Achievable rate for a multi-source relaying system

Jun Li*, Jin Xu†, Zihuai Lin*, Branka Vucetic*
*School of Electrical and Information Engineering, The University of Sydney, NSW, 2006, AUSTRALIA
Email:{jun.li,zihuai.lin,branka.vucetic}@sydney.edu.au
†Wireless Technology Pre-research Department, Product R&D System, ZTE, Shenzhen, 518055, P.R.CHINA
Email:xu.jin7@zte.com.cn

***Abstract*—In this work we determine the achievable rate in a multi-source relaying system with Gaussian phase-fading channels. In our system, M sources simultaneously transmit their messages to a common destination in M separate frequency bands with the help of a single relay (an $M-1-1$ system). The achievable rates of both a separate processing scheme at the relay, and a network coding scheme at the relay, are considered. For the separate processing scheme, we propose an new constrained water-filling algorithm which determines the power allocation at the relay in order to obtain the achievable rate. For the network coding scheme we derive the achievable rate based on the use of a new Galois field rate-splitting theorem, and discuss why power allocation at the relay in this scheme can be set using a traditional water-filling algorithm. We show how our network coding scheme will always obtain higher achievable rates relative to those obtained from a separate processing scheme.**

I. Introduction

In cooperative communications, the triangle model (one source, one relay and one destination) has been widely studied. However, in realistic scenarios, such as a cellular relaying network, a relay forwards multiple sources' information to the common destination (e.g., base station). Recently, much research has focused on the $M-1-1$ system, composed of M sources, one relay and one destination. For the relay, there are two processing methods. One method is separate processing at the relay, in which the relay separately processes and forwards the messages of the sources individually. The other method is joint processing at the relay, in which the relay combines the information of the sources in some coding scheme before retransmission. In fact, joint processing in the form of network coding [1] at the relay in cooperative wireless communications has recently fueled a surge of research activity [2–6]. In these latter works it is shown how network coding can provide for diversity gain with less time slots. Note that, in a network coding scheme the relay combines the messages of the sources in either the complex field [2], where the messages are superposed in the symbol level, or Galois field [3–6], where the messages are XORed in the digit level. Here we only focus on the Galois field network coding (henceforth simply referred to as network coding).

In this work, we determine the achievable rate of the $M-1-1$ system in an AWGN channel with uniform phase-fading, for both the separate processing scheme and the network coding scheme. The achievable rate in the case of separate processing is obtained using a new constrained water filling algorithm. With regard to the achievable rate in the network coding scheme, we propose the Galois field rate-splitting theorem which determines how many bits of information at the relay is allocated to each user. Our results show that the achievable rate in the network coding scheme is always higher than that in the separate processing scheme.

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II. System Model

Consider an $M-1-1$ system with M sources, 1 relay and 1 destination as shown in Fig. 1, where the sources $s_1, \cdots, s_M$, transmit their information to the destination d simultaneously with the help of a full-duplex relay r. All the channels are assumed to be AWGN with uniform phase-fading. The sources transmit using frequency division multiple access (FDMA) with the same block length n. The relay receives and transmits in all M frequencies utilized by the sources. The physical locations of the sources are assumed to be randomly distributed. Since the relay does not know the phase of the source-to-destination channel, there is no coherent transmission between a source and the relay.

As shown in Fig. 1, suppose that the m-th source s_m, ($m = 1, \cdots, M$), transmits its messages in the frequency band f_m, r has M parts $r_1, \cdots, r_M$ receiving and transmitting at $f_1, \cdots, f_M$, respectively, and d also receives signals at these M orthogonal frequency bands. With these assumptions, the multi-source system can be viewed as M independent parallel $1-1-1$ triangle systems. As such, the achievable rate of the $M-1-1$ system is the maximal summation of all the $1-1-1$ system's rates under various power allocation schemes. Each $1-1-1$ system's rate with a given power allocation can be directly obtained from [7, 8].

Let X_m be the signal transmitted by s_m, which has the average power P_m, and X_{1m} is the signal transmitted by r_m, which has the average power P_{1m}. All the receivers are assumed to have the same noise power N. The distance between s_m and the relay, the distance between s_m and the destination, and the distance between the relay and the destination are denoted as d_m^{sr}, d_m^{sd} and d^r, respectively. We assume that $d_m^{sr} < d_m^{sd}$ for all the m. The pass losses of all the channels are related to their distances with the same attenuation exponent α. So the channel coefficients between s_m and r_m, s_m and the destination, r_m and the destination,

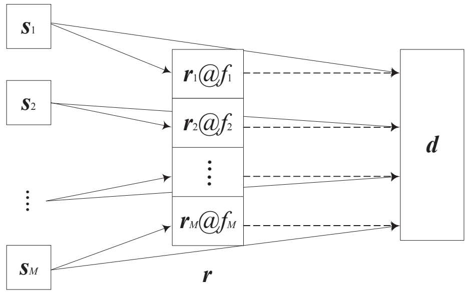

Fig. 1. The working mode of FDMA $M-1-1$ system. The relay r has M parts $r_1, \cdots, r_M$ receiving and transmitting at $f_1, \cdots, f_M$, respectively.

are denoted as,

$$h_m^{sr} = \frac{e^{j\phi_m^{sr}}}{\sqrt{(d_m^{sr})^\alpha}},\ h_m^{sd} = \frac{e^{j\phi_m^{sd}}}{\sqrt{(d_m^{sd})^\alpha}},\ h_m^{rd} = \frac{e^{j\phi_m^{rd}}}{\sqrt{(d^r)^\alpha}}, \quad (1)$$

where ϕ_m^{sr}, ϕ_m^{sd} and ϕ^r are the received signal phases between the s_m-to-relay channel, the s_m-to-destination channel, and the relay-to-destination channel, respectively.

III. Achievable Rate Analysis

A. Separate Processing Scheme

Let us briefly review the achievable rate in the $1-1-1$ system [7, 8], where the binning scheme is utilized to derive the achievable rate. In this scheme, s_m selects a new message $\omega_{im} \in \{1, \cdots, 2^{nR_m}\}$ in each block i. The set of s_m messages $\{1, \cdots, 2^{nR_m}\}$ is randomly partitioned into $2^{nR_1^m}$ bins ($R_m^1 \leq R_m$) of size $2^{n(R_m - R_m^1)}$. The message transmitted by the relay at f_m, i.e., r_m in block i is represented by ϕ_{im}, which is the bin index of $\omega_{(i-1)m}$, the message of s_m in the block $i-1$.

Suppose that $\bar{\mathbf{X}}_m(\omega_{im})$ and $\mathbf{X}_{1m}(\phi_{im})$ encode ω_{im} and ϕ_{im} via random codebooks of sizes 2^{nR_m} and $2^{nR_1^m}$, respectively. s_m divides its total power P_m into two fractions. One fraction, $\alpha_m P_m$ $(0 < \alpha_m \leq 1)$, is for the new message ω_{im}, which means that each element $\bar{X}_m^\eta(\omega_{im})$, $(\eta = 1, \cdots, n)$, in the vector $\bar{\mathbf{X}}_m(\omega_{im})$ has probability distribution $\mathcal{N}(0, \alpha_m P_m)$. The second fraction, $(1-\alpha_m)P_m$ is for the bin index ϕ_{im} of the previous codeword $\omega_{(i-1)m}$, which means that each element $X_{1m}^\eta(\phi_{im})$ in the vector $\mathbf{X}_{1m}(\phi_{im})$ has a probability distribution $\mathcal{N}(0, P_{1m})$. According to [7], we adopt $\alpha_m = 1$ in order to obtain the achievable rate for an AWGN channel with uniform phase-fading. This rate can be written as

$$R_m = \min\left\{\log\left(1 + \frac{P_m}{(d_m^{sr})^\alpha N}\right),\ \log\left(1 + \frac{P_m}{(d_m^{sd})^\alpha N} + \frac{P_{1m}}{(d^r)^\alpha N}\right)\right\}. \quad (2)$$

The first item of the right hand side (RHS) in the above equation is the message rate received at r_m, which is denoted as R_m^+. The second item of the RHS is the sum of R_m^1 and R_m^-, where R_m^1 is the rate of the bin index messages and R_m^- is the source's message rate received at the destination, and

$$\begin{aligned} R_m^1 &= \log\left(1 + \frac{P_{1m}}{(d^r)^\alpha \left(P_m/(d_m^{sd})^\alpha + N\right)}\right), \\ R_m^- &= \log\left(1 + \frac{P_m}{(d_m^{sd})^\alpha N}\right). \end{aligned} \quad (3)$$

We now consider the achievable rate R of the whole system composed of M parallel $1-1-1$ systems under the assumptions that there is no power allocation among the sources, and the power allocation between the M parts of the relay is subjected to a fixed total power P_{10}. Since the M $1-1-1$ systems are parallel, we have $R = \sum_{m=1}^M R_m$, and

$$\begin{aligned} R &= \sum_{m=1}^M \min\left\{R_m^+, R_m^1 + R_m^-\right\} \\ &\leq \min\left\{\sum_{m=1}^M R_m^+,\ \sum_{m=1}^M R_m^1 + \sum_{m=1}^M R_m^-\right\}. \end{aligned} \quad (4)$$

We notice that for a given location of the relay, the items $\sum_{m=1}^M R_m^+$ and $\sum_{m=1}^M R_m^-$ are fixed no matter what power allocation schemes are applied to the relay. Intuitively, there are two cases according to the location of the relay.

The first case is that the relay is near to the destination. In this case, we have $\sum_{m=1}^M R_m^+ \leq \sum_{m=1}^M R_m^1 + \sum_{m=1}^M R_m^-$. By satisfying $R_m^1 = R_m^+ - R_m^-$, for all m, under the power constraint $\sum_{m=1}^M P_{1m} = P_{10}$ at the relay, the upper bound $\sum_{m=1}^M R_m^+$ is achieved.

In the second case, the relay is near to the sources, then we have $\sum_{m=1}^M R_m^+ > \sum_{m=1}^M R_m^1 + \sum_{m=1}^M R_m^-$. So $\sum_{m=1}^M R_m^1 + \sum_{m=1}^M R_m^-$ dominates the rate R. The water filling principle [9–11] is utilized to enhance the total rate of the bin index messages, $\sum_{m=1}^M R_m^1$. This is a consequence of the use of FDMA for all M channels between the relay and the destination. Different from the traditional usage of this principal, extra constraints $R_m^1 \leq R_m^+ - R_m^-$, for all m, should be satisfied so as that the upper bound $\sum_{m=1}^M R_m^1 + \sum_{m=1}^M R_m^-$ can be achieved. Combining the two cases we propose an optimal power allocation scheme as the follows.

Theorem 1: the power allocation at the relay that obtains the achievable rate in an $M-1-1$ system is

$$P_{1m} = \begin{cases} 0 & \frac{\log e}{\tau} \leq \frac{1}{p_m} \\ \frac{\log e}{\tau} - \frac{1}{p_m} & \frac{1}{p_m} < \frac{\log e}{\tau} < d_m P_m + \frac{1}{p_m} \\ d_m P_m & \frac{\log e}{\tau} \geq d_m P_m + \frac{1}{p_m}, \end{cases} \quad (5)$$

where,

$$\begin{aligned} d_m &= \left(\frac{d^r}{d_m^{sd} d_m^{sr}}\right)^\alpha \left(\left(d_m^{sd}\right)^\alpha - (d_m^{sr})^\alpha\right), \\ p_m &= \frac{1}{(d^r)^\alpha \left(P_m/(d_m^{sd})^\alpha + N\right)}, \\ \tau &= \frac{W \log e}{\sum_{w=1}^W \frac{1}{p_w} + P_{10}}, \end{aligned} \quad (6)$$

Here, $W \leq M$ is number of positive $\log e/\tau - 1/p_m$.

Proof: The proof is similar to that given for a traditional water-filling algorithm, such as that given in [11], and as such details of the proof are omitted here.

An algorithm, which we refer to as the *constrained water filling algorithm*, that executes the power allocation described in *Theorem 1* is as follows.

Constrained water filling algorithm:

Step 1: Initialize the channel set $\mathcal{M}_{exe}$ that includes all the M channels, and total power $P_{exe} = P_{10}$.

Step 2: Utilize the traditional water filling algorithm with $\mathcal{M}_{exe}$ and P_{exe} to set the values P_{1m}, and also to determine W which in turns sets the value of τ in equation (6).

Step 3: Find the channel subset $\mathcal{M} \in \mathcal{M}_{exe}$, where there is $\log e/\tau \geq d_m P_m + 1/p_m$ for $m \in \mathcal{M}$. If the number of the elements in $\mathcal{M}$ equals to either 0 or M, then exit.

Step 4: Let $P_{1m} = d_m P_m$ for $m \in \mathcal{M}$, and calculate the extra power $P_{ext} = \sum_{m\in\mathcal{M}}(\log e/\tau - 1/p_m - d_m P_m)$.

Step 5: Obtain the channel set $\bar{\mathcal{M}}$ that is the supplementary set of $\mathcal{M}$ and the total power $P_{\bar{\mathcal{M}}}$ of the channels that are in the set $\bar{\mathcal{M}}$.

Step 6: Calculate the remaining power $P_{rem} = P_{ext} + P_{\bar{\mathcal{M}}}$, set the power $P_{exe} = P_{rem}$ and the channel set $\mathcal{M}_{exe} = \bar{\mathcal{M}}$, and goto **Step 2**.

B. Network Coding Scheme

In the network coding scheme, the received information at all the frequency bands is network coded before retransmission by the relay. The bin index signals $\mathbf{X}_{11}(\phi_{i1}), \cdots, \mathbf{X}_{1M}(\phi_{iM})$ at the relay are firstly decoded to M bit streams, then these bit streams are network coded in the Galois field by some coding scheme $\mathbb{C}_{\oplus}$, and remodulated to a new vector $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$. This vector can be seen as the bin index of the super block composed of all the sources' blocks. This process is described as

$$\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM}) = \mathbb{C}_{\oplus}(\mathbf{X}_{11}(\phi_{i1}), \cdots, \mathbf{X}_{1M}(\phi_{iM})). \quad (7)$$

The destination decodes the bin index signal $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ by treating $\bar{\mathbf{X}}_1(\omega_{i1}), \cdots, \bar{\mathbf{X}}_M(\omega_{iM})$ as noise. We shall now show how network coding at the relay obtains higher achievable rate than that in the separate processing scheme.

Before the derivation of the achievable rate, we firstly discuss a new Galois field rate-splitting theorem. A rate-splitting approach was originally proposed by [12], in order to determine the achievable rates of sources in a Gaussian multiple-access channel, where source signals are superposed at the destination. In the network coding scheme we study here, coded digits from the sources are superposed in the Galois field and compose a new message $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ at the relay. We propose the Galois field rate-splitting theorem to determine the number of check digits in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ allocated to each source.

Suppose that all the blocks have the length n, and the number of digits in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ is $nR^{1,nc} = n\sum_{m=1}^{M} R_m^{1,nc}$, in which $nR_m^{1,nc}$ digits are allocated to $\mathbf{X}_{1m}(\phi_{im})$. Different from the separate processing scheme, the number of digits allocated to $\mathbf{X}_{1m}(\phi_{im})$ is not straightforward since each digit

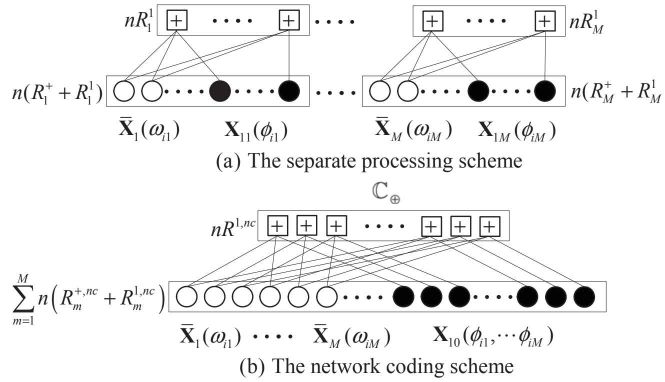

Fig. 2. The comparison of the separate processing scheme and the network coding scheme.

in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ is shared by more than one source. We define $nR_m^{1,nc}$ as the effective number of digits for s_m.

The determination of $nR_m^{1,nc}$ can be found from consideration of the degree distributions associated with a Tanner graph. The variable nodes of s_m are related to the digits in both $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ and $\bar{\mathbf{X}}_m(\omega_{im})$, $m = 1, \cdots, M$. The constraints in the coding scheme $\mathbb{C}_{\oplus}$ produces the check nodes. Fig. 2 compares the separate processing scheme with the network coding scheme. Note that the variable nodes with solid circles, which are are related to the digits in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$, all have one degree. We now propose our Galois field rate-splitting theorem.

Theorem 2: In the network coding scheme, the bin index signals $\mathbf{X}_{11}(\phi_{i1}), \cdots, \mathbf{X}_{1M}(\phi_{iM})$ at the relay are network coded (i.e., superposed in Galois field) by a coding scheme $\mathbb{C}_{\oplus}$. Assume the relay transmits the network coded signal $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ to the destination. So the network coded signal can be seen as the bin index signal for the super block composed of all sources' frames. Suppose $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ contains $nR^{1,nc} = n\sum_{m=1}^{M} R_m^{1,nc}$ digits. Then the effective digits allocated to s_m, i.e., $nR_m^{1,nc}$, is computed as

$$nR_m^{1,nc} = \sum_{j_m=1}^{j} \sum_{j=1}^{d_c} \frac{j_m + \frac{R_m^{1,nc}}{\sum_{w=1}^{M} R_w^{1,nc}}}{j} \vartheta_m^j, \quad (8)$$

where j is the degree of the check nodes related to $\mathbb{C}_{\oplus}$ (with maximum value d_c), j_m is the number of edges emanating from a degree-j check node and connected to the variable nodes related to the digits in $\bar{\mathbf{X}}_m(\omega_{im})$, and ϑ_m^j is the number of the degree-j check nodes with the j_m edges connected to the variable nodes related to the digits in $\bar{\mathbf{X}}_m(\omega_{im})$.

Proof: The graph in Fig. 2(b) can be decomposed into M sub-graphs. Each sub-graph is the Tanner graph for a source. The variable nodes in the m-th sub-graph are related to both the digits in $\bar{\mathbf{X}}_m(\omega_{im})$ (represented by the vector $\bar{\mathbf{b}}_m$) and the effective digits for s_m in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ (represented by the vector $\mathbf{b}_m^{eff}$). Since all the digits in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ (represented by the vector $\mathbf{b}$) are shared by more than one source, it is not so obvious how to determine $\mathbf{b}_m^{eff}$ for s_m. So we define the m-th effective sub-graph, which is obtained in the following steps.

(1) The check nodes in the m-th effective sub-graph are the same as that in the whole graph.

(2) The variable nodes in the m-th effective sub-graph are composed of the variable nodes related to the digits in both $\bar{\mathbf{b}}_m$ and $\mathbf{b}$.

(3) The connections between check nodes and the variable nodes in the m-th effective sub-graph are kept the same as that in the whole graph.

Then we determine the code rate of the m-th effective sub-graph. Without loss of generality, we randomly pick up a parity check node c in the whole graph. Suppose there are j edges emanate from the node c. Among these edges, j_m are connected to the variable nodes related to $\bar{\mathbf{b}}_m$, and one edge is connected to a variable node related to $\mathbf{b}$. We have $j = \sum_{m=1}^{M} j_m + 1$, which means that there are j variable nodes sharing one bit information provided by c. Since the variable nodes related to $\mathbf{b}$ belong to s_m with the probability $R_m^{1,nc}/R^{1,nc}$, there is $(j_m + R_m^{1,nc}/R^{1,nc})/j$ bit information of c allocated to s_m. In this sense, we count the number of c as $(j_m + R_m^{1,nc}/R^{1,nc})/j$ in the m-th effective sub-graph. By summing all the numbers of the parity check nodes in the m-th effective sub-graph, we obtain the effective parity check nodes allocated to s_m.

Since ϑ_m^j is denoted as the number of the degree-j parity check nodes which has j_m edges in the m-th effective sub-graph, the number of these parity check nodes in the m-th effective sub-graph is counted as $\vartheta_m^j(j_m + R_m^{1,nc}/R^{1,nc})/j$. Then we can easily get the the number of effective parity check nodes allocated to s_m in (8) and complete the proof. ■

To have a better understanding of *Theorem 2*, we provide below an example for a two sources case.

Example 1: Consider a two sources case where each source has 4 digits of information. The digits in the two sources are combined according to the coding scheme $\mathbb{C}_{\oplus}$ and produce 4 network coded digits. According to *Theorem 2*, we split the whole graph into two subgraphs as shown in Fig. 3.

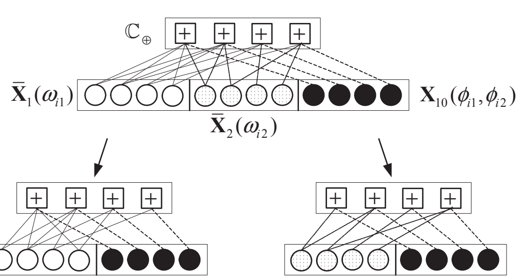

Fig. 3. Decomposition of the graph into two sub-graphs in two sources case.

It is shown in Fig. 3 that there are $nR^{1,nc} = nR_1^{1,nc} + nR_2^{1,nc} = 4$ check nodes and we can easily determine their degrees as $j = 5, 6, 4, 4$. Note that for the m-th sub-graph, when we calculate j_m, the edge connected to the variable node related to the digit in $\mathbf{X}_{10}(\phi_{i1}, \cdots, \phi_{iM})$ (dashed line in Fig. 3) is counted as $nR_m^{1,nc}/nR^{1,nc}$. So we get that the numerator $j_1 + R_1^{1,nc}/(R_1^{1,nc} + R_2^{1,nc})$ in (8) for the first source is $3 + R_1^{1,nc}/R^{1,nc}$, $3 + R_1^{1,nc}/R^{1,nc}$, $2 + R_1^{1,nc}/R^{1,nc}$ and $1 + R_1^{1,nc}/R^{1,nc}$ (corresponding to the 4 check nodes in the first sub-graph). Similarly, we get that the numerator $j_2 + R_2^{1,nc}/(R_1^{1,nc} + R_2^{1,nc})$ in (8) for the second source is $1 + R_2^{1,nc}/R^{1,nc}$, $2 + R_2^{1,nc}/R^{1,nc}$, $1 + R_2^{1,nc}/R^{1,nc}$ and $2 + R_2^{1,nc}/R^{1,nc}$ (corresponding to the 4 check nodes in the second sub-graph). For the first sub-graph,

$$nR_1^{1,nc} = \frac{3 + \frac{R_1^{1,nc}}{R^{1,nc}}}{5} + \frac{3 + \frac{R_1^{1,nc}}{R^{1,nc}}}{6} + \frac{2 + \frac{R_1^{1,nc}}{R^{1,nc}}}{4} + \frac{1 + \frac{R_1^{1,nc}}{R^{1,nc}}}{4}. \tag{9}$$

Then we get the effective number of digits allocated to s_1 is $nR_1^{1,nc} = 2.3617$. For the second sub-graph,

$$nR_2^{1,nc} = \frac{1 + \frac{R_2^{1,nc}}{R^{1,nc}}}{5} + \frac{2 + \frac{R_2^{1,nc}}{R^{1,nc}}}{6} + \frac{1 + \frac{R_2^{1,nc}}{R^{1,nc}}}{4} + \frac{2 + \frac{R_2^{1,nc}}{R^{1,nc}}}{4}. \tag{10}$$

Then we get the effective number of digits allocated to s_2 is $nR_2^{1,nc} = 1.6382$.

We now turn to the achievable rate for the network coding scheme. Recall that in our *constrained water filling algorithm*, the constraints $R_m^{1,nc} \le R_m^{+,nc} - R_m^{-,nc}$, are met by the power allocation at the relay. However, in the network coding scenario, the coding scheme $\mathbb{C}_{\oplus}$ is designed so as to satisfy these constraints according to the Galois field rate-splitting scheme. So we can apply the traditional water filling algorithm to the relay to maximize $\sum_{m=1}^{M} R_m^{1,nc}$ without any extra constraints. Obviously, the value of $\sum_{m=1}^{M} R_m^{1,nc}$ achieved under the traditional water filling algorithm is larger than that of the *constrained water filling algorithm*, which has the extra constraints as $R_m^{1,nc} \le R_m^{+,nc} - R_m^{-,nc}$. In the network coding case, after we obtain the optimal $\sum_{m=1}^{M} R_m^{1,nc}$, there is always a coding scheme $\mathbb{C}_{\oplus}$ to guarantee that $R = \min\left\{\sum_{m=1}^{M} R_m^{+,nc},\ \sum_{m=1}^{M} R_m^{1,nc} + \sum_{m=1}^{M} R_m^{-,nc}\right\}$.

IV. Numerical Results

As an example in Fig. 4, we adopt $M = 5$ and an attenuation exponent $\alpha = 2$. All sources are randomly distributed in a circle with radius 0.5, where distance between the centre of the circle and the destination is normalized to 1. We also assume that the initial position of the relay is at the centre of the circle, and can only move along the line from the centre of the circle to the destination. The value of d_m^{sr} is uniformly distributed in the range $(0, 0.5)$ and the angle φ_m is uniformly distributed in the range $(0, 2\pi]$ when the relay is at its initial position. We consider the total power of the relay as $P_{10} = 1, 6, 11$ and the noise power $N = 1$. We study the achievable rate with various locations of the relay.

In Fig. 5, the power of each source is assumed to have the same value as $P_m = 5$, $m = 1, \cdots, 5$. While in Fig. 6, the power of each source is uniformly distributed from 1 to 10. Fig. 5 and Fig. 6 show the achievable rates of 4 different schemes - which are outlined below (1) Separate processing scheme with constrained water filling algorithm (SP, CWF); (2) Separate processing scheme with average power allocation scheme (SP, APA); (3) Separate processing

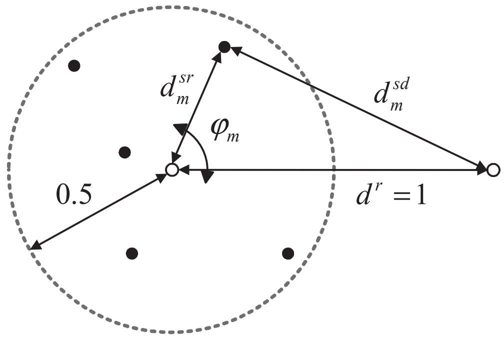

Fig. 4. The geographical distributions of the sources, the relay and the destination. The distance between the relay and the destination is 1. All the sources are randomly distributed in a circle with the radius 0.5.

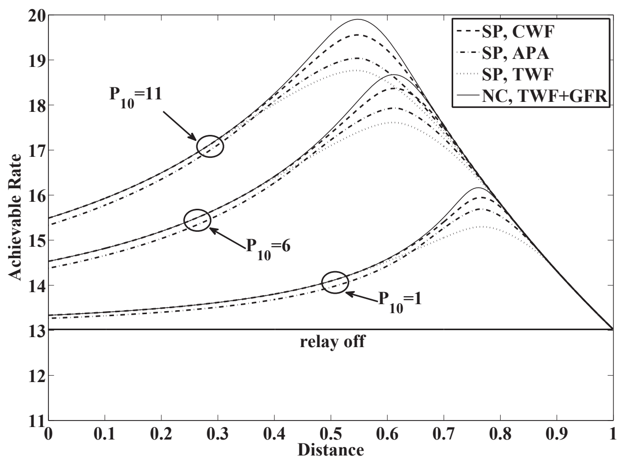

Fig. 5. The power of each source is equally allocated as $P_m = 5$, $m = 1, \cdots, 5$. The horizontal axis represents the distance between the relay and the centre of the circle. The vertical axis represents the achievable rate.

scheme with traditional water filling algorithm (SP, TWF); (4) Network coding scheme with traditional water filling algorithm plus the Galois field rate-splitting approach (NC, TWF+GFR).

From the simulations, we can see that the achievable rate of the network coding scheme is the highest among all the other schemes. Our proposed constrained water filling algorithm for the separate processing scheme obtains higher achievable rate relative to both the traditional water filling algorithm and an equal power allocation scheme.

V. Conclusions

In this work we have determined the achievable rates in an FDMA $M-1-1$ relay system under the assumption of Gaussian phase-fading channels. Two different schemes were investigated, namely, a separate processing scheme at the relay, and a network coding scheme at the relay. For the separate processing scheme, we proposed an new constrained water-filling algorithm which determined the power allocation at the relay needed to obtain the achievable rate. For the network coding scheme we derived the achievable rate based on the use of a Galois field rate-splitting theorem. We discussed

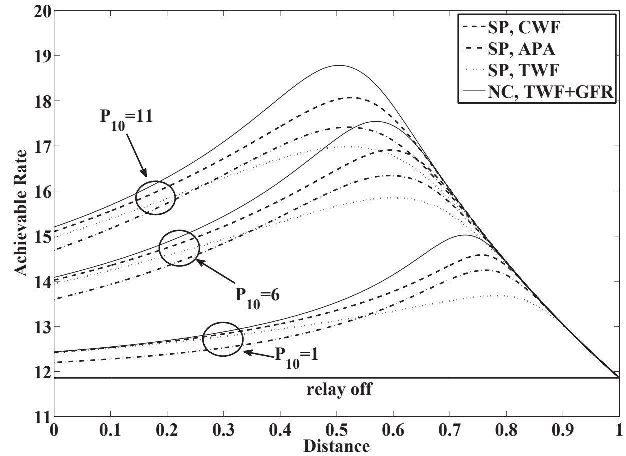

Fig. 6. The power of each source is uniformly distributed from 1 to 10. The horizontal axis represents the distance between the relay and the centre of the circle. The vertical axis represents the achievable rate.

how, in the network coding scheme, a traditional water-filling algorithm can be deployed to set the power allocations at the relay. Finally, we show how network coding schemes will always obtain higher achievable rates relative to those obtained from a separate processing schemes.

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