Robust Synthesis Method for Secure Directional Modulation With Imperfect Direction Angle

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***Abstract*— Directional modulation (DM) is a secure transmission technology that is able to retain the original constellation of transmitted signals along the desired direction, while distort the constellation in the undesired directions at the same time. In this letter, we develop novel and robust DM synthesis methods for enhancing the transmission performance. Specifically, we first propose a low-complexity dynamic DM synthesis method. In this method, we derive a closed-form expression for the null space of conjugate transpose of the steering vector in the desired direction. Based on the expression derived, we construct a projection matrix in order to form artificial noises to those undesired directions. Then, we focus our attention on more practical scenarios, where there is uncertainty in the estimated direction angle. This uncertainty will cause estimation errors and seriously jeopardize the receiving performance in the desired direction. To mitigate the uncertainty effect, we further propose a robust DM synthesis method based on conditional minimum mean square error. The proposed method aims to minimize the distortion of the constellation points along the desired direction. Simulation results show that our proposed robust DM method is capable of substantially improving the bit error rate performance compared with the state-of-the-art methods.**

***Index Terms*— Directional modulation, secure, robust, conditional minimum mean square error.**

I. Introduction

Directional modulation (DM), as a promising technique without encryption for physical layer security, has attracted extensive studies in recent years. In [1], the authors realized a DM synthesis method based on the time-varying changes in the antenna near-field boundary condition. A similar DM concept was proposed in [2], where the signal can be produced in a given direction by shifting the phase of each antenna element. In [3], the authors developed a method, in which the in-phase and quadrature baseband signals are separately radiated from the antenna array and then combined in the pre-specified spatial direction. However, in these previous works, the DM synthesis is implemented on the radio frequency (RF) frontend, which lacks flexibility and makes the design of constellation diagram very complicated.

To solve these problems, recent works transfer the design of DM synthesis from the RF frontend to the baseband [4], [5]. Specifically, the authors in [4] proposed an orthogonal vector approach to implement the DM synthesis. Compared with the design in the RF, this approach enables dynamic DM transmissions by sending the same constellation point with a different pattern at each time, thereby improving the information secrecy. However, this method is of a bit high complexity, since there is no closed-form expression for generating the orthogonal basis utilized to carry artificial noises.

Furthermore, although DM synthesis has been investigated intensively, almost all of the previous studies about DM synthesis assume perfect estimations on the direction angles at the DM transmitter. However, in practice, there always exist estimation uncertainties, which may cause a performance degradation of the desired receiver.

In this letter, we 1) propose a novel DM synthesis method with a lower computational complexity compared to [4], and 2) develop, in the first time, a robust solution to combat the estimation errors of the direction angles. Specifically, our DM synthesis method is achieved with a closed-form expression by projecting the artificial vector to null space of conjugate transpose of the steering vector along the desired direction. Furthermore, in our robust solution, given a distribution of the estimation errors, we minimize distortion of the constellation points along the desired direction, based on the minimum mean square error (MMSE) criterion. Simulation results shows that our robust solution achieves a substantial performance gain over the state-of-the-art methods.

The remainder of this letter is organized as follows. In Section II, we present a low-complexity dynamic synthesis method for DM. Then a robust DM synthesis based on the MMSE method is proposed in Section III. The performance of the proposed method is evaluated in Section IV and conclusions are given in Section V.

II. Low-Complex DM Synthesis

In the DM system, the transmitter is deployed with an N-element linear antenna array. The normalized steering vector for the transmit antenna array is denoted by

$$\mathbf{h}(\theta) = \frac{1}{\sqrt{N}}[e^{j2\pi\Psi_\theta(1)} \ \dots \ e^{j2\pi\Psi_\theta(n)} \ \dots \ e^{j2\pi\Psi_\theta(N)}]^T, \quad (1)$$

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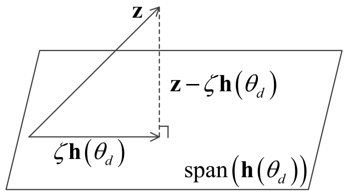

Fig. 1. Illustration of the construction process for the projection matrix **P**.

where $(\cdot)^T$ refers to a transpose operation, θ is the direction angle, d denotes the element spacing in the transmit antenna array, λ is the wavelength, and $\Psi_\theta(n)$ is defined by

$$\Psi_\theta(n) \triangleq -\frac{(n-(N+1)/2)d\cos\theta}{\lambda}, \quad n = 1, 2, \ldots, N. \tag{2}$$

The steering vector has the normalized value $\mathbf{h}^H(\theta)\mathbf{h}(\theta) = 1$, here, $(\cdot)^H$ denotes a conjugate transpose operation. To enhance the information secrecy, artificial noises are deliberately superimposed on the transmitted signals. In the following, we will propose a novel DM synthesis method by designing a projection matrix and projecting the artificial noises to the null space of conjugate transpose of the steering vector along the desired direction. By using this method, we expect that the signal along the desired direction will not be affected by the artificial noises, while the signals along undesired directions will be seriously distorted by them. Furthermore, our method is capable of changing signals dynamically such that the eavesdroppers can hardly decipher the useful messages.

The artificial noise vector $\mathbf{z}$ imposed consists of N complex Gaussian variables with power one, i.e., $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I}_N)$, here, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. As shown in Fig. 1, $\mathbf{z}$ generally does not lie in span($\mathbf{h}(\theta_d)$). The vector $\zeta\mathbf{h}(\theta_d) \in$ span($\mathbf{h}(\theta_d)$) closest to $\mathbf{z}$ in the Euclidean norm occurs when the residual vector $\mathbf{z}-\zeta\mathbf{h}(\theta_d)$ is orthogonal to span($\mathbf{h}(\theta_d)$). Thus, for the least squares solution ζ, the residual vector $\mathbf{z}-\zeta\mathbf{h}(\theta_d)$ must be orthogonal to $\mathbf{h}(\theta_d)$, and hence we have

$$\mathbf{h}^H(\theta_d)(\mathbf{z} - \zeta\mathbf{h}(\theta_d)) = 0. \tag{3}$$

Then we obtain the scaler $\zeta = \frac{\mathbf{h}^H(\theta_d)\mathbf{z}}{\mathbf{h}^H(\theta_d)\mathbf{h}(\theta_d)}$. By substituting ζ into $(\mathbf{z} - \zeta\mathbf{h}(\theta_d))$, we arrive at

$$\mathbf{z} - \zeta\mathbf{h}(\theta_d) = (\mathbf{I}_N - \mathbf{h}(\theta_d)\mathbf{h}^H(\theta_d))\mathbf{z}, \tag{4}$$

we notice that the matrix $(\mathbf{I}_N - \mathbf{h}(\theta_d)\mathbf{h}^H(\theta_d))$ can project $\mathbf{z}$ into the null space of $\mathbf{h}^H(\theta_d)$. We therefore define the project matrix as follows:

$$\mathbf{P}(\theta_d) \triangleq \mathbf{I}_N - \mathbf{h}(\theta_d)\mathbf{h}^H(\theta_d). \tag{5}$$

The baseband transmit signal can be expressed as

$$\mathbf{s} = \beta_1\sqrt{\mathrm{P_s}}\mathbf{v}x + \beta_2\sqrt{\mathrm{P_s}}\mathbf{P}(\theta_d)\mathbf{z}, \tag{6}$$

where x is a symbol chosen from the complex signal constellation with average power $\mathbb{E}[\|x\|^2] = 1$. Here, $\mathbb{E}[\cdot]$ refers to the expectation operation and $\|\cdot\|$ denotes the norm of a complex number. Also in Eq. (6), $\mathrm{P_s}$ is the average transmit power, β_1 and β_2 are the power allocation factors for the transmitted signals and artificial noises, respectively, with the constraint $\beta_1^2 + \beta_2^2 = 1$. Furthermore, $\mathbf{v}$ is defined as the excitation signal vector, namely, a vector for the purpose of preserving the transmitted standard constellation pattern along θ_d. Since the steering vector of the desired direction angle is $\mathbf{h}(\theta_d)$, we therefore set $\mathbf{v} = \mathbf{h}(\theta_d)$.

We assume that all the channels are line-of-sight (LoS) ones, i.e., the channels between the transmitter and the receivers are unobstructed. Without loss of generality, we also assume that channel gain is equal to 1 and phase compensation is employed at the receivers. Then the received signal for the desired receiver in the direction angle θ_d is

$$\begin{aligned}\mathbf{y}(\theta_d) &= \mathbf{h}^H(\theta_d)\mathbf{s} + \mathbf{n}_d \\ &= \beta_1\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_d)\mathbf{v}x + \beta_2\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_d)\mathbf{P}(\theta_d)\mathbf{z} + \mathbf{n}_d \\ &= \beta_1\sqrt{\mathrm{P_s}}x + \mathbf{n}_d,\end{aligned} \tag{7}$$

where $\mathbf{n}_d$ is the received $N \times 1$ complex additive white Gaussian noise (AWGN) vector, distributed as $\mathbf{n}_d \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_N)$. For those undesired directions, e.g., θ_u, the received signal can be expressed as

$$\begin{aligned}\mathbf{y}(\theta_u) &= \mathbf{h}^H(\theta_u)\mathbf{s} + \mathbf{n}_u \\ &= \beta_1\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_u)\mathbf{v}x + \beta_2\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_u)\mathbf{P}(\theta_d)\mathbf{z} + \mathbf{n}_u,\end{aligned} \tag{8}$$

where $\mathbf{n}_u$ is the received noise vector with distribution as $\mathbf{n}_u \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_N)$, and $\mathbf{h}(\theta_u)$ is the steering vector along the undesired direction.

Furthermore, from Eq. (8), we can see that the item $\beta_1\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_u)\mathbf{v}$ distorts the amplitude and phase of the signals along the undesired directions. Additionally, since $\mathbf{h}(\theta_u)$ is not orthogonal to $\mathbf{P}(\theta_d)$, the item $\beta_2\sqrt{\mathrm{P_s}}\mathbf{h}^H(\theta_u)\mathbf{P}(\theta_d)\mathbf{z}$ is nonzero, which further distorts constellation of x.

According to Eq. (5), the calculation of the projection matrix $\mathbf{P}(\theta_d)$ requires N^2 multiplications and N^2 summations. Therefore, the computational complexity on calculating $\mathbf{P}(\theta_d)$ is $O(N^2)$. By contrast, the method in [4] needs to obtain the orthonomal basis of $\mathbf{h}^H(\theta_d)$, which has a higher computational complexity of $O(N^3)$. Comparing to [4], our method has a lower computational complexity. Furthermore, owing to the closed-form expression obtained from our method, we can further develop a robust DM synthesis. However, since the method in [4] does not have a closed-form expression, it is unable to fulfill a further derivation.

III. Robust DM Synthesis

Generally, to synthesize the DM, the transmitter needs to know the value of direction angle of the desired user in advance. In a practical system without GPS, the direction of departure (DoD) is usually obtained by some classic estimation algorithms such as Capon's method, MUSIC, and ESPRIT. However, a small error in the DoD estimation will cause angle mismatch between the steering vector $\mathbf{h}(\theta_d)$ and the excitation signal vector $\mathbf{v} = \mathbf{h}(\hat{\theta}_d)$, which severely degrade the performance of the DM system. To address this problem, in this section, we will propose a robust DM synthesis method to recalculate $\mathbf{v}$ of the transmitter, denoted by $\hat{\mathbf{v}}$, for combating the estimation errors of the direction angles.

We denote by $\hat{\theta}_d$ the estimated angle. Due to the estimation error, we have $\hat{\theta}_d = \theta_d + \triangle\theta_d$, where $\triangle\theta_d$ is the angle error.

In this letter, we assume $\triangle\theta_d$ is uniformly distributed over the interval $[-\triangle\theta_m, \triangle\theta_m]$. Note that $\triangle\theta_m$ represents the maximum angle error and is a positive value up to the beamwidth between first nulls [6]. Let $p(\triangle\theta_d)$ denote the probability distribution of $\triangle\theta_d$. We have

$$p(\triangle\theta_d) = \begin{cases} \frac{1}{2\triangle\theta_m}, & \text{for} -\triangle\theta_m \leq \triangle\theta_d \leq \triangle\theta_m, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Due to the effect of the angle error $\triangle\theta_d$, the estimated angle $\hat{\theta}_d$ can also be viewed as a variable with a uniform distribution. Correspondingly, we calculate the expectation of the projection matrix $\mathbf{P}(\theta_d)$ with respect to $\hat{\theta}_d$ as

$$\begin{aligned} \mathbb{E}_{\hat{\theta}_d}[\mathbf{P}(\hat{\theta}_d)] &= \mathbb{E}_{\hat{\theta}_d}[\mathbf{P}(\theta_d + \triangle\theta_d)] \\ &= \mathbf{I}_\mathrm{N} - \mathbb{E}_{\hat{\theta}_d}[\mathbf{h}(\theta_d + \triangle\theta_d)\mathbf{h}^H(\theta_d + \triangle\theta_d)]. \quad (10) \end{aligned}$$

We further define $\mathbf{R} \triangleq \mathbb{E}_{\hat{\theta}_d}[\mathbf{h}(\theta_d + \triangle\theta_d)\mathbf{h}^H(\theta_d + \triangle\theta_d)]$ and the calculation of each element in $\mathbf{R}$ is derived in Appendix A. Hence, the transmitted signal and received signal in Eq. (6) and (7) can be rewritten, respectively, as

$$\begin{aligned} \mathbf{s} &= \beta_1\sqrt{\mathrm{P_s}}\hat{\mathbf{v}}x + \beta_2\sqrt{\mathrm{P_s}}\mathbb{E}_{\hat{\theta}_d}[\mathbf{P}(\hat{\theta}_d)]\mathbf{z}, \\ \mathbf{y}(\hat{\theta}_d) &= \mathbf{h}^H(\hat{\theta}_d)\mathbf{s} + \mathbf{n}_d. \quad (11) \end{aligned}$$

In the following, we will propose a robust method based on the conditional MMSE [7] by optimizing the excitation signal vector $\hat{\mathbf{v}}$. The optimization problem can be formulated as

$$\begin{aligned} &\min_{\hat{\mathbf{v}}} \mathbb{E}_{\hat{\theta}_d}[\|\mathbf{y}(\hat{\theta}_d) - \beta_1\sqrt{\mathrm{P_s}}x\|^2] \\ &\text{s.t. } \hat{\mathbf{v}}^H\hat{\mathbf{v}} \leq 1. \quad (12) \end{aligned}$$

Theorem 1: According to the objective function in Eq. (12), the optimal solution of $\hat{\mathbf{v}}$ is expressed as

$$\hat{\mathbf{v}} = (\mathbf{R} + \frac{1}{\gamma}\mathbf{I}_N)^{-1}\mathbf{u}, \quad (13)$$

where $\gamma \triangleq \beta_1^2 P_s/\sigma^2$ is the signal-to-noise ratio (SNR) in the desired direction.

Proof: Please refer to Appendix B.

The received signal along the desired direction is

$$\begin{aligned} \mathbf{y}(\hat{\theta}_d) &= \mathbf{h}^H(\hat{\theta}_d)\mathbf{s} + \mathbf{n}_d \\ &= \beta_1\sqrt{\mathrm{P_s}}\mathbf{h}^H(\hat{\theta}_d)((\mathbf{R} + \frac{1}{\gamma}\mathbf{I}_\mathrm{N})^{-1}\mathbf{u})x \\ &\quad + \beta_2\sqrt{\mathrm{P_s}}\mathbf{h}^H(\hat{\theta}_d)(\mathbf{I}_N - \mathbf{R})\mathbf{z} + \mathbf{n}_d. \quad (14) \end{aligned}$$

As shown in Eq. (14), the SNR and angle error have been fully considered in our method. In the following, we conduct simulations to demonstrate how these two factors effect the performance of the DM system.

IV. Simulations

To evaluate the performance of our methods, the parameters and specifications in our simulation are used as follows. We adopt quadrature phase shift keying (QPSK) modulation. The element spacing is one half wavelength, i.e., $d = \lambda/2$. The number of elements is 16, i.e., $N = 16$. The desired direction θ_d is set to 45°, the transmit power is set to $P_s = 1$, and the power allocation factors are set to $\beta_1 = \sqrt{0.9}$ and $\beta_2 = \sqrt{0.1}$.

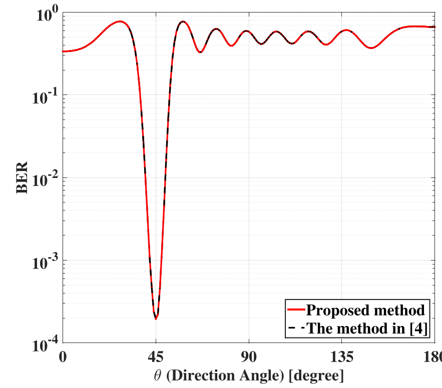

Fig. 2. Comparison of BER performance between our proposed method and the method in [4].

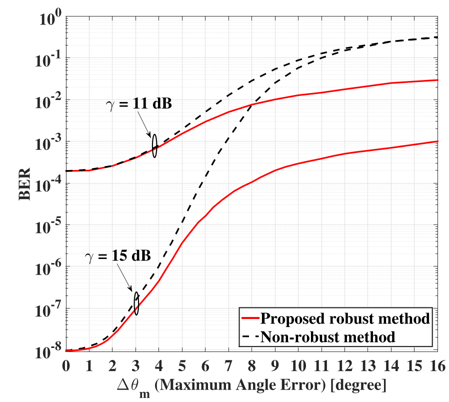

Fig. 3. The performance of BER versus maximum angle error.

In Fig. 2, we set the SNR γ to 11 dB. The figure shows the bit error rate (BER) performances of our proposed method (solid line) and the work in [4] (dotted line) versus various values of the direction angle θ from 0° to 180°. From the figure, we can see that the BER of our method can achieve 1.95×10^{-4} in the desired direction, while becomes worse rapidly in the undesired direction. This is because that the signal along the desired direction will not be affected by the the artificial noise and the constellation of the signal is not distorted, while the signals along the undesired directions are severely interfered by them. Furthermore, the BER of our method is same as that of the method proposed in [4].

Now we study the performance of the DM system versus the maximum estimated error $\triangle\theta_m$ of the direction angle. In Fig. 3, we set the SNR γ to 11dB and 15dB, and investigate BER performance under various values of $\triangle\theta_m$ from 0° to 16°. From the figure, we can see that the BER performance of the non-robust method (dotted line) becomes worse more rapidly compared to that of our proposed robust method (solid line). This means that our robust method is capable of suppressing the estimated errors more effectively than the non-robust method. As the SNR γ increases, the BER gap between the two methods becomes larger. Therefore, the proposed robust method performs better than the non-robust one with the increase of SNR. For further comparison, Fig. 4 shows the BER performance at a specified $\triangle\theta_m = 8°$ and the SNR γ is set to 15 dB, respectively. In the desired direction, it shows

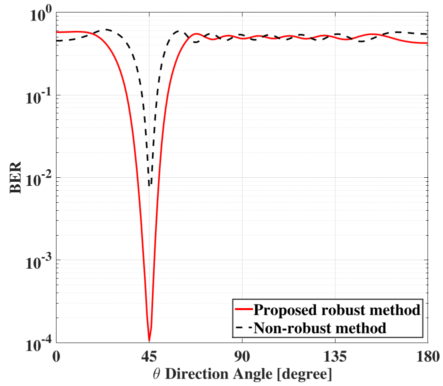

Fig. 4. The performance of BER versus direction angle.

that the BER performance of the proposed robust method is much better than that of the non-robust method.

V. Conclusion

In this letter, we first proposed a dynamic synthesis method for DM with a low computational complexity. Compared to previous methods, the advantage of this method is to provide a closed-form expression of the projection matrix for directly constructing the null space of conjugate transpose of the steering vector. In the scenarios where the estimation angle error presents, a robust DM synthesis method based on conditional MMSE was proposed to alleviate the performance degradation due to the estimation error. The excitation signal vector and projection matrix were developed according to the probability distribution of the angle error. From the analysis and simulations, our proposed robust method have much better BER performance than the non-robust ones.

Appendix A
Derivation of R

The matrix $\mathbf{R}$ is calculated through computing expectation with respect to estimated angle $\hat{\theta}_d$, in which the entry at the m-th row and the n-th column is computed as follows

$$\begin{aligned}\mathbf{R}_{mn} &= \mathbb{E}_{\hat{\theta}_d}[\mathbf{h}_m(\theta_d+\Delta\theta_d)\mathbf{h}_n^H(\theta_d+\Delta\theta_d)]\\ &= \frac{1}{N}\int_{-\Delta\theta_m}^{\Delta\theta_m} e^{\frac{j2\pi(m-n)d(\cos(\theta_d)\cos(\Delta\theta_d)-\sin(\theta_d)\sin(\Delta\theta_d))}{\lambda}}\\ &\quad\times p(\Delta\theta_d)\mathrm{d}(\Delta\theta_d). \end{aligned} \tag{15}$$

In order to simplify Eq. (15), let

$$\begin{aligned} a_{mn} &\triangleq j2\pi(m-n)d\cos(\theta_d)\lambda^{-1},\\ b_{mn} &\triangleq -j2\pi(m-n)d\sin(\theta_d)\lambda^{-1},\\ c &\triangleq \Delta\theta_m\pi^{-1}. \end{aligned} \tag{16}$$

Substituting Eq. (9) and Eq. (16) in Eq. (15) yields

$$\begin{aligned}\mathbf{R}_{mn} &= \frac{1}{2\pi N}\int_{-\pi}^{\pi} e^{(a_{mn}\cos(cx)+b_{mn}\sin(cx))}\mathrm{d}x\\ &= \frac{1}{N}g_I(a_{mn},b_{mn},c), \end{aligned} \tag{17}$$

where $g_I(\cdot)$ is the extension of the modified Bessel function of the first kind with the integer order 0 [8]. This completes the derivation. ■

Appendix B
Solution to the Optimization Problem

The objective function in Eq. (12) is expanded as

$$\begin{aligned}&\mathbb{E}_{\hat{\theta}_d}[\|\mathbf{y}(\hat{\theta}_d)-\beta_1\sqrt{\mathrm{P_s}}x\|^2]\\ &\quad= \mathbb{E}_{\hat{\theta}_d}[\|\mathbf{h}^H(\hat{\theta}_d)\mathbf{s}-\beta_1\sqrt{\mathrm{P_s}}x\|^2]+\sigma^2. \end{aligned} \tag{18}$$

Since that the receiver noise $\mathbf{n_d}$ are independent of the signal $\mathbf{s}$, minimizing Eq. (18) is equivalent to minimizing

$$\mathbb{E}_{\hat{\theta}_d}[\|\mathbf{h}^H(\hat{\theta}_d)\mathbf{s}-\beta_1\sqrt{\mathrm{P_s}}x\|^2]+\sigma^2\hat{\mathbf{v}}^H\hat{\mathbf{v}}, \tag{19}$$

Here, we use $f(\hat{\mathbf{v}})$ to represent the objective function of the optimization problem Eq. (19), and we further expand the $f(\hat{\mathbf{v}})$ as

$$\begin{aligned} f(\hat{\mathbf{v}}) &= \beta_1^2\mathrm{P_s}\hat{\mathbf{v}}^H\mathbf{R}\hat{\mathbf{v}}+\sigma^2\hat{\mathbf{v}}^H\hat{\mathbf{v}}\\ &\quad-\beta_1^2\mathrm{P_s}\mathbb{E}_{\hat{\theta}_d}[\mathbf{h}^H(\theta_d+\Delta\theta_d)]\hat{\mathbf{v}}\\ &\quad-\beta_1^2\mathrm{P_s}\hat{\mathbf{v}}^H\mathbb{E}_{\hat{\theta}_d}[\mathbf{h}(\theta_d+\Delta\theta_d)]+\beta_1^2\mathrm{P_s}+\beta_2^2\mathrm{P_s}\|\mathbf{R}\|^2. \end{aligned} \tag{20}$$

We define $\mathbf{u}\triangleq\mathbb{E}_{\hat{\theta}_d}[\mathbf{h}(\theta_d+\Delta\theta_d)]$, and the calculation of $\mathbf{u}$ is similar to that of $\mathbf{R}$.

To obtain the optimal excitation signal vector, we need to compute the derivative of $f(\hat{\mathbf{v}})$ with respect to $\hat{\mathbf{v}}$,

$$\frac{\partial f(\hat{\mathbf{v}})}{\partial\hat{\mathbf{v}}} = 2\beta_1^2\mathrm{P_s}\mathbf{R}\hat{\mathbf{v}}+2\sigma^2\hat{\mathbf{v}}-2\beta_1^2\mathrm{P_s}\mathbf{u}. \tag{21}$$

It is easy to see from Eq. (21) that the $\beta_1^2\mathrm{P_s}\mathbf{R}+\sigma^2\mathbf{I}_N$ is invertible. Let $\frac{\partial f(\hat{\mathbf{v}})}{\partial\hat{\mathbf{v}}}=\mathbf{0}$, we obtain

$$\hat{\mathbf{v}} = \beta_1^2\mathrm{P_s}(\beta_1^2\mathrm{P_s}\mathbf{R}+\sigma^2\mathbf{I}_N)^{-1}\mathbf{u}. \tag{22}$$

This completes the proof. ■

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