde{x}_R}(-\Lambda - \eta | x_R = -1) \\ &= p_{\tilde{n}}(-\Lambda - \eta + \eta | x_R = -1) \\ &= p_{\tilde{n}}(-\Lambda | x_R = -1). \end{aligned} \tag{15}$$

where in the second line we have used (14). ■

The following corollary shows that the PDF of the soft error variable possesses even symmetry.

Corollary 1. *The PDF of the soft error variable satisfies* $p_{\tilde{n}}(\Lambda) = p_{\tilde{n}}(-\Lambda)$ *for all* $\Lambda \in \mathbb{R}$.

Proof: We have

$$\begin{aligned} & p_{\tilde{n}}(\Lambda) \\ &= p_{\tilde{n}}(\Lambda | x_R = 1) p(x_R = 1) + p_{\tilde{n}}(\Lambda | x_R = -1) p(x_R = -1) \\ &= \frac{1}{2} p_{\tilde{n}}(\Lambda | x_R = 1) + \frac{1}{2} p_{\tilde{n}}(-\Lambda | x_R = 1) \\ &= p_{\tilde{n}}(-\Lambda). \end{aligned} \tag{16}$$

■

The following lemma proves that the soft scalar η is independent of conditioning on x_R.

Lemma 2. $\mathbb{E}(\tilde{x}_R | x_R = +1) = -\mathbb{E}(\tilde{x}_R | x_R = -1) = \eta$.

Proof: We have

$$\begin{aligned} \mathbb{E}(\tilde{x}_R | x_R = +1) &= \int_{-\infty}^{\infty} p_{\tilde{x}_R}(\Lambda | x_R = 1) \Lambda d\Lambda \\ &= \int_{-\infty}^{\infty} p_{\tilde{x}_R}(-\Lambda | x_R = -1) \Lambda d\Lambda \\ &= -\int_{-\infty}^{\infty} p_{\tilde{x}_R}(\Lambda | x_R = -1) \Lambda d\Lambda \\ &= -\mathbb{E}(\tilde{x}_R | x_R = -1) \,, \end{aligned} \tag{17}$$

where in the second line we have used (14). This may be rewritten as $\mathbb{E}(x_R \tilde{x}_R | x_R = +1) = \mathbb{E}(x_R \tilde{x}_R | x_R = -1)$, and therefore both of these quantities are equal to $\frac{1}{2}[\mathbb{E}(x_R \tilde{x}_R | x_R = +1) + \mathbb{E}(x_R \tilde{x}_R | x_R = -1)] = \mathbb{E}(x_R \tilde{x}_R) = \eta$. ■

Next, note that the soft error variance may be expressed as

$$\begin{aligned}\mathbb{E}(\tilde{n}^2) &= \mathbb{E}((\tilde{x}_R - \eta x_R)^2), \\ &= \mathbb{E}(\tilde{x}_R^2) - 2\eta\mathbb{E}(x_R\tilde{x}_R) + \eta^2, \\ &= \mathbb{E}(\tilde{x}_R^2) - \eta^2 \; . \end{aligned} \quad (18)$$

Next, from lemma 2, we can directly derive the following upper bound for soft error variance

$$\sigma_{\tilde{n}}^2 \leq \mathbb{E}(\tilde{x}_R^2), \quad (19)$$

where η^2 is a positive value.

Next, define $\mu^+ = \mathbb{E}(\tilde{x}_R | x_R = +1)$. The next result shows that a very simple formula for the soft error variance can be derived if we make the assumption (commonly adopted at high SNR, c.f. [10]) that the mean of the LLR, conditioned on the transmission of the symbol $+1$, is equal to twice its variance, i.e.,

$$\mathbb{E}((\tilde{x}_R - \mu^+)^2 | x_R = +1) = 2\mu^+ \; . \quad (20)$$

Theorem 1. *Under the assumption of* (20)*, the soft error variance can be expressed as* $\sigma_{\tilde{n}}^2 = 2\eta$*.*

Proof: First note that $\mu^+ = \eta$ by Lemma 2. Also,

$$\begin{aligned}\mathbb{E}(\tilde{n}^2 | x_R = +1) &= \int_{-\infty}^{\infty} p_{\tilde{n}}(\Lambda | x_R = 1)\Lambda^2 d\Lambda \quad (21) \\ &= \int_{-\infty}^{\infty} p_{\tilde{n}}(-\Lambda | x_R = -1)\Lambda^2 d\Lambda \\ &= \int_{-\infty}^{\infty} p_{\tilde{n}}(\Lambda | x_R = -1)\Lambda^2 d\Lambda \\ &= \mathbb{E}(\tilde{n}^2 | x_R = -1) \; ,\end{aligned}$$

and so $\mathbb{E}(\tilde{n}^2 | x_R = +1) = \mathbb{E}(\tilde{n}^2 | x_R = -1) = \sigma_{\tilde{n}}^2$. Therefore,

$$\begin{aligned}\sigma_{\tilde{n}}^2 &= \mathbb{E}(\tilde{n}^2 | x_R = 1) \\ &= \mathbb{E}((\tilde{x}_R - \eta)^2 | x_R = 1)) \\ &= \mathbb{E}((\tilde{x}_R - \mu^+)^2 | x_R = 1)) \\ &= 2\mu^+ \\ &= 2\eta \; ,\end{aligned} \quad (22)$$

where in the fourth line we have used (20). ■

The expression (18) can be used to directly estimate the soft error variance, as the two terms involved can be estimated at the receiving node. However, if the assumption of (20) holds, Theorem 1 provides a very computationally efficient means (via (22)) to compute the soft error variance. This expression is easy to compute on-the-fly, and has very low implementation complexity.

Note that estimation of the parameter $\eta = \mathbb{E}[x_R\tilde{x}_R]$ requires knowledge of the transmitted data symbols at the relay. Therefore, in practice estimation of η could be initiated by a training phase, and during data transmission η could be updated every time a frame is correctly decoded (using a syndrome check at the relay). This also allows for automatic adaptation of η to the channel conditions. On the other hand, (22) is data independent and computationally efficient. Finally, as another alternative, the upper bound on the soft error variance formed by the first term of (18) could be used as it does not require any knowledge of the data symbols.

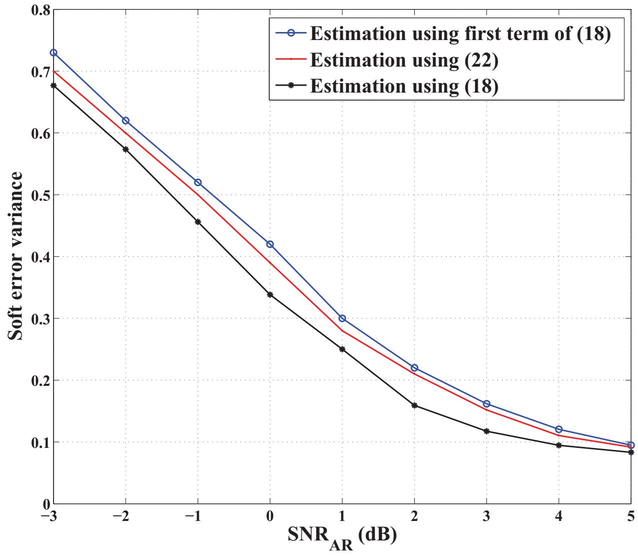

Figure 2. The soft error variance as estimated using (18), using (22), and using the first term of (18).

IV. Simulation Results and Discussion

In this section, we evaluate the performance of the proposed SDF scheme. The relevant dimensions of the parity-check matrix are $N = 816$ and $K = 408$. The simulations assume BPSK and a power normalization of $P_A = P_B = P_R = 1$. We simulate the scenario where all links experience Rayleigh fading and AWGN. All simulations assume $\text{SNR}_{AR} = \text{SNR}_{BR}$.

Fig. 2 compares the estimated value of $\sigma_{\tilde{n}}^2$ obtained using (18) against that obtained using (22). In the same figure, the upper bound on the soft error variance as obtained by using only the first term in (19) is also illustrated. The upper-bound-based result becomes closer to those of (18) and (22) in the high SNR regime. Note that the upper bound can be computed and tracked without knowledge of the data symbols at the relay, but that the other two results are more accurate fit to the soft scalar model.

Fig. 3 compares the performance of the LDPC code based SDF scheme with soft error variance estimated using (18) with that where the soft error variance is estimated using (22). As a benchmark scheme, the hard network coded two-way relay system is also depicted here. The SNR of each source-relay link was fixed at 3dB to represent a poor source-relay channel condition. The SDF scheme with soft error variance estimated using (18) yields approximately 0.6dB performance gain over that using the simplified form of the soft error variance as given by (22). Both of these schemes show a significant improvement over the hard DF scheme. The SDF protocol yields considerable gain with respect to the hard DF scheme in the regime of low source-relay SNR, regardless of the method of estimation of η, since in the latter scheme

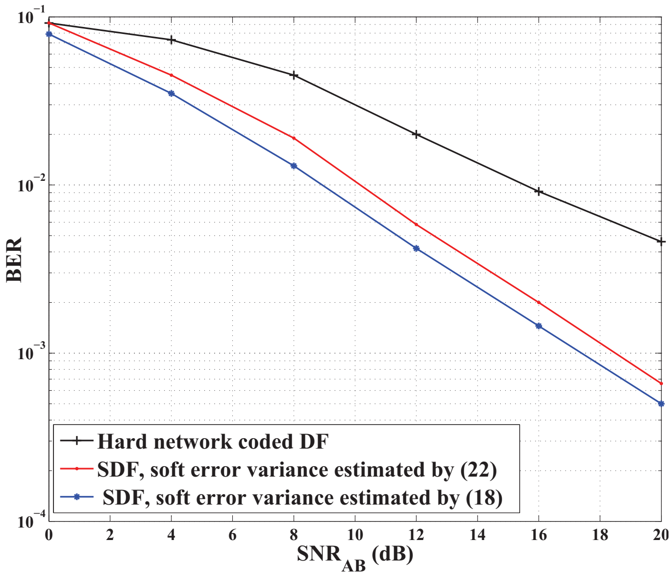

Figure 3. A comparison of the BER performance of the proposed LDPC coded SDF scheme with that of hard network-coded DF relaying. The source-relay link SNRs were both kept constant at 3dB.

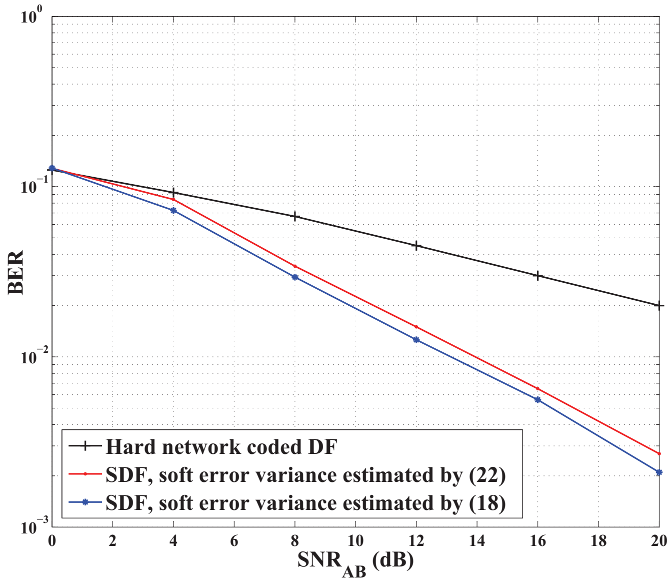

Figure 4. A comparison of the BER performance of the proposed uncoded SDF scheme with that of uncoded hard network-coded DF relaying. The source-relay link SNRs were both kept constant at 3dB.

erroneous decoding results in the communication of incorrect symbols to the destination.

Fig. 4 shows the error performance for uncoded SDF scheme with soft error variance estimated using (18) and with (22). As a competing scheme, the uncoded hard network coded two-way relay system is also depicted here. From Fig. 4 we can see that uncoded 'Hard network coded DF' can only achieve a diversity of one, while all the other protocols can achieve the full diversity gain of the system, i.e., a diversity of two. Our uncoded SDF with soft error variance estimated using (18) and (22) are outperformed uncoded 'Hard network coded DF' scheme in terms of BER performance.

However, there is a trade-off between the error performance and the computational complexity. The soft error variance expression (22) is computationally simpler to estimate than the soft error variance as given by (18). For both methods, the computation of $\sigma_{\tilde{n}}^2$ starts in a training phase and then is updated on-the-fly. Note that both forms of the soft network coded SDF system offer clear advantages for a source-relay channel which exhibits strong variations in source-relay SNR.

V. Conclusion

This paper investigates an LDPC code based SDF relay protocol in the TWRC. We have derived an analytical expression for soft error variance, as well as a simplified expression. Both forms facilitate the estimation of the soft error variance without having to access the actual or estimated information signal of the sources. The computation of the simplified expression for soft error variance is easy to adjust on-the-fly, and has a very low implementation complexity. This makes the LLR computation at the destination more precise as well as adaptable to changing channel conditions.

Acknowledgment

This work was supported by the Telecommunications Graduate Initiative (TGI), which is funded by the Irish Higher Education Authority (HEA) under the Programme for Research in Third-Level Institutions (PRTLI) Cycle 5.

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