The Optimal Performance of Cooperative Communication Systems as a Function of Location Information Quality

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***Abstract*—In this work we investigate the use of location information in the context of emerging 4G wireless communication systems, in which cooperative communication among mobile nodes is utilized. We first determine the optimal throughput achievable in such systems under the assumption of perfect knowledge on the locations of the mobile nodes. We then investigate how the system throughput is significantly affected by the introduction of realistic errors on the reported locations of the nodes. The analytical framework, and the supporting simulations, we provide allow 4G cooperative communication systems to be properly designed for realistic working conditions. Our work also provides key insight into the critical trade-off between the lifetime and performance of wireless cooperative communication systems. Knowledge of this trade-off in the context of cross-layer design is discussed.**

I. Introduction

The use of location information in wireless networks is rapidly becoming ubiquitous, not only in the development of new protocols and applications at the higher network layers, but also in the design of new lower-layer protocols aimed at efficient use of scarce wireless spectrum. In this latter context, the use of location information at the physical layer, in order to deliver increased performance via the use of spatial diversity, has garnered much interest in the recent research literature [1]-[5].

In this work we will be particularly interested in the use of location information at the physical layer within the context of emerging Fourth Generation (4G) wireless networks. Such networks are anticipated to dominate new mobile broadband services as well as allowing for seamless switching between different transmission schemes [2]. The emerging standards for 4G networks introduce the use of additional relay stations in order to assist in communication performance [3]. The relay, in conjunction with cooperation between the mobile nodes, allows for an increase in the spatial diversity available to the wireless system. This additional diversity can be exploited to provide for much higher network performance in the form of increased *throughput*. The location of the relay and the locations of the mobile nodes are key to the delivery of this improved throughput.

Many recent works have investigated the optimal throughput that can be expected in cooperative relay systems of emerging 4G networks [6]-[10]. However, none of these works have studied the impact of the quality of the location information utilized in delivering the optimal throughput. It is the main purpose of this work to remedy that shortcoming. We will find that the input of realistic location error into the analysis has a major effect on the optimal performance.

High accuracy location information of a mobile node is normally obtained through the use of an on-board location system such as GPS, or via use of the wireless network signals through triangulation procedures based on signal strength, timing or directional information. Once acquired constant and rapid communication of the location information throughout the network must be achieved. However, acquiring GPS location information has a non-negligible impact on the battery sources of a mobile device [11]. The acquisition of position information through triangulation is also surprisingly costly in terms of battery resources if mobile nodes are used in the cooperative triangulation [12][13].

Depletion of battery resources obviously impact the lifetime (time before recharge) of the cooperative wireless system. As the mobile nodes move around the network, a natural trade-off arises. One can increase the longevity of the network at the cost of decreasing the location accuracy of the location information provided. It is therefore extremely important to properly assess the cost of using impaired location information on network throughput if we are to intelligently use such a trade-off.

The paper is organized as follows. In section II our 4G system model is introduced and we analyze the optimal throughput of the M-1-1 system in the presence of location error. In section III, we use our results to investigate how the optimal throughput is affected in the context of a well known motion model. Finally, in Section VI we provide discussion on the use of our results and draw our conclusions.

II. Achievable Rate in Presence of Location Errors

We consider a multi-source relay system, where sources $S_m, m \in \{1, ...M\}$ transmit their information to the destina-

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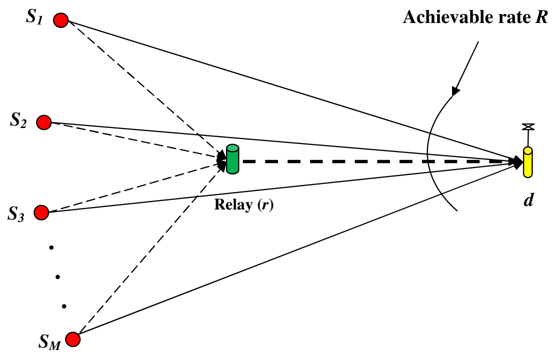

Fig. 1: Multi-source $S_m, m = \{1, ..., M\}$, with single relay and destination (M-1-1) system.

tion d (a base station) simultaneously with the help of a full-duplex relay r. A bin indexing scheme as in [6] was assumed to transmit information and parity bits. The conventional DF relaying with orthogonal transmission[1] through FDMA is assumed. With these constraints, the multi-source system can be viewed as M independent parallel 1 - 1 - 1 triangle systems. All channels are assumed to undergo Rayleigh fading and are corrupted with Additive White Gaussian Noise (AWGN).

We will first investigate the achievable rate R of the M-1-1 system and then investigate the effect of location errors on R. We make the following assumptions in our analysis. A non-interfering transmission scheme is adapted by source and relay such that, the M-1-1 system can be considered as M independent parallel 1-1-1 systems. The source to relay channel SNR is always greater than the source to destination channel SNR. The distances S_m to d, S_m to r and r to d are denoted by d_m^{sd}, d_m^{sr} and d_m^{rd}, respectively.

The source S_m transmits its message $w \in \{1, ..., K\}$ in the frequency bands f_m to the destination. The message w is encoded into n symbols $x_1[1], ..., x_1[n]$ and transmitted over the channel, under the power constraint $\frac{1}{n}\sum_{i=1}^{n} x_1[i]^2 \leq P_s$, where P_s is the maximum transmit power available at each source. The relay decodes and forwards a new message $x_2[i]$ to aid the communication between source and destination. $x_2[i]$ is also encoded into n symbols subject to the power constraint $\frac{1}{n}\sum_{i=1}^{n} x_2[i]^2 \leq P_{rm}$, where P_{rm} is the power allocated to the mth source by the relay. The received signal at the relay y_{rm} and the destination y_{dm} are given by,

$$y_{rm} = h_m^{sr} x_1[i] + N_r \qquad (1)$$

$$y_{dm} = h_m^{sd} x_1[i] + h_m^{rd} x_2[i] + N_d, \qquad (2)$$

where N_r and N_d are independent AWGN's with zero mean and unit variance, h_m^{sr}, h_m^{sd} and h_m^{rd} are complex fading random variables. For a Rayleigh channel, the real and imaginary parts of the complex fading variables are Gaussian distributed having zero mean and variance 1/2.

[1]Here, by orthogonal transmission we mean there is no interference at the destination due to transmissions from multiple sources and relay.

A. Achievable rate of M-1-1 DF relay system with zero location errors

The mth source (isolated from the M sources), the relay and the destination, form a 1-1-1 system as shown in Fig. 1. The instantaneous achievable rate for such a 1-1-1 system can be expressed as [15],

$$R_m^i = \min\left\{\log\left(1 + \frac{|h_m^{sr}|^2 P_s}{(d_m^{sr})^\alpha N_r}\right), \log\left(1 + \frac{|h_m^{sd}|^2 P_s}{(d_m^{sd})^\alpha N_d} + \frac{|h_m^{rd}|^2 P_{rm}}{(d_m^{rd})^\alpha N_d}\right)\right\}. \qquad (3)$$

We now investigate the achievable rate of a 1-1-1 system averaged over all channel states and extend it to the M-1-1 system. We first define $\gamma_m^{sr} = |h_m^{sr}|^2 \frac{P_s}{(d_m^{sr})^\alpha N_r}$, $\gamma_m^{sd} = |h_m^{sd}|^2 \frac{P_s}{(d_m^{sd})^\alpha N_d}$ and $\gamma_m^{rd} = |h_m^{rd}|^2 \frac{P_{rm}}{(d_m^{rd})^\alpha N_d}$. The achievable rate of 1-1-1 system for the Rayleigh fading can then be written as,

$$R_m = \min\{R_{1m}, R_{2m}\}, \qquad (4)$$

where,

$$R_{1m} = \int_0^\infty \log(1 + \gamma_m^{sr})\, g_1(\cdot) d\gamma_m^{sr} \qquad (5)$$

and

$$R_{2m} = \int_0^\infty\int_0^\infty \log(1 + \gamma_m^{sd} + \gamma_m^{rd})\, g_2(\cdot) g_3(\cdot) d\gamma_m^{sd} d\gamma_m^{rd}, \qquad (6)$$

where $g_1(\cdot)$, $g_2(\cdot)$ and $g_3(\cdot)$ defines the probability density functions (pdf) (χ^2 distributed with two degrees of freedom) of γ_m^{sr}, γ_m^{sd} and γ_m^{rd}, respectively. We define, $k_m^{sr} = \frac{P_s}{(d_m^{sr})^\alpha N_r}$, $k_m^{sd} = \frac{P_s}{(d_m^{sd})^\alpha N_d}$ and $k_m^{rd} = \frac{P_{rm}}{(d_m^{rd})^\alpha N_d}$. The integral in Eq. (5) can be solved by parts and the final result is given by,

$$R_{1m} = \frac{1}{\ln(2)}\left[\exp\left(\frac{1}{k_m^{sr}}\right) E_1\left(\frac{1}{k_m^{sr}}\right)\right], \qquad (7)$$

where $E_1(\cdot)$ is the exponential integral defined as $E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt, (x > 0)$. Eq. (6) was first solved in [14] and R_{2m} is given by,

$$R_{2m} = R_{2m}^+ + R_{2m}^-, \qquad (8)$$

where

$$R_{2m}^+ = k_m^{rd}\left[\frac{\exp\left(\frac{1}{k_m^{rd}}\right) E_1\left(\frac{1}{k_m^{rd}}\right) - \exp\left(\frac{1}{k_m^{sd}}\right) E_1\left(\frac{1}{k_m^{sd}}\right)}{\ln(2)(k_m^{rd} - k_m^{sd})}\right] \qquad (9)$$

and

$$R_{2m}^- = \frac{\exp\left(\frac{1}{k_m^{sd}}\right) E_1\left(\frac{1}{k_m^{sd}}\right)}{\ln(2)}. \qquad (10)$$

Since the transmissions from M sources and the relay are non-interfering at the destination, the M-1-1 system can be considered as independent 1-1-1 systems. The achievable rate for the M-1-1 DF system with Rayleigh fading is then given by,

$$R = \min\left\{\sum_{m=1}^{M} R_{1m}, \sum_{m=1}^{M} R_{2m}^+ + \sum_{m=1}^{M} R_{2m}^-\right\}. \qquad (11)$$

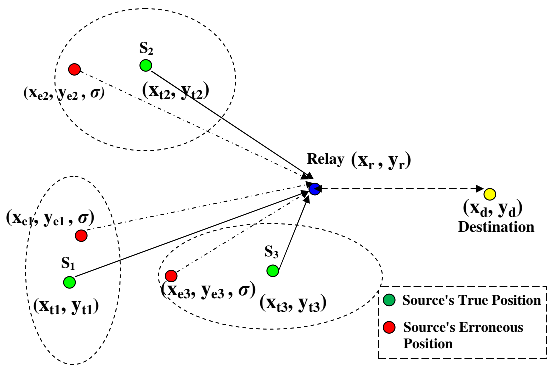

Fig. 2: Location Error Scenario.

We consider the power allocation scheme at the relay which obtains the achievable rate R in Eq. (11). Note that in Eq. (11), the terms $\sum_{m=1}^{M} R_{1m}$ and $\sum_{m=1}^{M} R_{2m}^{-}$, are independent of the power allocation scheme at the relay for a given location. Also due to the minimization term in Eq. (11), the second term $\sum_{m=1}^{M} R_{2m}^{+} + \sum_{m=1}^{M} R_{2m}^{-}$ should be always less than the first term $\sum_{m=1}^{M} R_{1m}$, for the power allocation at the relay to be useful. Therefore, any power allocation scheme at the relay must be under the constraint of $\sum_{m=1}^{M} R_{2m}^{+} \leq \sum_{m=1}^{M} R_{1m} - \sum_{m=1}^{M} R_{2m}^{-}$. The relay has a total available power of P_r and needs to allocate the power among M users to obtain the rate R. Obtaining R turns out be maximization of R_{2m}^{+}. The power allocation vector $P_{r1}, ..., P_{rM}$ can be obtained by solving the following convex optimization problem with the objective function and constraints listed as follows.

$$\max_{P_{r1},...,P_{rM}} \sum_{m=1}^{M} R_{2m}^{+} \tag{12}$$

subject to,

1) $\sum_{m=1}^{M} P_{rm} = P_r$,
2) $P_{rm} \geq 0, m = 1, \cdots, M$,
3) $R_{2m}^{+} - R_{1m} + R_{2m}^{-} \leq 0, m = 1, \cdots, M$.

Due to the integration in Eq. (5) and Eq. (6), there is no closed form expression for the power allocation vector $P_{r1}, ..., P_{rM}$ in Eq. (12). However, the problem can be solved numerically using any optimization toolbox available (e.g. MATLAB). We use the power allocation scheme to investigate the achievable rate with location errors in the following section.

B. Impact of Location Errors on Achievable Rate

In this section we investigate the impact of location errors on the achievable rate R. Specifically, we introduce errors in the true position coordinates (x_{tm}, y_{tm}) of the source S_m. Consider a location error scenario in a M-1-1 system as in Fig.1, for M = 3. Since in most of the distributed localization techniques the error characteristics are elliptic in nature (i.e., $\sigma_{x_m} \neq \sigma_{y_m}$) the location algorithm would localize a node anywhere within the ellipse around its true position. The major and minor axes of the ellipse represents the error deviation in terms of the normalized distance d_m^{rd}.

For simplification, we assume that the relay and destination has an error free location at (x_r, y_r) and (x_d, y_d); the true position of the source *m* is at (x_{tm}, y_{tm}) with its erroneous position at (x_m, y_m) , with x_m and y_m being distributed following a 2-D Gaussian distribution with mean centered on (x_{tm}, y_{tm}) and variance $\sigma_m^2 = \sigma_{x_m}^2 + \sigma_{y_m}^2$. We assume the standard deviation of location error σ_m to be the same across all sources. The erroneous distance between the source m and the relay node is then given by,

$$d_m^{sr}(x_m, y_m) = \sqrt{(x_r - x_m)^2 + (y_r - y_m)^2}. \tag{13}$$

Similarly, the erroneous distance between the source m and the destination node is given by,

$$d_m^{sd}(x_m, y_m) = \sqrt{(x_d - x_m)^2 + (y_d - y_m)^2}. \tag{14}$$

The location co-ordinates x_{tm} and y_{tm} of the source node *m* experience independent standard deviation in error, σ_{x_m} and σ_{y_m} respectively. The average rates $\bar{R}_{1m}$ and $\bar{R}_{2m}$ with location errors are obtained by integrating R_{1m} defined in Eq. (7) and R_{2m} defined in Eq. (8) as,

$$\bar{R}_{1m} = \int_0^{\infty} \int_0^{\infty} R_{1m} \left[d_m^{sr}(x_m, y_m) \right] p(x_m) p(y_m) dx_m dy_m, \tag{15}$$

and

$$\bar{R}_{2m} = \int_0^{\infty} \int_0^{\infty} R_{2m} \left[d_m^{sd}(x_m, y_m) \right] p(x_m) p(y_m) dx_m dy_m, \tag{16}$$

where, $p(x_m)$ and $p(y_m)$ are the pdf of x_m and y_m, respectively and are given by,

$$p(x_m) = \frac{1}{\sqrt{2\pi\sigma_{x_m}^2}} \exp\left(-\frac{(x_m - x_{tm})^2}{2\sigma_{x_m}^2} \right), \tag{17}$$

and

$$p(y_m) = \frac{1}{\sqrt{2\pi\sigma_{y_m}^2}} \exp\left(-\frac{(y_m - y_{tm})^2}{2\sigma_{y_m}^2} \right). \tag{18}$$

The average achievable rate $\bar{R}$ with location errors is then given by,

$$\bar{R} = \min\left\{ \bar{R}_{1m}, \bar{R}_{2m} \right\}. \tag{19}$$

Note that an estimation of σ_m for source *m* is dependent on the wireless network technology and the location information metric used (e.g. signal strength, time-of-arrival, *etc*). The most widely deployed wireless location algorithms adopt signal strength as the metric to utilize, due to the ease with which it can be measured. Assuming a log-normal shadowing model it is straightforward to derive the Cramer-Rao Bound (CRB) on the location uncertainty for particular node geometries. For a source *m*, this is given as (e.g. [13])

$$\sigma_m^2 = \frac{\sum_{i=1}^{N_R} \frac{1}{d_i^2}}{\frac{10\mu}{\sigma_{db} \ln 10} \sum_{i=1}^{N_R - 1} \sum_{j=i+1}^{N_R} cos^2 \frac{(\phi_i - \phi_j)}{d_i^2 d_j^2}}, \tag{20}$$

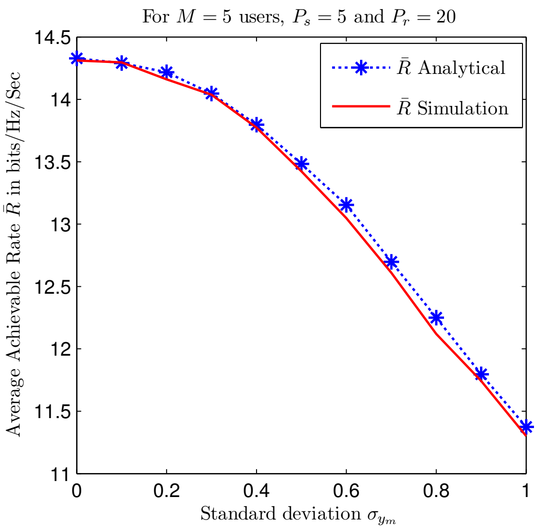

Fig. 3: Average achievable rate $\bar{R}$ plotted against σ_{y_m}, with $\sigma_{x_m} = 0.4$ for M = 5 users.

where N_R is number of a reference nodes of known position, d_i is the distance from the node of unknown position to a reference node, ϕ_{ij} is the angle formed between the node of unknown position and the reference nodes, and σ_{db} is the standard deviation of the log normal shadowing. For typical node densities used in our simulations, and typical shadowing environments, the above relation lead to standard deviations on the location error in the range 20%-60% of the true distances. GPS location errors would be $\approx 10\%$ of the true distances.

Using the above equations we can now determine the achievable rate as a function of the standard deviation of the location error. We let error deviations at a source vary between 0 and 1 i.e., $\sigma_{x_m} = 0.4$ means a 40% deviation on the true position x_{mt}. The average achievable rate $\bar{R}$ is plotted in Fig. 3. Also shown here is the simulation result.

We can see the dramatic impact the location error has on the achievable rate. After values of σ_m greater than about ≈ 0.44 the throughput of the system will fall dramatically. Similar results are found for a range of system configurations leading us to the conclusion that $\sigma_m > 0.44$ indicates point where throughput predictions based on zero-location error inputs should no longer be trusted.

III. Discussion

In this section we investigate the impact of mobility of sources on the achievable rate of the M-1-1 system. Here, motion models are used to aid location predictions. More specifically, we will investigate the error in the achievable rates predicted when we utilize true location information (zero error) as compared to predicted locations (*i.e.* locations possessing error). This provides another means of gauging the impact of location error on 4G throughput expectations. We consider a random way point (RWP) mobility model in which sources move around randomly within the transmission

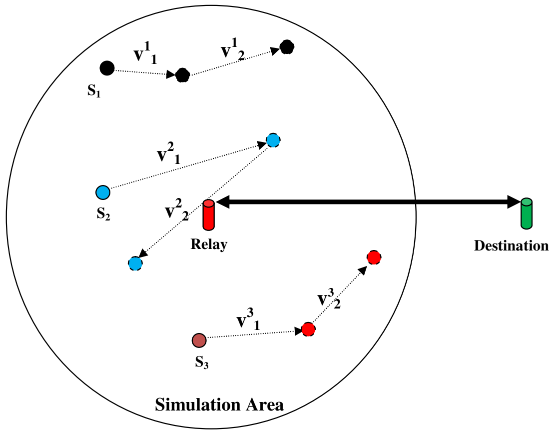

Fig. 4: Simulation with RWP Model.

range of both relay and destination. The start-point and end-point of all the sources are chosen randomly from a uniform distribution. The sources moves from the start-point to the end-point with a uniform velocity in the range of 1.25m/sec to 1.5m/sec (walking speed). After reaching the end-point, with zero pause time, they select another end-point randomly and move with a uniform velocity. The model is illustrated in Fig. 4

A. Mobility with Random Way Point Model

Initially, we assume the relay is aware of the location of all the sources and the location information is updated every T_u seconds at the relay. We perform a power allocation at the relay every T_u seconds and compute the throughput T of the M-1-1 system. T is then averaged over the number of update periods and is denoted by T_{avg}. To maximize T_{avg}, the location information of all sources is required at the relay for the power allocation at the relay to be useful. We define $\Delta R = R - T_{avg}$ and show through simulations that $\Delta R \to 0$ as $T_u \to 0$.

B. Mobility with Pathway Model

Mobile nodes, in the RWP model, are allowed to move freely and randomly anywhere in the simulation field. However, in most real-world applications, we observe that a node's movement is subject to path constraints. In reality, nodes move in a pseudo-random way on predefined pathways. Such constraints was integrated in the mobility models in [16]. One way to integrate geographic constraints into the mobility model is to restrict the node movement to the pathways according to a predefined map. The map can be either randomly generated or carefully defined based on a real city. The vertices of the map represent the buildings of the city, and the edges model the streets between those buildings. The model is illustrated in Fig. 5.

Initially, the nodes are placed randomly on the edges of the graph. Then for each node a end-point is randomly chosen and the node moves towards this end-point through the shortest path along the edges. Upon arrival, the node pauses for $T = 0$ time and then chooses a new end-point. This procedure is

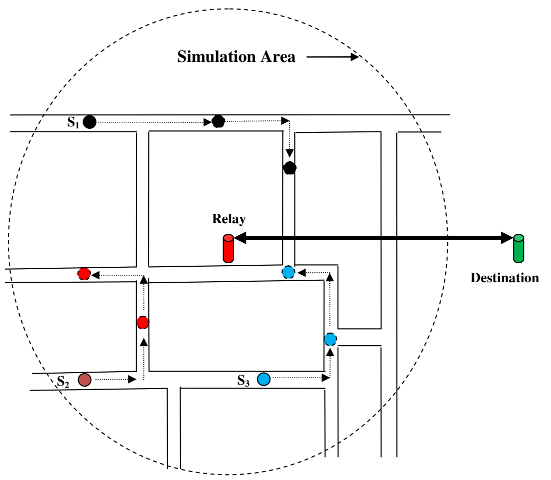

Fig. 5: Mobility Simulation With Pathway Model with sources S_1, S_2 and S_3 restricted to a geographical area.

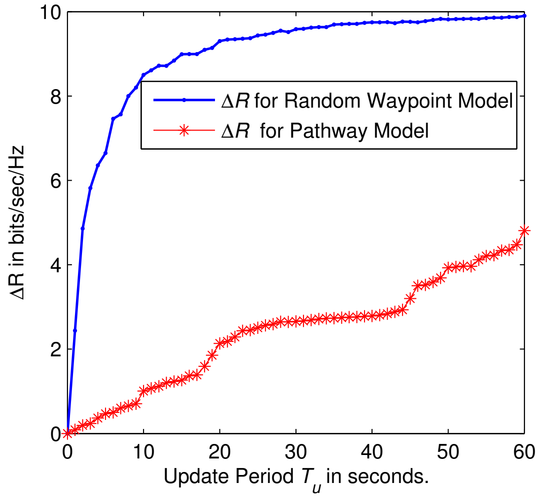

Fig. 6: ΔR plotted against update period T_u.

repeated until the end of simulation. Unlike the RWP model where the nodes can move freely, the mobile nodes in this model are only allowed to travel on predefined pathways. However, since the end-point of each motion phase is randomly chosen, a certain level of randomness still exists for this model. So, in this graph-based mobility model, the nodes are traveling in a pseudo-random fashion on the pathways.

The results of the difference in achievable rates (ΔR) is shown in Fig. 6. Here the rates obtained using the true position of the nodes are shown minus the rates obtained using the reported locations (as determined by the motion model). The results are shown for both motion models as a function of the update period. We can clearly see the benefits of the Pathway model with its better predictive properties. Lower update periods of course provide more accurate location information. However, as mentioned earlier, the trade-off here is the extra battery resources that will be used to obtain such information. Although, higher throughputs will be achieved for faster update periods, the lifetime of the network will be much smaller. Update periods of order 5-10 seconds appear most-useful in the trade-off sense. Information on the required update period can then be considered as an input to layer 2 protocols and layer 3 management systems, such as Mobile IP (internet protocol).

IV. Conclusions

In this work we have explored the use of location information in the context of emerging 4G wireless communication systems, in which cooperative communication among mobile nodes is utilized. Most importantly, we have detailed how the system throughput depends on the quality of the information on the reported locations of the nodes. We have also provided a detailed simulation of our work in the context of well known motion models. The work presented here will be of value to the design and implementation of real-world wireless cooperative communication systems. This work was supported by Australian Research Council (ARC) Grant number DP0879401.

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