Performance Analysis of Relay Selection for Bi-Directional Cooperative Network

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***Abstract*—In this paper, we compare three different relay schemes and propose a mixed relay selection algorithm in a bi-directional cooperative networks. In the multiple access phase, both terminals transmit their messages to *N* half-duplex relay nodes in an orthogonal way. Then, the relays take the mixed scheme, a threshold-based adaptive relay selection scheme combining conventional partial decode-and-forward and compress-and-forward, to optimize the relay function while satisfying a given outage requirement. Simulation results show that the proposed mixed relay selection strategy optimize the relay function via functional analysis such that the average probability of error is minimized in the high signal-to-noise ratio (SNR) regime.**

***Index Terms*—Cooperative relaying, bi-directional communication, relay selection, decode-and-forward, compress-and-forward**

I. INTRODUCTION

Relay-assisted communication is a promising solution for the high system throughput and greater coverage. Recently, Bletsas et al. [1] did an interesting work utilizing selection diversity among cooperative users, in which, an algorithm of opportunistic relaying as an efficient cooperative diversity scheme has been proposed. In concrete, the "best" relay between source and destination is selected based on instantaneous channel state information (CSI), while neither topology information nor communication among the relays is needed.

In the literatures, numerous protocols has been presented to choose the best relay among a collection of available relays. In [2], the best relays are selected based on geographic information. In [4], Luo considered a best-selection relay scheme, in which, only the relay that has received the transmission data from the source correctly and has the highest mean signal-to-noise ratio (SNR) to the destination, is chosen to forward the source's data. In [5], Ibrahim propose a relay-selection scheme for single-relay decode-and-forward cooperative systems, in which, the source decides whether or not employ the relay to forward its data based on the instantaneous information of the source-destination and source-relay channels gain.

As a cooperative relay scheme, the bi-directional relay system has attracted considerable attention recently [6], in which two terminals simultaneously exchange their information to each other. In this paper, we focus on the two-hop bi-directional cooperative system with two terminals and N relays. From fig. 1, we can see T_1 (or T_2) is out of the transmission range of T_2 (or T_1). The shared relays can help T_1 and T_2 exchange their information. Let $h_{i,j}$ denote the channel between *i* and *j*. We assume that $|h_{i,j}|$ is a Rayleigh-distributed random variable, with zero mean and unit variance. In time slot 1, terminal 1 and terminal 2 broadcast their information to N relays simultaneously. Thus, at the end of time slot 1, relay r_i receives

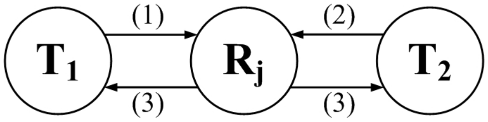

Fig. 1. Bi-directional cooperative network. (a) In time slot 1, both terminal 1 and terminal 2 transmit the message simultaneously to the relays denoted by (1) and (2); In time slot 2, the relay group broadcast the processed signals to two terminals, denoted by (3).

$$y_{r_i,1} = h_{t_1,r_i}x_{t_1} + h_{t_2,r_i}x_{t_2} + n_{r_i,1}, \tag{1}$$

where x_{t_1}, x_{t_2} and $y_{r_i,1}$ denote the transmitted signal of terminal 1, terminal 2 and the signals received at the i_{th} relay in time slot 1, respectively. h_{t_1,r_i} and h_{t_2,r_i} are channel coefficients of the terminal 1 to relay *i* and terminal 2 to relay *i*. $n_{r_i,1}$ is additive circularly symmetric white Gaussian noise (AWGN) in the corresponding channel, with variance $N_{r_i,1}$.

We will perform outage analysis for the the two-hop bi-directional system. According to the instantaneous channel information, we divide the relay subsets, and derive an approximation formulation of outage probability in high-SNR regime in terms of distance. Furthermore a novel relay selection greedy algorithm based on statistic CSI (distance information between terminal node and relay node) and instantaneous CSI (channel information between relay node and destination terminal node) are proposed, respectively. Simulation results shows that the proposed subset power allocation method and Greedy Algorithm based relay selection system yield optimal rate with instantaneous CSI. And Power allocation factor ξ has a great impact on rate performance.

II. OUTAGE ANALYSIS

The destination and all candidate relay nodes employ typical set decoding to decode x_{t_1}, x_{t_2}. The candidate relays

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decode both x_{t_1} and x_{t_2} simultaneously. Two channel SNR thresholds, η_1 and η_2, are chosen to determine the set of received rates for this two-level coding strategy. The two-way communication consists of two phases, namely a multiple access channel (MAC) phase and a broadcast channel (BC) phase. An achievable rate region for such a relay channel was proven in [6]. It consists of all rate pairs $[R_1, R_2]$ satisfying

$$R_1 \leq \min\left[\alpha I(X_1;Y_R|X_2,Q), \beta I(X_R;Y_2)\right]. \tag{2}$$

$$R_2 \leq \min\left[\alpha I(X_2;Y_R|X_1,Q), \beta I(X_R;Y_1)\right]. \tag{3}$$

$$R_1 + R_2 \leq \alpha I(X_1,X_2;Y_R|Q). \tag{4}$$

for some joint probability distributions $p(x_R)p_2(y_1,y_2|x_R)$ and $p(q)p(x_1|q)p(x_2|q)p_1(y_R|x_1,x_2)$ and some $\alpha,\beta > 0$ with $\alpha+\beta = 1$. This region was obtained under the assumption that the relay nodes decode the messages of both the terminals in the MAC phase.

Then we can categorize the relays into four subsets for tranmission.

- κ_0: For relay *i*, if $|h_{t_1,i}| < |\eta_1|$ and $|h_{t_2},i| < |\eta_2|$, then it can not decode neither x_{t_1} nor x_{t_2}, and it does not transmit in time slot 2 .
- κ_1: If $|h_{t_1,i}| > |\eta_1|$ and $|h_{t_2},i| < |\eta_2|$, then a selected relay *i* can only decode x_{t_1}, and will forward x_{t_1} only to the terminal 2 in time slot 2.
- κ_2: If $|h_{t_1,i}| < |\eta_1|$ and $|h_{t_2},i| > |\eta_2|$, then the selected relay *i* can only decode x_{t_2}, and will forward x_{t_2} to the terminal 1 in time slot 2.
- κ_3: If $|h_{t_1,i}| > |\eta_1|$ and $|h_{t_2},i| > |\eta_2|$, then a selected relay *i* can decode both x_{t_1} and x_{t_2}, then will forward x_{t_1} and x_{t_2} to both terminals in time slot 2.

Suppose that the total transmission power is P_t, and the source allocate power ξP_t to terminal 1 and $(1-\xi)P_t$ to terminal 2, respectively, where $\xi \in [0,1]$. Thus, power P_{R_1}, P_{R_2} and $P_{R_{12}}$ are respectively allocated to relay subset κ_1, κ_2 and κ_3 by the following rules.

1. Relaying Protocols

(1) Fixed Relay Selection Schemes

A. DF-based cooperative diversity scheme: when source-relay link has perfect CSI, DF relay protocol can achieve optimal throughput. Thus relay nodes in subsets κ_1 and κ_2 partial decode and forward the received messages to the two terminals. While relay nodes in subset κ_3 decode and forward the signal transmitted from the terminals. We do not consider relay nodes in subset κ_0 in this scheme.

B. EF-based cooperative diversity scheme: when source-relay link has close CSI as relay-destination link, and relay-destination link has perfect CSI, EF relay protocol can achieve optimal throughput. Thus relay nodes in subset κ_0 compress the received signal and broadcast it to two terminals. Like in part A, we do not consider relay nodes in subset κ_1, κ_2 and κ_3 in this scheme.

(2) Threshold-based Adaptive Mixed Relay Selection Scheme

We begin by considering BPSK signaling at the terminals. Each terminal transmits X_{t_i}

$$x_{r,i} = \begin{cases} 0, & \text{if } |h_{t_1,i}| < |\eta_1|,|h_{t_2},i| < |\eta_2|, \\ \sqrt{\frac{P_{R_1}}{\xi\, P_t}} x_{t_{1,i}}, & \text{if } |h_{t_1,i}| > |\eta_1|,|h_{t_2},i| < |\eta_2|, \\ \sqrt{\frac{P_{R_2}}{(1-\xi)P_t}}\, x_{t_{2,i}}, & \text{if } |h_{t_1,i}| < |\eta_1|,|h_{t_2},i| > |\eta_2|, \\ \sqrt{\frac{P_{R_{1,2}}}{P_t}}(x_{t_{1,i}} + x_{t_{2,i}}), & \text{if } |h_{t_1,i}| > |\eta_1|,|h_{t_2},i| > |\eta_2|, \end{cases} \tag{5}$$

where $x_{r,i}$ are the power allocated symbols for relay i. Then $\cup_{l=1}^{3}\kappa_l$ is the set of all relays that transmit in time slot 2,

Our proposed multiple-relay selection algorithm chooses $\cup_{l=1}^{3}\kappa_l$ to maximize the expected rate subject to a sum power constraint over all relays $l \in \cup_{l=1}^{3}\kappa_l$. The maximum average mutual information between node i and node j in this case, achieved by independent and identically distributed ($i.i.d.$) zero-mean, circularly symmetric complex Gaussian inputs, is given by

$$I_{i,j} = \log\left(1 + SNR|h_{i,j}|^2\right). \tag{6}$$

Thus, the outage event for spectral efficiency R is given by $I_{i,j} < R$, where R is the transmission rate, and equivalent to the event [3]

$$|h_{i,j}|^2 < \frac{2^R - 1}{SNR}. \tag{7}$$

The maximum average mutual information for repetition-coded Decode-and-Forward cooperative transmission can be shown to be

$$I_{DF} = \min(\max(\min(\phi_1), \min(\phi_2)), \max(\min(\phi_3), \min(\phi_4))), \tag{8}$$

where ϕ_1 is the maximum rates at which terminal 2 can reliably decode the selected relays messages from subset κ_1; ϕ_2 is the maximum rates at which terminal 2 can reliably decode the selected relays messages from subset κ_3; ϕ_3 represents the maximum rates at which terminal 1 can reliably decode the selected relays messages from subset κ_2; ϕ_4 represents the maximum rates at which terminal 1 can reliably decode the selected relays messages from subset κ_3. Specifically, $\phi_1 = (C_{s_1r_1}, C_{r_1s_2})$,$\phi_2 = (C_{s_1r_{12}}, C_{r_3s_2})$, $\phi_3 = (C_{s_2r_2}, C_{r_2s_1})$, and $\phi_4 = (C_{s_2r_3}, C_{r_3s_1})$.

The outage event for spectral efficiency R is given by $I_{DF} < R$ and is equivalent to the following events

$$\begin{aligned} \min(|h_{s_1r_1}|^2, |h_{r_1s_2}|^2) &< \frac{2^{2R_1}-1}{SNR}, \\ \min(|h_{s_1r_3}|^2, |h_{r_3s_2}|^2) &< \frac{2^{2R_{12}}-1}{SNR}, \\ \min(|h_{s_2r_2}|^2, |h_{r_2s_1}|^2) &< \frac{2^{2R_2}-1}{SNR}, \\ \min(|h_{s_2r_3}|^2, |h_{r_3s_1}|^2) &< \frac{2^{2R_{12}}-1}{SNR}. \end{aligned} \tag{9}$$

For Rayleigh fading, the outage probability for repetition-coded decode-and-forward can be computed according to [3]

$$\begin{aligned} & P_1^{out}(SNR, R) \\ & = P_r[I_1 < R_1] \\ & = P_r[|h_{s_1 r_1}| < Q(SNR)] \\ & + P_r[|h_{s_1 r_1}| \geq Q(SNR)] P_r[|h_{r_1 s_2}| < Q(SNR)], \end{aligned} \tag{10}$$

where $Q(SNR) = [2^{2R} - 1]/SNR$. The first term represents that terminal 1 cannot communicate with selected relays in subset 1. And the second term represents that although terminal 1 successfully communicate with selected relays in subset 1, but fail to communicate with terminal 2. Similarly, we can derive the representations of P_2^{out}, P_3^{out} and P_4^{out} similarly. Although we may readily compute a close-form expression for (10), for compactness we at first study behavior of them at large SNR by computing the limit

$$\begin{aligned} & \frac{1}{Q(SNR)} P_1^{out}(SNR, R) \\ & = \frac{1}{Q(SNR)} P_r[|h_{s_1 r_1}| < Q(SNR)] \\ & + P_r[|h_{s_1 r_1}| \geq Q(SNR)] \frac{1}{Q(SNR)} P_r[|h_{r_1 s_2}| < Q(SNR)] \end{aligned} \tag{11}$$

as $SNR \rightarrow \infty$. Using the results of Facts 1 and 2 in [1], we conclude that

$$\begin{aligned} P_{DF}^{out} = & P_1^{out} P_2^{out} + P_3^{out} P_4^{out} \\ = & \left(\frac{1}{\sigma_{s_1 r_1}^2} \frac{2^{2R_1 - 1}}{SNR} + \frac{d^\alpha}{\sigma_{r_1 s_2}^2} \frac{2^{2R_1 - 1}}{SNR}\right) \\ & \left(\frac{1}{\sigma_{s_1 r_3}^2} \frac{2^{2R_{12} - 1}}{SNR} + \frac{d^\alpha}{\sigma_{r_3 s_2}^2} \frac{2^{2R_{12} - 1}}{SNR}\right) \\ + & \left(\frac{1}{\sigma_{s_2 r_2}^2} \frac{2^{2R_2 - 1}}{SNR} + \frac{d^\alpha}{\sigma_{r_2 s_1}^2} \frac{2^{2R_2 - 1}}{SNR}\right) \\ & \left(\frac{1}{\sigma_{s_2 r_3}^2} \frac{2^{2R_{12} - 1}}{SNR} + \frac{d^\alpha}{\sigma_{r_3 s_1}^2} \frac{2^{2R_{12} - 1}}{SNR}\right) \end{aligned} \tag{12}$$

We can see that P_{DF}^{out} is a *signomial function* of distance in the high-SNR regime. Thus we consider an optimal relay placement algorithm to minimize the outage probability or transmission rate of the bi-directional system.

III. RATE-MAXIMIZING RELAY SELECTION

Consider the case where a subset of the available relay nodes $(1, 2, ..., N)$ are selected to assist the sources. Let $h_{i,j}$ denote the channel between a transmitting node and a receiving node. The received rate at a receiving node via decoding x_{t_1} is

$$C_1(|h|^2) = \frac{1}{2} \log_2 \left(1 + SNR \cdot |h_{t_1,i}|^2\right), \tag{13}$$

and the received rate at a receiving node via decoding x_{t_2} is

$$C_2(|h|^2) = \frac{1}{2} \log_2 \left(1 + SNR \cdot |h_{t_2,i}|^2\right). \tag{14}$$

Finally, the received rate at a receiving node via decoding both x_{t_1} and x_{t_2} is

$$C_{1,2}(|h|^2) = \frac{1}{2} \log_2 \left(1 + SNR \cdot |h_{t_1,i}|^2 + SNR \cdot |h_{t_2,i}|^2\right). \tag{15}$$

The expected rate of the superposition coding strategy is

$$\begin{aligned} \bar{R}(N) = & (1 - P_{out}(R_1, \kappa_1)) P_{out}(R_2, \kappa_2) R_1 \\ & + (1 - P_{out}(R_2, \kappa_2)) P_{out}(R_1, \kappa_1) R_2 \\ & + (1 - P_{out}(R_{1,2}, \kappa_3)) R_{1,2}, \end{aligned} \tag{16}$$

So the relay selection problem can be formulated as follows.

$$\begin{aligned} & \max_{\kappa \subseteq \{1,2,\cdots,N\}} \bar{R}(\kappa), \\ & \text{subject to} \sum_{i \in \kappa} P_i \leq P_{max}, \\ & \text{and} \quad 0 \leq P_i \leq P_{i,max}, \quad \forall i \in \kappa. \end{aligned} \tag{17}$$

The previous analysis shows that in the high-SNR regime, $\bar{R}(N)$ is a nonlinear function of P_i for $i \in \kappa$. So we approximate the relay selection problem as a relay position problem. Here we hypothetically place n relays in the positions that would maximize $\bar{R}(N)$. Then, n relays that are close to the rate-maximizing positions are selected.

$$\begin{aligned} (x_1^*, y_1^*, \cdots, x_n^*, y_n^*) = arg & \max_{x_1, y_1, \cdots, x_n, y_n} \bar{R}(\kappa), \\ & \text{subject to} \quad ||\kappa|| = n, \\ & \sum_{i \in \kappa} P_i \leq P_{max}, \\ & \text{and} \quad 0 \leq P_i \leq P_{i,max}, \quad \forall i \in \kappa. \end{aligned} \tag{18}$$

We assume that $-a < x_i < a$ for each relay i since the relays are interspersed throughout the region between the two terminals. Let the n selected relays be located at $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$. In the high-SNR regime, $\bar{R}(N)$ is a *polynomial function* of $(x_1, y_1, \cdots, x_n, y_n)$.

Now we consider a simple case, a line network. Assume that the relay is located at $(d_i, 0)$, and distributed in the x-axis uniformly, which means d_i is a uniform-distributed random variable. Then $d_{t_1,i} = |d_i + a|$ and $d_{t_2,i} = |d_i - a|$. The rate-maximizing locations are derived from the following *proximity-based greedy algorithm*.

1) A candidate set, from which a solution is created. We first define the relay location vector $[D_1, D_2, ..., D_n] = [(x_1, y_1), (x_2, y_2)..., (x_n, y_n)]$ and the channel information vector $[H_1, H_2, ..., H_n] = [(|h_{s_1 r_1}|^2, |h_{r_1 s_2}|^2), (|h_{s_1 r_2}|^2, |h_{r_2 s_2}|^2), ..., (|h_{s_1 r_n}|^2, |h_{r_n s_2}|^2)]$. Then the effect of statistic CSI factor with instantaneous CSI factor are combined. $(D_1, H_1), (D_2, H_2), ..., (D_n, H_n)$ are the candidate sets, where $n \in (1, 2, 3, ..., N)$,
2) A selection function, which chooses the best candidate to be added to the solution. Here we define the function $f(D_i, H_i)$ to be our selection function.
3) A feasibility function, which is used to determine if a candidate can be used to contribute to a solution. Here

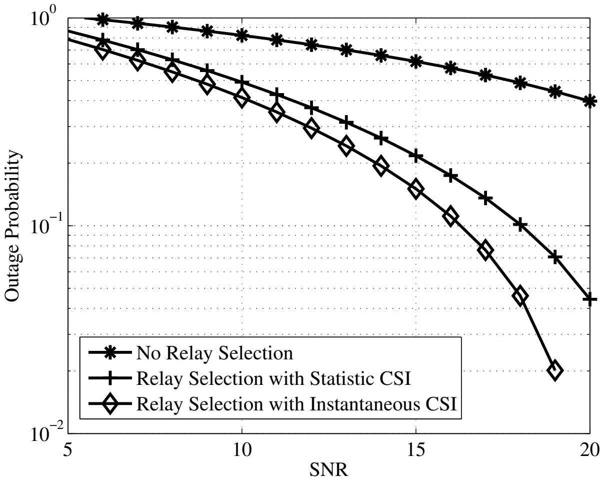

Fig. 2. Outage Probability

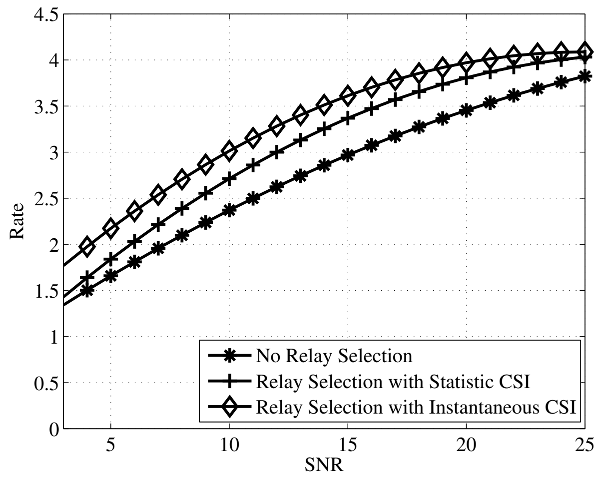

Fig. 3. Rate as a function of average received SNR

$P_j = P_{max}/n$ is used as a feasibility function, where P_j denotes the power of jth relay, P_{max} is the total power of the system, and n is the total number of relays we want to choose from the three subsets.

4) An objective function, which assigns a value to a solution, or a partial solution.
5) A solution function, which will indicate when we have discovered a complete solution. We define $(D_1^*, H_1^*), (D_2^*, H_2^*), ..., (D_n^*, H_n^*)$ as the optimal solution to maximize the received rate $\bar{R}_N$.

IV. NUMERICAL RESULTS

We place the terminal 1 at $(-20, 0)$ and the terminal 2 at $(20, 0)$. We set the variance of noise to be $\sigma^2 = -104dBm$. We also have a carrier frequency $f_c = 2.4GHz$ along with a reference distance $d_0 = 1m$ and a path loss exponent $\mu = 3$. We randomly place $N = 50$ relays in the region between the two terminals.

Fig. 2 shows the outage probability as a function of the average SNR. In contrast to the performance of no relay selection, outage probabilities with statistic CSI and instantaneous CSI are better performed. Furthermore, although outage probability with instantaneous CSI is greater than the statistic CSI one at first, we can observe that, as SNR goes up after 11dB, the outage probability with instantaneous CSI goes down quickly, which implies that relay selection based on instantaneous channel information has better communication performance.

Fig. 3 shows how expected rate $\bar{R}(N)$ varies with the averaged received SNR for different relay selection algorithms. We see that as the averaged received SNR increases, the expected rate increase for all three algorithms. Rate with no relay selection first increases faster than the others. However, after SNR approaching 14dB, it begins to decreases, while the other two, rates with relay selection based on instantaneous CSI and statistic CSI increase. And rate with relay selection based on instantaneous CSI increase faster than the statistic CSI rate.

V. CONCLUSION

In this paper, we studied the bi-directional cooperative model in which the relays are configured to implement both user cooperation and network coding. We use power allocation method according to the instantaneous channel information to divide the relay subsets, and derive an approximation formulation of outage probability in high-SNR regime in terms of distance. Furthermore a novel relay positioning greedy algorithm based on both statistic CSI (distance condition) and instantaneous CSI (channel information between relay and destination terminal) is proposed. Simulation results show that the outage probability is decreased by SNR using the proposed relay selection method, and sum rate is maximized compared with the previous protocols.

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