# Joint Orthogonal Coding and Modulation Scheme for Soft Information in Bi-Directional Networks

Sha Wei*, Jun Li†, and Hang Su*,
*Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai, China, 200240
Email: {venessa724, Hmilyanjohn}@sjtu.edu.cn
†School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, NSW 2052, AUSTRALIA
Email: jun.li@unsw.edu.au

***Abstract*—In this paper, the authors propose a confidence-based joint orthogonal coding and modulation algorithm for soft information cooperation in the time-division broadcast bi-directional relaying networks. We first compute the log-likelihood ratio (LLR) of the received signals at relay nodes in the first two time slots, then derive the probability distribution of the scaled LLR. Inspired by the space-time coding, the quadrature components of the orthogonal coded signal is composed by the soft information from two terminals. Later, to reduce the computation complexity of the traditional cluster partition, we quantize the orthogonal coded signal to the lattice points, and propose a confidence-based cluster partition algorithm which is adaptive to the modulation scheme and bandwidth limitation. Moreover, we derive the bit error rate (BER) expression for the proposed algorithm. From the simulation results, it is shown that with multiple relays assisting the transmission, the proposed scheme is outperforms AF, DF and CF strategies in BER metric; while for single relay case, the BER performance of the proposed scheme is comparable with AF, DF and CF strategies.**

***Index Terms*—Soft information, bi-directional relay, lattice quantization, confidence-based partition, orthogonal coding**

## I. Introduction

To design an effective cooperation protocol which can benefit from soft information LLRs and energy-saving is nowadays becoming an attractive topic.

The quantize-and-forward (QF) scheme [1] requires lower computational complexity comparing with DF since channel decoding is not required, and it requires lower hardware complexity comparing with AF thanks to digital operation. Furthermore, the bandwidth consumption comparing with Amplify-and-Forward (AF) and Decode-and-Forward (DF) strategies is also a sensitive concern [2]. Besides, utilizing the probability distribution of soft information and the benefit of its confidence can enhance the advantages of LLR-based soft information strategy comparing with CF and QF [3]. Finally, since the terminals cannot derive the exact self soft information computed at relay node, removing the self soft information in an appropriate way requires more attention.

Based on the above three design criteria, we propose a joint orthogonal coding and modulation algorithm (JOCMA) that is adaptive to different channel state information. In addition, a confidence based partition algorithm, which utilizes the probability distribution of orthogonal soft information and the importance of soft information confidence, is analyzed.

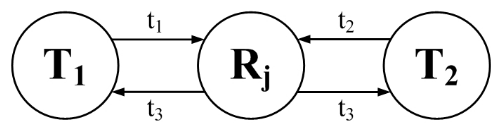


Fig. 1. Bi-directional cooperative networks. In the first two time slots $t_1$ and $t_2$, terminal $T_1$ and terminal $T_2$ transmit the messages sequentially to the $N$ relays. Here, the $j$-th relay is shown as an example. In time slot $t_3$, the relay nodes broadcast the processed signals to two terminals.

Moreover, We derive the BER performance of the proposed algorithm. Simulation results show that in full SNR regime, our proposed soft information cooperation algorithm has better performance compared with AF, DF and CF.

## II. System Model

Consider a wireless bi-directional relaying networks where two terminals, $T_1$ and $T_2$, exchange information with each other with the help of $N$ candidate relays $\mathcal{R} = \{R_1, R_2, \ldots, R_N\}$, see in figure 1. In the first two time slots, terminal $T_1$ and $T_2$ sequentially transmit data to relay nodes; in the third time slot, the relay nodes process the received signals, and broadcast the processed signals to the two terminal nodes.

Let $X_i$ for $i = 1, 2$ be an equiprobable bipolar signal with amplitude $\sqrt{E_t}$ and modulated by Binary Phase Shift Keying (BPSK) scheme, where $E_t$ is the transmitting power of each terminal $i$

$$P_r(X_i = +\sqrt{E_t}) = P_r(X_i = -\sqrt{E_t}) = \frac{1}{2}. \tag{1}$$

To be practical, all the nodes are considered to be half-duplex. In addition, the Rayleigh channels between $T_i$ and $\mathcal{R}$ are denoted by $\mathbf{G}_i = [g_{i1}, g_{i2}, \cdots, g_{iN}]$ for $i = 1, 2$, where $g_{ij}$ is modeled as a zero mean complex Gaussian random variable with variance $1/2$ per real dimension, i.e., $g_{ij} \sim \mathcal{CN}(0, 1)$ and $j = 1, 2, ..., N$. For simplicity, we assume that the channels are reciprocal, and the received nodes know perfect channel state information (CSI).

In the TDBC scheme, the received signals in the first and second time slots at the relay nodes can be written as

$$\mathbf{Y}_i = \mathbf{G}_i X_i + \mathbf{Z}_i, \ \text{ for } \ i = 1, 2, \tag{2}$$

978-1-4244-6252-0/11/$26.00 ©2011 IEEE

where $\mathbf{Z}_i = [z_{i1}, z_{i2}, ..., z_{iN}]$ are independent and identically distributed (*i.i.d.*) additive white Gaussian noise (AWGN), with distribution given by $z_{ij} \sim \mathcal{CN}(0, \delta^2)$ for $j = 1, 2, ..., N$. The received signals in the third time slot at terminal $T_i$ are given by

$$\mathbf{Y}_{T_i} = \mathbf{a}\mathbf{G}_i^T \mathbf{X}_r + \mathbf{V}_i, \quad \text{for} \quad i = 1, 2, \tag{3}$$

where $\mathbf{a}$ is an $N \times 1$ power allocation vector satisfying the power constrains at relays, which is determined by the instantaneous CSI, the noise statistics and the transmitting power $E_r$ of relay nodes; $\mathbf{X}_r$ is an $N \times 1$ vector which represent the relay processed signal, and the $2 \times 1$ vector $\mathbf{V}_i$ is also an i.i.d. AWGN.

## III. Joint Orthogonal Coding and Modulation Algorithm for Soft Information

### A. Soft Information Calculation and Orthogonal Coding

Without loss of generality, we will take the received signal $Y_{ij}$ at relay node $j$ from terminal $i$ in the first time slot as an example. According to the definition of LLR, we have

$$\begin{aligned} \ell_{ij} &= \log \frac{P_r\left\{X_i = +\sqrt{E_t}|Y_{ij}, g_{ij}\right\}}{P_r\left\{X_i = -\sqrt{E_t}|Y_{ij}, g_{ij}\right\}} \\ &= \log \frac{\frac{1}{2\pi\delta^2}\exp\frac{-|Y_{ij}-g_{ij}\sqrt{E_t}|^2}{2\delta^2}}{\frac{1}{2\pi\delta^2}\exp\frac{-|Y_{ij}+g_{ij}\sqrt{E_t}|^2}{2\delta^2}} \\ &= \frac{2|g_{ij}|^2 X_1}{\delta^2} + 2\left(\Re\{g_{ij}\}\Re\{z_{ij}\} + \Im\{g_{ij}\}\Im\{z_{ij}\}\right), \end{aligned} \tag{4}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary part of $\{\cdot\}$, respectively. Later, we will drop the subscript $j$ for expression simplicity. Since the real and imaginary part of noise $z_i$ are two independent Gaussian random variables with zero mean and variance $\delta^2/2$, the probability distribution of $\ell_i$ is modeled as a mixture Gaussian distribution

$$f(\ell_i) = \frac{1}{2\sqrt{2\pi\upsilon}}\left[\exp\left(\frac{-(\ell_i - \mu)^2}{2\upsilon}\right) + \exp\left(\frac{-(\ell_i + \mu)^2}{2\upsilon}\right)\right], \tag{5}$$

where the mean $\mu$ is equal to $\frac{2\sqrt{E_t}}{\delta^2}$ and variance is $\upsilon = 2/\delta^2$.

Inspired by the space-time coding in MIMO system, we propose the orthogonal coding strategy, which form an orthogonal signal $\zeta = (\ell_1, \ell_2)$ with horizontal component $\ell_1$ and vertical component $\ell_2$. And we choose arc tangent as our scaling function, which can scale the soft information values in the specified range $[-\alpha, \alpha]$ as equation (6) shows,

$$\begin{aligned} &\ell_i \xrightarrow{\text{Scaling}} l_i = \frac{2\alpha}{\pi}\arctan(\ell_i), \\ &\zeta_i \xrightarrow{\text{Scaling}} \zeta_i' = (l_1, l_2), \\ &\text{for} \quad \ell_i \in (-\infty, \infty) \quad \text{and} \quad l_i \in (-\alpha, \alpha). \end{aligned} \tag{6}$$

Thus the two independent random variable $l_1$ and $l_2$ have the joint distribution as

$$\begin{aligned} f_{L_i}(l_i) &= \frac{\pi}{2\alpha} f\left(\tan\left(\frac{\pi l_i}{2\alpha}\right)\right), \\ f(\zeta') &= f(l_1, l_2) = f(l_1) f(l_2). \end{aligned} \tag{7}$$

### B. Modulation Algorithm

In this subsection, we first quantize the training sequence to lattice points for complicated computations of the direct partition [4], and then propose a confidence-based cluster partition algorithm. The centroid of each new-formed cluster is the codeword of the system. And the number of partitions is determined by the constellation size of the constellation diagram. With the designed codewords, only a nearest neighbor quantization is needed for practical input signals.

Now, we implement the lattice quantization for the training sequence which can dramatically reduce the computation complexity. An n-dimensional lattice $\Lambda$ is composed of all integer combinations of the columns of an $n \times n$ matrix G called the generator matrix of the lattice [5]

$$\Lambda = u \in \mathbb{R}^n : u = \beta G \cdot i \quad \text{for some} \quad i \in \mathbb{Z}^n, \tag{8}$$

where $\beta$ is an adjustment parameter that determines the size of the Voronoi cell of lattice. Lattice $\Lambda$ can be viewed as an infinite discrete subgroup of the Euclidean space $\mathbb{R}^n$.

In our algorithm, lattice $A_2$ is chosen for its quantization optimality in two-dimensional space. The scaled soft information of training sequence is quantized to the nearest lattice centroid of its corresponding Voronoi cell. In particular, the input point $\zeta'$ gets mapped into a closest lattice point $\lambda_i$. The optimal encoder, selects the codeword $\lambda_i$ if: $d(\zeta', \lambda_i) \le d(\zeta', \lambda_j), \forall j$, where $d(\cdot, \cdot)$ denotes the Euclidean norm $L^2$. That is

$$Q_\Lambda(\mathbf{x}) \triangleq \lambda_i \in \Lambda \quad \text{where} \quad ||\zeta' - \lambda_i|| \le ||\zeta' - \lambda_j|| \quad \text{for all} \quad \lambda_j \in \Lambda, \tag{9}$$

where the distortion measure $||\cdot|| \triangleq \frac{1}{L}\zeta'^T\zeta$ is chosen to be the normalized 2-norm.

Some important definitions are introduced before staring to introduce the cluster partition algorithm. The distance functions between a point and a set is defined as follows. A vector $\mathbf{x}$ is assigned to a cluster $\mathcal{C}$ taking into account the distance between $\mathbf{x}$ and $\mathcal{C}$, $\rho(\mathbf{x}, \mathcal{C})$. We implement the minimum distance function

$$\rho(\mathbf{x}, \mathcal{C}) = \min_{y \in \mathcal{C}} \rho(\mathbf{x}, \mathbf{y}), \tag{10}$$

where $\mathbf{y}$ is the centroid of the set $\mathcal{C}$.

Another important definition is the confidence factor of each centroid of Voronoi cell in lattice, which represents the reliability of the lattice point. An example definition of confidence $w_i$ is

$$w_i = \frac{1}{n}\sum_{i=1}^{n} |l_i|, \quad \text{for} \quad l_i \in \Lambda_i, \tag{11}$$

where $\Lambda_i$ is the $i$th Voronoi cell of lattice, $n$ is the number of scaled soft information in the corresponding Voronoi cell, $|\cdot|$ stands for the absolute value. The stop metric of the partition iteration is defined as

$$\eta \ge \sum_{k=1}^{m} w_k, \tag{12}$$

where $\eta$ is the maximum sum confidence, and $m$ is the total number of voronoi cells inside the new partition.

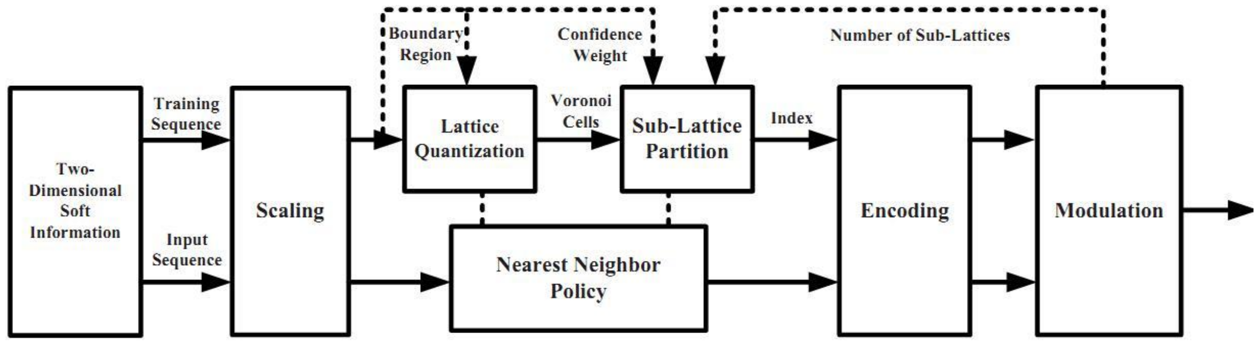


Fig. 2. Joint Orthogonal Coding and Modulation Algorithm Based on Lattice Codes

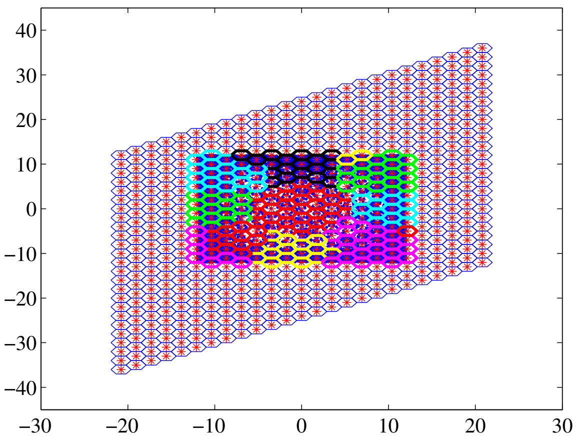


Fig. 3. Simulation Result of the Confidence-based Cluster Partition Algorithm

Consider the confidence of the soft information, we propose a novel cluster partition algorithm to obtain the codewords of the system, see Algorithm 1.

Given the codebook, $\mathcal{C} = \mathbf{c}_i, i = 1, \cdots, N$, which is the centroid of the new formed partition, we implement the nearest neighbor condition to find the partition the input vector belongs to

$$\Lambda = \mathbf{c}_i, \ \text{if} \ \Lambda \in \Omega_i. \tag{13}$$

Then $\Lambda$ is transmitted to two terminals in the form of a constellation point that $\mathbf{c}_i$ corresponding to.

### *C. Confidence-based Decoding Algorithm*

The relays broadcast the modulated signal $\mathbf{X}_r$ to the two terminals in the third time slot. By applying the principle of zero-forcing, we get

$$\begin{aligned} T &= (G_i^* G_i)^{-1} G_i^*, \\ Y'_{T_i} &= T \cdot Y_{T_i} = X_r + T \cdot V_i. \end{aligned} \tag{14}$$

Since the terminals know the modulation size of relay nodes, it can reconstruct the partition that the relays implement. Each demodulated signal is mapped to the centroid according to the

**Algorithm 1** Confidence-based Cluster Partition Algorithm

**Begin initialization**:
*Initialize Global Parameters*
$\mathbf{M} = 2^m$ for $m$ is even: the objective number of partition lattice points
$N_T = 0$: the number of clusters that has been generated
$\eta$: threshold of confidence sum, and is the stop sign of the iteration
$\alpha$: the specified range bound of the soft information values, as referred in equation (6)
$\beta$: the adjustment parameter of the voronoi cell size in lattice
*Initialize all the soft information*
Compute the confidence of all vertex: $\mathcal{W} = \{w_1, \cdots, \omega_{N_T}\}$
**End initialization**
**Begin partition**
**while** $\sum_{k=1}^{m} \omega_k < \eta$ **do**
  Find the vertex with the minimum confidence, denote it by $\lambda_0$, and its confidence is equal to $\omega_0$
  Find a vertex $\lambda_i$ which is closest to the partitioned set $\mathcal{C}$, i.e., $d(\lambda_i, \mathcal{C}) = \min_{r,s=1,\cdots,N, r\neq s} d(\lambda_r, \lambda_s)$;
  Merge $\lambda_i$ and $\mathcal{C}$ into a single partition $\mathcal{C}'$, and remove $\lambda_i$ from the set containing vertex which has not been processed
  Confidence of the new partition is equal to the sum of vertex confidence $\omega_i$ and set weight $\omega_j$
  Remove the new formed set generated in iteration $t$ from the set in iteration $t-1$, i.e., $\mathcal{R}_{t-1} - \mathcal{R}_t$;
  **if** $\sum_{k=1}^{m} \omega_k > \eta$ **then**
    The new partition includes the first $m-1$ voronoi cells
  **else**
    The new partition include the $m$ voronoi cells
  **end if**
  $N_T = N_T + 1$
**end while**
**End partition**

nearest neighbor policy. Then, the terminal $T_i$ can obtain the corresponding orthogonal component of its counterpart.

## IV. Performance Analysis

Now, we analyze the bit error rate (BER) performance for the proposed joint network-channel coding scheme for soft information. First, we classify different scenarios that determine the decoding consequence. Next, the probability that each scenario occurs is determined. Then, we derive an average BER formulation of the cooperation system by taking into account all the possible scenarios.

There are two events that jointly determine the decoding result of the combiner output. The first scenario, $\xi_1$, corresponds to the case when the relays receive the signal $[Y_1, Y_2]$, the scaled soft information $l = (l_1, l_2)$ are mapped to the correct partition. For example, given $X_1 = X_2 = +\sqrt{E_t}$, the probability distribution of $\ell_i$ is degraded to a normal distribution with mean $+\mu$ and variance $\upsilon$. Thus the probability that $l$ is partitioned to the region $\Omega_i$ is

$$P_i = \iint_{\Omega_i} f_L(l) d\Omega_i, \quad \text{for} \quad i = 1, \cdots, N_t, \qquad (15)$$

where $\Omega_i$ is the $i$-th space resulted in the proposed confidence-based cluster partition algorithm.

The second scenario, $\xi_2$, corresponds to the case when $\mathbf{X}_r$ is decoded correctly at terminals. This probability is determined by the modulation scheme that relays choose. For $M$-QAM, the symbol error probability for coded $M$-QAM over $N$ i.i.d. Rayleigh fading channel and with maximal ratio combining (MRC) reception is

$$\begin{aligned} P(\xi_2|\xi_1) = 2\left(1 - \tfrac{1}{\sqrt{M}}\right) \sum_{i=1}^{N} \rho_i \left(1 - \sqrt{\tfrac{g_{QAM}\overline{\gamma_i}}{1+g_{QAM}\overline{\gamma_i}}}\right) + \left(1 - \tfrac{1}{\sqrt{M}}\right)^2 \\ \times \left[\tfrac{4}{\pi} \sum_{i=1}^{N} \rho_i \sqrt{\tfrac{g_{QAM}\overline{\gamma_i}}{1+g_{QAM}\overline{\gamma_i}}} \tan^{-1}\left(\sqrt{\tfrac{1+g_{QAM}\overline{\gamma_i}}{g_{QAM}\overline{\gamma_i}}}\right) - \sum_{i=1}^{N} \rho_i\right], \end{aligned} \qquad (16)$$

where $g_{QAM} = \frac{3}{2}(M-1)$

Then the successful probability and the bit error rate of the system are written as

$$\begin{aligned} P_r &= P(\xi_1)P(\xi_2|\xi_1)P(\xi_3|\xi_2, \xi_1), \\ P_e &= 1 - P_r. \end{aligned} \qquad (17)$$

## V. Simulations

In this section, we present the numerate results on the BER performance via Monte-Carlo simulations, comparing the proposed joint orthogonal coding and modulation algorithm with AF, DF and CF protocols under the conditions that multiple relays or single relay assist the transmission between two terminals. All channel coefficients are generated by assuming that $h_{ij}$ is independent complex Gaussian random variable with zero mean and unit variance. As mentioned earlier, we assume that the CSIs are available at the receivers. In addition, we assume that the maximum power constraints at two terminals and relay nodes are the same, i.e. $E_t = E_r$. Fig. 4 compares the BER of the JOCMA with traditional AF, DF and CF relay protocols under the condition that multiple relays assist the transmission between two terminals. It turns out that the proposed algorithm yields a significant lower BER performance than AF and CF protocols. And in low SNR regime, i.e., from 1dB to 10dB, the BER performance of JOCMA is comparable with DF protocol, while in high SNR regime, i.e., from 11dB to 20dB, the BER performance of JOCMA is superior to DF protocol, since the accuracy of high confidence results is improved in high SNR regime.

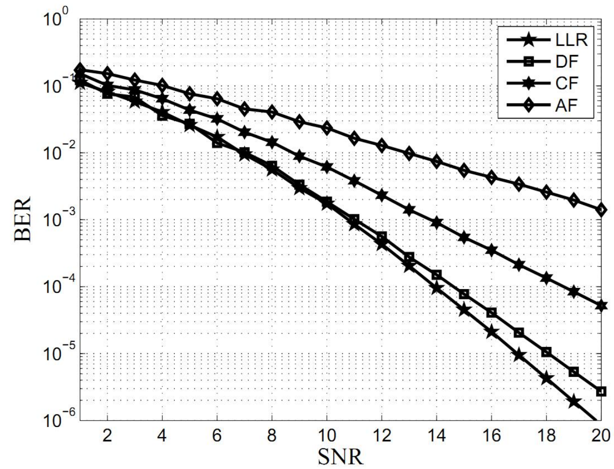


Fig. 4. BER Performance of Multiple Relays

## VI. Conclusions

The authors propose a confidence-based joint orthogonal coding and modulation algorithm for soft information cooperation in bi-directional relaying networks. Consider the joint probability distribution of scaled soft information $l_1$ and $l_2$, the authors propose an orthogonal coding strategy, which can protect the soft information from both terminals and is easy to recover in the decoding process. Then to reduce the computation complexity, the author quantize the orthogonal coded signal to the lattice points and propose a modulation scheme which is adaptive to the bandwidth limitation. The simulation results show that the proposed algorithm is optimal when multiple relays assist the communication between two terminals.

## References


[1] Stefan Schwandter and Gerald Matz, ”A practical forwarding scheme for wireless relay channels based on the quantization of log-likelihood ratios”, in Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on, pp. 2502C2505.

[2] Yuexing Peng et al., ”Cooperative network coding with soft information relaying in two-way relay cchannels”, Journal of Communications, vol. 4, no. 11, pp. 849C855, Dec 2009.

[3] R. Dabora and S. D. Servetto, ”On the role of estimate-and-forward with time sharing in cooperative communication”, Information Theory, IEEE Transactions on, vol. 54, no. 10, pp. 4409C4431, 2008.

[4] Robert; Friedman Jerome Hastie, Trevor; Tibshirani, *The Elements of Statistical Learning*, chapter 14.3.12 Hierarchical clustering, p. pp. 520528, Springer Publishers, New York, USA, 2nd ed. edition, 2009.

[5] J. H. Conway, N. J. A. Sloane, and E. Bannai, *Sphere-packings, lattices, and groups*, Springer-Verlag New York, Inc., New York, NY, USA, 1987.