A Stackelberg Game Approach for Caching Video in Small-Cell Cellular Networks

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Abstract—**In this paper, a game-theoretical approach is proposed for efficiency on pricing and resource allocation in commercial caching system, which consists of a video retailer (VR) and multiple network service providers (NSPs). A two-level Stackelberg game is employed to jointly consider the benefits of NSPs and VR in which NSPs are models as the leaders and VR is modeled as the follower, respectively. Each NSP first sets a proper price on leasing its small-cell base stations (SBSs) to the VR for the purpose of maximizing utilities, followed by the responses of the VR to the leaders' strategies and rents an optimal fraction of SBSs to provide better local video services to the mobile users (MUs) for gaining profit. First, we develop the system model and measure the downloading performance based on stochastic geometry theory. Then, we formulate the optimization problems and investigate the Stackelberg equilibrium. Numerical results are provided for quantifying the proposed framework by showing its efficiency on pricing and resource allocation**

Keywords-wireless caching; small-cell networks; stochastic geometry; Stackelberg game

I. INTRODUCTION

Wireless data traffic, especially the multimedia traffic, has been increasing dramatically in recent years driven by the enhancing capabilities of smart-phones and tablets. An important observation regarding the mobile traffic increase is that a large amount of traffic is caused by the duplicated download of a small portion of popular contents, such as popular movies and videos. There are numerous repetitive requests on the same videos from the MUs, such as online movies, leading to redundant transmissions. Fortunately, this redundancy can be reduced by locally storing popular videos, known as caching, into the memory of intermediate network nodes, effectively forming a local caching system [1,2]. This local caching brings video content closer to the MUs and alleviates redundant data transmissions via redirecting the downloading requests to the intermediate nodes.

As a large number of small-cell base stations (SBS) have already been deployed these years and the number of SBSs is expected to keep increasing, this dense deployment of SBSs provides a good basis for caching, referred to as small-cell caching. Compared with wireless caching in D2D networks [3], small-cell caching has several advantages, such as abundant power supply, less security issues and more reliable data delivery.

In [4], a small-cell caching scheme, called `Femto-caching', is proposed for a cellular network embedded with SBSs, where the data placement at the SBSs is optimized in a centralized manner for reducing the transmission delay imposed. In [5], the small-cell caching is investigated in the context of stochastic networks. The average performance is developed via stochastic geometry [6,7], where the distribution of network nodes are modeled by Poisson point process (PPP) [8].

In this paper, we propose a novel commercial caching system. We first model the MUs and SBSs as two different ties of a Poisson point process (PPP) based on the stochastic geometry theory. Under this network model, we derive the probability of the event that an MU obtains the requested video directly from the storage of an nearest SBS with a closed-form expression. Next, a two-level Stackelberg game is employed to jointly consider the benefits of NSPs and VR in which NSPs are models as the leaders and VR is modeled as the follower, respectively. Then, we formulate the optimization problems based on the derived probability. Furthermore, we investigate the Stackelberg equilibrium by solving the optimization problems in two cases, i.e., whether or not the VR has a deliberate budget plan on spending how much money in renting the SBSs. Numerical results are finally provided to quantify the proposed framework by showing its efficiency on pricing and resource allocation.

II. SYSTEM MODEL AND PRELIMINARIES

In this paper, we consider a small-cell network consisting of L NSPs, one VR and multiple MUs. Each NSP $\mathcal{N}_i$, $i=1,\cdots,L$, owns a number of SBSs. These SBSs owned by $\mathcal{N}_i$ are equipped with a transmission power P_i and they are spatially distributed as a homogeneous PPP (HPPP) Φ_i of intensity λ_i. Each MU is affiliated with an NSP and connects to one of its SBSs for accessing network services. These MUs affiliated with $\mathcal{N}_i$ are spatially distributed as a independent HPPP Ψ_i of intensity ζ_i. Here, the intensity represents the average number of SBSs/MUs in per unit area. Then, the VR rents a fraction τ_i of SBSs from $\mathcal{N}_i$ to cache its video files. We assume that the rented SBSs of $\mathcal{N}_i$ are uniformly selected by the VR, according to the thinning theorem in HPPP, thus the distribution of these SBSs can be modeled as a "thinned" HPPP with the intensity of $\tau_i\lambda_i$.

We consider a content library of M different files with the same size. Note that, M does not represent the number of files available on the Internet, but the number of popular files that MUs tend to access. We denote by q_m the probability that a particular file will be requested. By stacking q_m into

$\{q_m : m=1,\cdots,M\}$, we can get the request probability mass function (PMF) of the M files. According to [9], we can model the request PMF of the files as a Zipf distribution. More specifically, for the $m-th$ file, its request probability q_m is written as

$$q_m = \frac{1/m^{\beta}}{\sum_{j=1}^{M} 1/j^{\beta}}, \quad \forall m, . \tag{1}$$

where the exponent β is a positive value, characterizing the file popularity. A large β corresponds to uneven popularity among these files.

Due to the limited storage of SBSs, each SBS cannot cache the entire file library. Therefore, we assume that the SBSs owned by $\mathcal{N}_i$ are equipped with a specified storage of Q_i, i.e., each SBS of $\mathcal{N}_i$ can store Q_i video files. Meanwhile, each SBS of $\mathcal{N}_i$ is required to cache the most popular Q_i files $\mathcal{F}_1,\cdots,\mathcal{F}_{Q_i}$ and we denote by $\mathcal{G}_i$ these files. The probability that an MU requests a file in $\mathcal{G}_i$, denoted by R_i, is thus given by

$$R_i = \sum_{m=1}^{Q_i} q_m \, . \tag{2}$$

Generally, an MU will connect to the nearest SBS that caches the requested files. Assume that wireless the down-link channels spanning from the SBSs to the MUs are independent and identically distributed (i,i,d), and modeled as the combination of path-loss and Rayleigh fading. Without a loss of generality, we conduct our analysis on a typical MU $\mathcal{M}$ located at the origin. The path-loss between $\mathcal{M}$ and an SBS located at z is denoted by $\|z\|^{-\alpha}$, where α is the path-loss exponent. The channel power of the Rayleigh fading between them is denoted by h_z, where $h_z \sim \exp(1)$. The noise at each MU is Gaussian distributed with variance σ^2. We assume that the SBSs from the same NSP transmit on the same channel, causing mutual interferences. At the same time, different NSPs are allocated with orthogonal channels. Hence, there are no interferences across NSPs. To simplify the notation, we utilize the location z of a rented SBS from $\mathcal{N}_i$ to represent a point in the HPPP $\Phi_i, \forall i$. The received signal-to-interference-plus-noise ratio (SINR) at a typical MU $\mathcal{M}$ in Ψ_i from an SBS $\mathcal{B}$ in Φ_i, located at z, can be expressed as

$$\gamma_i(z) = \frac{P_i h_z \| z\|^{-\alpha}}{\sum_{x\in\Phi_i\backslash z} P_i h_x \| x\|^{-\alpha} + \sigma^2} \, . \tag{3}$$

where $\sum_{x\in\Phi_i\backslash z} P_i h_x \| x\|^{-\alpha}$ represents the interference from the SBSs of Φ_i except for $\mathcal{B}$.

When an MU $\mathcal{M}$ affiliated with $\mathcal{N}_i$ demands $\mathcal{F}_m$, the request will be redirected to the nearest SBS $\mathcal{B}$ in Φ_i that caches $\mathcal{F}_m$. We assume that only when the received SINR at the MU is above a prescribed threshold δ, can the requested file be successfully downloaded. The requested video can be obtained directly from $\mathcal{B}$, and we define such an event by $\mathcal{D}_{i,m}$. Regarding the probability $\Pr(\mathcal{D}_{i,m})$ of the event $\mathcal{D}_{i,m}$, we have the following theorem based on the stochastic geometry theory.

Theorem 1: The probability $\Pr(\mathcal{D}_{i,m})$, $\forall i,m$, can be expressed as

$$\Pr(\mathcal{D}_{i,m}) = \begin{cases} \int_0^{\infty} \exp\left(-\pi(1-\tau_i)\lambda_i C(\delta,\alpha) z^2\right) \\ \pi\tau_i\lambda_i \exp\left(-\pi\tau_i\lambda_i A(\delta,\alpha) z^2\right) \\ \exp(-\frac{z^{\alpha}\delta}{P_i}\sigma^2)\exp\left(-\pi\tau_i\lambda_i z^2\right) \mathrm{d}z^2 \qquad m \le Q_i, \\ 0 \qquad M \ge m > Q_i, \end{cases} \tag{4}$$

where we have

$$\begin{aligned} A(\delta,\alpha) &\triangleq \frac{2\delta}{\alpha-2}\, {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\delta\right), \\ C(\delta,\alpha) &\triangleq \frac{2}{\alpha}\delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1-\frac{2}{\alpha}\right). \end{aligned} \tag{5}$$

Furthermore, ${}_2F_1(\cdot)$ in the function $A(\delta,\alpha)$ is the hypergeometric function and the Beta function in $C(\delta,\alpha)$ is formulated as $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$.

Proof : In this work we omit all of our proofs due to space considerations. Full proofs will appear in an extended version of this paper.

If the event $\mathcal{D}_{i,m}$ does not occur, the MU $\mathcal{M}$ will connect to the central server of VR, posited at backbone networks, for the requested video clip, causing a high transmission latency.

Generally, the power of interference in a network is much greater than that of the noises. By assuming that $\frac{\sigma^2}{P_i}$ goes to zero, and for simplicity $A = A(\delta,\alpha)$, $C = C(\delta,\alpha)$. We can further rewrite Eq.(4), then we have

$$\Pr(\mathcal{D}_{i,m}) = \frac{\tau_i}{(A-C+1)\tau_i + C} \, . \tag{6}$$

III. GAME MODEL

Stackelberg game is a strategic game consisting of leaders and followers competing with each other for certain resources [10]. The leaders move first and the followers move subsequently. In our small-cell caching system, we model the NSPs as the leaders, and the VR as the follower. The NSP $\mathcal{N}_i, \forall i$, will set a proper price p_i for leasing one of its SBSs to the VR during a unit period and the VR will respond to the NSPs' strategy and perform the optimal SBSs allocation.

A. NSPs Level Model

The income of the NSPs comes from leasing their SBSs. Meanwhile, the NSPs need to pay for the cost of maintaining the local caching system. We denote by c_i such cost on each rented SBS of $\mathcal{N}_i$ during a unit period. Then the net-profit of $\mathcal{N}_i$ can be expressed as

$$U_{\mathcal{N}_i} = (p_i - c_i)\tau_i\lambda_i . \tag{7}$$

Then, the objective of the NSP $\mathcal{N}_i$ is to maximize its profit $U_{\mathcal{N}_i}$ formulated in Eq.(7). Note that the fraction τ_i is a function of the price p_i under the Stackelberg game formulation. This means that the fraction of the SBSs that the VR is willing to rent depends on the specific price charged for renting an SBS. If the price p_i is too high, the VR will choose not to rent any SBS from $\mathcal{N}_i$. At the same time, if p_i is set too low, $\mathcal{N}_i$ cannot make any profit. Since the maximum payment made by the VR is *U*, the NSPs have to compete with each other on the price such that they can be selected by the VR whilst keep their profit maximized.

Problem 1: The optimization problem for each NSP can be summarized as follows.

$$\max_{p_i \geq 0} \quad U_{\mathcal{N}_i}(p_i, \tau_i), \quad \forall i . \tag{8}$$

B. VR Level Model

For the VR, the revenue gained, with the help of local caching, is from VR's providing fast downloading services as well as mitigating the traffic from its central server. We denote by s the profit acquired by the VR when an MU downloads a video clip from the local caching system, and denote by *K* the number of video requests from each MU on average within an unit period. Then the overall income per unit area and per unit period (*/UAP*) of VR is calculated as

$$U_{cache} = \sum_{i=1}^{L} K\zeta_i R_i \Pr(\mathcal{D}_{i,m}) s. . \tag{9}$$

Meanwhile, the VR needs to pay for renting the SBSs. We denote by p_i the price for renting an SBS from $\mathcal{N}_i$ during an unit period. Then the overall payment*/UAP* made by the VR can be expressed as

$$U_{rent} = \sum_{i=1}^{L} \lambda_i \tau_i p_i . . \tag{10}$$

We assume that the VR may make a budget on how much payment it will make in renting the SBSs. The budget is defined by U/UAP, i.e., $U = U_{rent}$. Then the net profit*/UAP* obtained by the VR is

$$\begin{aligned} U_{VR} &= U_{cache} - U_{rent} \\ &= \sum_{i=1}^{L} \frac{Ks\zeta_i R_i \tau_i}{(A-C+1)\tau_i + C} - \sum_{i=1}^{L} \lambda_i \tau_i p_i . \end{aligned} \tag{11}$$

We can see from Eq. (11) that once the price vector $\mathbf{p}$ is fixed, the profit of VR depends on τ_i, $\forall i$. If VR increases the fraction τ_i, it will gain more profit, while at the same time, VR has to pay for renting more SBSs. Therefore, τ_i needs to be optimized for maximizing the profit of VR. By defining the price vector $\mathbf{p} \triangleq [p_1, p_2, \cdots, p_L]$ and the fraction vector $\boldsymbol{\tau} \triangleq [\tau_1, \cdots, \tau_L]$, this optimization can be formulated as follows.

Problem 2: The optimization problem of maximizing VR's profit can be written as

$$\max_{0 \leq \boldsymbol{\tau} \leq 1} \quad U_{VR}(\mathbf{p}, \boldsymbol{\tau}) . \tag{12}$$

Furthermore, in the case that the VR has a budget plan, there is the constraint $\sum_{i=1}^{L} \tau_i \lambda_i p_i = U$.

IV. Analysis of the Proposed Stackelberg Game

In the following, we will have an in-depth investigation on this game-theoretic optimization. Specifically, we consider two cases in the optimization. In the first case, we assume that the VR does not have a concrete budget plan. That is, the VR will not take into account how much money it spends in renting SBSs. Thus the constraint $\sum_{i=1}^{L} \tau_i \lambda_i p_i = U$ is released during the optimization. In the second case, we assume that the VR has a budget by setting $\sum_{i=1}^{L} \tau_i \lambda_i p_i = U$ in the optimization.

A. Optimization Without Payment Constraint

Let us define $\mathbf{p}^\star \triangleq [p_1^\star, p_2^\star, \cdots, p_L^\star]$,where $p_i^\star$, $\forall i$, is a solution for *Problem 1* and define $\boldsymbol{\tau}^\star \triangleq [\tau_1^\star, \tau_2^\star, \cdots, \tau_L^\star]$ a solution for *Problem 2*. First, we will solve the optimization problem in our game by assuming that the constraint $\sum_{i=1}^{L} \tau_i \lambda_i p_i = U$ is released. We present the following lemma.

Lemma 1: Given a price vector $\mathbf{p} = [p_1, \cdots, p_L]$, the optimum solution of *Problem 2*, without considering the constraint $\sum_{i=1}^{L} \tau_i \lambda_i p_i = U$, can be expressed as

$$\tau_i^\star(p_i) = \left[\frac{1}{A-C+1}\left(\sqrt{\frac{KsC\zeta_i R_i}{\lambda_i p_i}} - C\right)\right]^{\pm}, \forall i \tag{13}$$

where $[w]^{\pm}$ represents $0 \leq w \leq 1$.

We can see from *Lemma 1* that $\tau_i^\star(p_i) \propto p_i^{-\frac{1}{2}}$. If the price p_i is too high such that

$$p_i \geq p_i^{upper} \triangleq \frac{Ks\zeta_i R_i}{C\lambda_i}. \quad (14)$$

where p_i^{upper} is calculated by letting $\tau^\star = 0$, then there is $\tau^\star \leq 0$. This means that, given the price $p_i \geq p_i^{upper}$, the VR will opt out renting any SBS from the NSP $\mathcal{N}_i$ due to the high price charged. In this case, the NSP $\mathcal{N}_i$ may need to lower the price p_i to some extent for ensuring $\tau_i^\star(p_i) > 0$. Meanwhile, considering the cost c_i, $\mathcal{N}_i$ will opt out leasing any SBS to the VR if there is $p_i \leq c_i$.

On the other hand, if the price p_i given is too low such that

$$p_i \leq p_i^{lower} \triangleq \frac{KsC\zeta_i R_i}{(A+1)^2\lambda_i}. \quad (15)$$

where p_i^{lower} is calculated by letting $\tau_i^\star = 1$, then there is $\tau_i^\star \geq 1$. This means that the price given is too low such that the VR is willing to rent all the SBSs of $\mathcal{N}_i$. In this case, the NSP $\mathcal{N}_i$ may need to increase the price p_i for acquiring more profit. Substitute $\tau_i^\star(p_i)$ of *Lemma 1* into Eq.(7) , and we have the following lemma regarding the optimum price $p_i^\star$.

Lemma 2: Given the expression of $\tau_i^\star$ in Eq.(13), the optimum solution of *Problem 1* can be expressed as

$$p_i^\star = \max\left\{\frac{1}{\left(x_i^{\frac{1}{3}} - \frac{1}{3c_i}x_i^{-\frac{1}{3}}\right)^2}, p_i^{lower}\right\}, \forall i. \quad (16)$$

where

$$x_i \triangleq \frac{C}{c_i y_i} + \sqrt{\left(\frac{C}{c_i y_i}\right)^2 + \frac{1}{27c_i^3}}, \quad y_i \triangleq \sqrt{\frac{KsC\zeta_i R_i}{\lambda_i}}. \quad (17)$$

Remark 1: The optimum solution $\mathbf{p}^\star$ in Eq.(16), combined with the solution of $\boldsymbol{\tau}^\star$ given by Eq.(13), constitutes the SE for the Stackelberg game.

From the above discussions, there exist no competitions among the NSPs when no payment constraint is imposed by the VR, since $p_i^\star$ and $\tau_i^\star$ only depend on the parameters of $\mathcal{N}_i$.

B. Optimization Under Budget Plan

We now focus our attention on the game theoretic optimization with the budget plan $\sum_{i=1}^{L}\tau_i\lambda_i p_i = U$. Usually, in this case, the VR does not have a sufficient U to rent enough SBSs. Thus, the money spent by the VR needs to be deliberately planed for renting the SBSs from those competent NSPs.

Regarding the optimum solution at the VR's side, we have the following theorem.

Theorem 2: Given a price vector $\mathbf{p} = [p_1, \cdots, p_L]$, the optimum solution $\tau_i^\star$, $\forall i$ of *Problem 2* can be expressed as

$$\tau_i^\star(\mathbf{p}) = \begin{cases} 0 & \xi > \frac{Ks\zeta_i R_i}{\lambda_i p_i C} - 1, \\ 1 & \xi < \frac{KsC\zeta_i R_i}{\lambda_i p_i (A+1)^2} - 1, \\ \frac{1}{A-C+1}\left(\sqrt{\frac{KsC\zeta_i R_i}{\lambda_i p_i (1+\xi)}} - C\right) & \text{otherwise}, \end{cases} \quad (18)$$

where

$$\xi = \left(\frac{\sum_{j\in\mathcal{S}_2}\sqrt{KsC\lambda_j p_j \zeta_j R_j}}{(A-C+1)(S - \sum_{j\in\mathcal{S}_1}\lambda_j p_j) + \sum_{j\in\mathcal{S}_2}\lambda_j p_j C}\right)^2 - 1. \quad (19)$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ are two sets of the subscripts of τ_i. For $\mathcal{S}_1$, we have $\tau_j = 1$, $\forall j \in \mathcal{S}_1$. For $\mathcal{S}_2$, we have $0 < \tau_j < 1$, $\forall j \in \mathcal{S}_2$. The calculation of *Theorem 2* can be implemented in the following procedure. First, we utilize the conventional water-filling algorithm to optimize τ_i, $\forall i$ without considering the constraint $\tau_i \leq 1$. Second, we check the optimization result. If, for example, there exists i' such that $\tau_{i'} > 1$, then we set $\tau_{i'} = 1$, and modify the original payment constraint to $\sum_{i=1, i\neq i'}^{L}\tau_i\lambda_i p_i = U - \lambda_{i'} p_{i'}$. Next, we execute a second round water-filling process on $\tau_{i'} > 1$, $\forall i$, $i \neq i'$ with the updated constraint. By conducting this procedure iteratively, we finally achieve the optimum solution in *Theorem 2*.

Substituting Eq.(18) into Eq.(7), can be rewritten as

$$\max_{p_i > 0} \quad U_{\mathcal{N}_i} = (p_i - c_i)\lambda_i \tau_i^\star(\mathbf{p}), \forall i. \quad (20)$$

Note that Eq.(20) is a non-cooperative game by the NSPs. Due to the insufficient payment of the VR, the optimal pricing strategy of $\mathcal{N}_i$ depends on other NSPs' pricing strategies, causing competitions among the NSPs.

By taking the derivation of $U_{\mathcal{N}_i}$ to p_i and equating it to zero, we have

$$\frac{\partial U_{\mathcal{N}_i}}{\partial p_i} = \lambda_i \tau_i^\star(\mathbf{p}) + (p_i - c_i)\lambda_i \frac{\partial \tau_i^\star(\mathbf{p})}{\partial p_i} = 0, \forall i. \quad (21)$$

Solving the above equations, we obtain the optimal price $p_i^\star$. However, we can see that there is no closed-form for $p_i^\star$, since each optimal price is related to other prices. One NSP needs to update its own price after the other NSPs change their prices. As such, the optimization process for the NSPs and the VR has to be conducted in an iterative manner.

The iterative process can be summarized as follows. First, after rearranging Eq.(20), we have

$$p_i = I_i(\mathbf{p}) \triangleq c_i - \frac{\tau_i^\star(\mathbf{p})}{\partial \tau_i^\star(\mathbf{p}) / \partial p_i}. \quad (22)$$

In order to calculate p_i in Eq.(22), each NSP communicates with the VR to obtain the values of $\tau_i^\star(\mathbf{p})$ and $\partial\tau_i^\star(\mathbf{p})/\partial p_i$. Then the updating of the NSPs' prices can be described by a vector of the form $\mathbf{p}=\mathbf{I}(\mathbf{p})$, where $\mathbf{I}(\mathbf{p})=[I_1(\mathbf{p}),\cdots,I_L(\mathbf{p})]$. Consequently, an iterative method can be utilized to achieve the optimal solutions, expressed as

$$\mathbf{p}^{(t+1)} = \mathbf{I}^{(t)}(\mathbf{p}^{(t)}) \quad (23)$$

where the superscription t represents the $t-th$ iteration.

Note that $\mathbf{I}^{(t)}$ may change in different iterations, since the two sets $\mathcal{S}_1$ and $\mathcal{S}_2$ can vary. To be specific in the optimization of $\boldsymbol{\tau}$ at the VR in the $t-th$ iteration, if, for example, there is $\tau_i > 1$, the value of τ_i will be set to one and $p_i^\star$ will be set to $p_i^{(t)}$. In this case, it is considered that the NSP $\mathcal{N}_i$ achieves its equilibrium and will quit the iteration for price update.

Remark 2: Once the iterative optimization process converges, the optimum solution $\mathbf{p}^\star$, combined with the solution of $\boldsymbol{\tau}^\star$ given by Eq.(18), constitutes the SE for the game.

V. Numerical Results

In this section, we evaluate the performance of our proposed approach. In our system, the parameters are set as follows: $F=500$, $\beta=0.5$, $s=1$, $K=50$, $\alpha=5$, $\delta=0.1$. Assume there are four NSPs: $c_1=10$, $c_2=12$, $c_3=14$, $c_4=16$; $\lambda_1=10/km^2$, $\lambda_2=25/km^2$, $\lambda_3=40/km^2$, $\lambda_4=50/km^2$, $\zeta_1=100/km^2$, $\zeta_2=140/km^2$, $\zeta_3=180/km^2$, $\zeta_4=220/km^2$; $Q_1=20$, $Q_2=30$, $Q_3=40$, $Q_4=50$.

We first investigate pricing scheme based on storage size. For simplify, we can change the storage size Q_2. Fig.1 shows various prices versus the storage size Q_2 with the budget plan $U=1000$. We can find that the optimal price of $\mathcal{N}_2$ is increasing but the optimal prices of other NSPs are all decreasing. Since there are competitions between these NSPs, while the storage size Q_2 increases, the NSP $\mathcal{N}_2$ becomes more competitive and it can charge a higher price, other NSPs will be behind the competition with fixed storage.

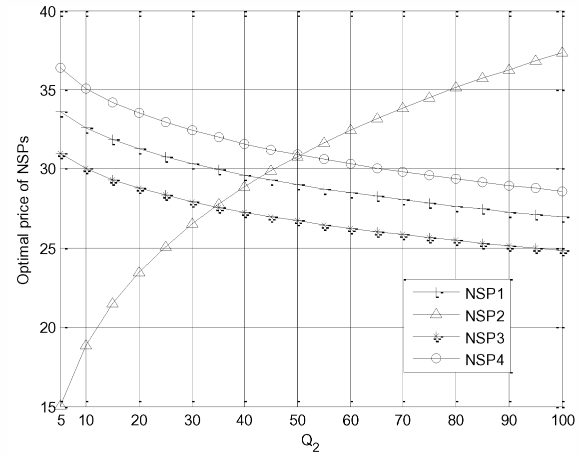

Figure 1. Optimal prices versus Q_2 with the budget plan U = 1000.

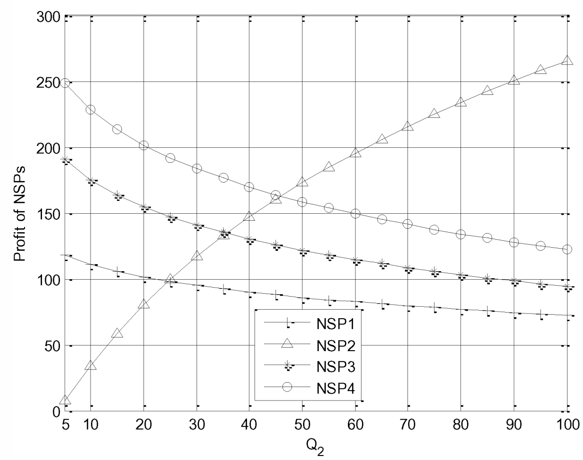

Figure 2. Profit of each NSP versus Q_2 with budget plan U = 1000.

Then, we study the profit of the NSPs with the change storage size under the same scenario. Fig.2 shows that the profit of $\mathcal{N}_2$ is increasing but the profits of other NSPs are all decreasing with Q_2. This is consistent with the change of prices with the storage size Q_2 in Fig.1. Therefore, we can find that for the fixed other parameters, larger storage size Q results in higher price charged by NSP and profit.

Furthermore, we demonstrate the convergence of the iterative optimization process. Fig.3 shows the price updating of the NSPs with budget plans $U=500{,}1000$. From the figure, in the case of budget plan $U=1000$, we can find that the curves all begin to converge when the number of iterations is about 7. In the other case of budget plan $U=500$, we can find that the curves all begin to converge when the number of iterations is about 11. Therefore, we demonstrate the convergence of our proposed optimization algorithm.

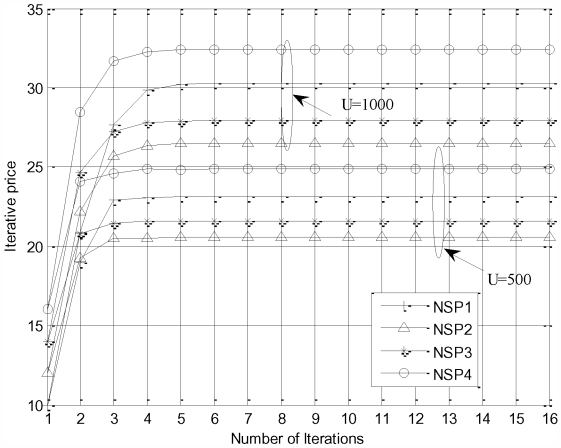

Figure 3. The updating of the prices for NSPs versus the number of iterations. The budget plans are set to U = 500,1000.

VI. CONCLUSIONS

In this paper, we have considered a commercial small-cell caching system consisting of multiple NSPs and one VR, where the NSPs lease their SBSs to the VR for gaining profits, while the VR, after storing popular videos to the rented SBSs, can provide faster transmissions to the MUs, hence gaining more profits. We first modeled the MUs and SBSs using two independent PPPs with the aid of stochastic geometry, and developed the probability expression of successful downloading. Then, we set up the game models and formulated a Stackelberg game for maximizing the profit of the NSPs as well as the VR. We also investigated the Stackelberg Equilibrium for the two cases by solving a series of optimization problems. Finally, we provided several numerical results for showing that the proposed schemes are effective in both pricing and SBSs allocation.

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