Performance Analysis of Soft Information Cooperation in Bi-directional Networks

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***Abstract*—In this paper, we propose the soft information cooperation scheme in the bi-directional wireless relay networks. We first compute the Log-Likelihood ratio (LLR), referred to as the soft information of relay received signal in Rayleigh fading channel, and then propose an efficient LLR-based network coding scheme. Moreover, for computational simplicity, we optimize the soft information cooperation by the defined self soft information at terminals. In addition, we analyze the system performance in terms of generalized signal-to-noise ratio (GSNR), which is shown to be an efficient metric to reveal the BER performance, for one-relay and multi-relay cases. The simulation results show that the soft information cooperation scheme outperforms the traditional amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) protocols in terms of GSNR and bit-error rate (BER) in the low-SNR regime.**

***Index Terms*—Bi-directional networks, Soft information, Log-likelihood ratio, Network Coding, Generalized SNR.**

I. Introduction

Soft iterative decoding using log-likelihood ratios (LLRs), associated with low-density parity-check (LDPC) codes [1] and turbo codes [2] has been shown to approach the Shannon limit on many channels, e.g., additive white Gaussian noise channel.

Compared with conventional relaying schemes, such as Amplify-and-Forward (AF) and Decode-and-Forward (DF), soft information will neither transmit the noise corrupted signal directly to the destinations, nor bring additional decoding error at relay nodes. In the soft information cooperation scheme, the relays compute the reliability of the received signal, i.e., the LLRs, and transmit such soft data to the destinations.

In this paper, we aim to design a cooperation protocol by taking the advantages of soft information, which can achieve better performance at low SNR regime. Specifically, we propose a soft information based network coding scheme in wireless bi-directional relay networks. The self LLR, defined as the LLR of transmitted signal at terminal nodes is introduced and used in decoding process. Furthermore, for non-linear property of the relay function and derivation complexity of closed-form BER or capacity expression, we implement Generalized Signal-to-Noise Ratio (GSNR) [4] to analyze the proposed network coding scheme, which is shown to be closely related to bit error rate (BER) performance and is easy to derive for non-linear relay function. GSNR performance at relay and terminal nodes are derived in the one-relay and multi-relay cases.

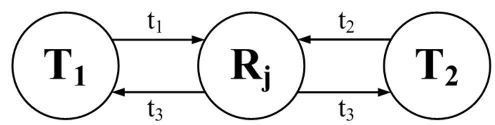

Fig. 1. Bi-directional cooperative networks. In the first two time slots t_1 and t_2, terminal T_1 and terminal T_2 transmit the messages sequentially to the N relays. Here, the j-th relay is shown as an example. In time slot t_3, the relay nodes broadcast the processed signals to two terminals.

The simulation results show that our proposed soft information relaying outperforms AF, DF and CF protocols at low SNR in terms of GSNR and BER for multiple cooperate relay case. With only one relay, the BER and GSNR performances of soft information relaying protocol are comparable with the other cooperation protocols.

In this paper, vector or matrix is denoted by bold letters. Transpose and Hermitian of a matrix $\mathbf{A}$ are denoted by $\mathbf{A}^T$ and $\mathbf{A}^H$ respectively. $|a|$ indicates the absolute value of a. $\Re(\cdot)$, $E(\cdot)$ and $P(\cdot)$ denote the real part, expectation and probability, respectively.

II. Wireless Network Model and Soft Information Cooperation

A. Networks and Channel Models

Consider a wireless bi-directional relaying networks where two terminals, T_1 and T_2, exchange information with each other by the help of N candidate relays $R = \{R_1, R_2, \ldots, R_N\}$, shown in Fig. 1. In particular, terminal T_1 and T_2 transmit, respectively, data x_1 and x_2 to relay nodes sequentially with equal power P_t in the first two time slots. In the third time slot, the relay nodes process the received signals, and broadcast the processed signals to the two terminal nodes simultaneously. The modulation is chosen to be binary phase-shift keying (BPSK) with coherent detection: x_i is $+\sqrt{P_t}$ or $-\sqrt{P_t}$ with equal probability, for $i = 1, 2$. To be practical, all the nodes are considered to be half-duplex. In addition, the Rayleigh channels between T_i and R are respectively denoted by $\mathbf{H} = [h_1, h_2, ..., h_N]$ and $\mathbf{G} = [g_1, g_2, ..., g_N]$ for $i = 1, 2$,

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where h_j and g_j are modeled as zero mean complex Gaussian random variable with variance $1/2$ per real dimension, i.e., $h_j, g_j \sim \mathcal{CN}(0,1)$ and $j = 1, 2, ..., N$. For simplicity, we assume that the channels are reciprocal, and the received nodes know perfect channel state information (CSI).

The received signals in the first and second time slots at the relay nodes can be respectively written as

$$\mathbf{r}_1 = \mathbf{H}x_1 + \mathbf{V} \text{ and } \mathbf{r}_2 = \mathbf{G}x_2 + \mathbf{Z}, \tag{1}$$

where $\mathbf{V} = [v_1, v_2, ..., v_N]$ and $\mathbf{Z} = [z_1, z_2, ..., z_N]$ are independent and identically distributed ($i.i.d.$) additive white Gaussian noise (AWGN), with distribution given by $v_j, z_j \sim \mathcal{CN}(0, \delta^2)$ for $j = 1, 2, ..., N$. The relays broadcast the $N \times 1$ processed vector $\mathbf{x}_r$ to the two terminals. Then the received signals in the third time slot at terminal T_1 and T_2 are given respectively by

$$\mathbf{y}_1 = \mathbf{a}\mathbf{H}^T\mathbf{x}_r + \mathbf{V}_r \text{ and } \mathbf{y}_2 = \mathbf{a}\mathbf{G}^T\mathbf{x}_r + \mathbf{Z}_r, \tag{2}$$

where $\mathbf{a}$ is an $N \times 1$ power allocation vector satisfying the power constrains at relays, which is determined by the instantaneous CSI, the noise statistics and the transmitting power P_r of relay nodes, and the $N \times 1$ vectors $\mathbf{V}_r$ and $\mathbf{Z}_r$ are $i.i.d.$ AWGN.

B. Soft Information Relaying

Now, we introduce the cooperation protocol at relay nodes using soft information, shown in Fig. 2. When the relay nodes receive $\mathbf{r}_1$ and $\mathbf{r}_2$ at first two time slots, they first implement the zero-forcing equalization to $\mathbf{r}_i$, i.e., compute the pseudo reverse of $\mathbf{H}$ and $\mathbf{G}$ respectively, i.e.,

$$\begin{aligned}\mathbf{T}_1 &= (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H = [t_{11}, t_{12}, ..., t_{1N}],\\ \mathbf{T}_2 &= (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H = [t_{21}, t_{22}, ..., t_{2N}].\end{aligned} \tag{3}$$

Then, we multiply $\mathbf{T}_1$ and $\mathbf{T}_2$ respectively to the received signals $\mathbf{r}_1$ and $\mathbf{r}_2$ to do zero-forcing, i.e.,

$$\mathbf{T}_1\mathbf{r}_1 = x_1 + \mathbf{T}_1\mathbf{V} \text{ and } \mathbf{T}_2\mathbf{r}_2 = x_2 + \mathbf{T}_2\mathbf{Z}, \tag{4}$$

where $t_{1j}v_j$ and $t_{2j}z_j$ are complex Gaussian random variables with zero mean and variance $|t_{1j}|^2\delta^2$ and $|t_{2j}|^2\delta^2$ respectively for $j = 1, 2, ..., N$.

Now we derive Log-Likelihood Ratio (LLR) Λ_{1j} for x_1 and Λ_{2j} for x_2 according to the definition, where Λ_{1j} and Λ_{2j} denote the LLR information of the j-th relay. We have

$$\begin{aligned}\Lambda_{1j} &= \ln \frac{P\left(x_1 = +\sqrt{P_t}|x_1 + t_{1j}v_j\right)}{P\left(x_1 = -\sqrt{P_t}|x_1 + t_{1j}v_j\right)}\\ &= \frac{2\sqrt{P_t}}{\delta^2}\frac{\Re\{x_1 + t_{1j}v_j\}}{|t_{1j}|^2},\\ \Lambda_{2j} &= \frac{2\sqrt{P_t}}{\delta^2}\frac{\Re\{x_2 + t_{2j}z_j\}}{|t_{2j}|^2}.\end{aligned} \tag{5}$$

The signs of Λ_{1j} and Λ_{2j} are the hard decision value, and the magnitudes of Λ_{1j} and Λ_{2j} are the reliability of hard decision.

For two LLRs Λ_1 and Λ_2, the LLR based network code Λ is defined in [5] as equation (6) shows

$$\Lambda = \Lambda_1 \oplus_\Lambda \Lambda_2 = \text{sign}(\Lambda_1)\,\text{sign}(\Lambda_2)\min(|\Lambda_1|, |\Lambda_2|), \tag{6}$$

with the additional computation rules as

$$\Lambda \oplus_\Lambda \infty = \Lambda, \;\; \Lambda \oplus_\Lambda -\infty = -\Lambda, \;\text{ and }\; \Lambda \oplus_\Lambda 0 = 0, \tag{7}$$

where the operation symbol $\oplus_\Lambda$ is used to distinguish with the regular XOR $\oplus$. Then the LLR based network coded symbol is

$$x_{r_j} = \Lambda_{1j} \oplus_\Lambda \Lambda_{2j} = \text{sign}(\Lambda_{1j})\,\text{sign}(\Lambda_{2j})\min(|\Lambda_{1j}|, |\Lambda_{2j}|), \tag{8}$$

where $\mathbf{x}_r = [x_{r_1}, x_{r_2}, ..., x_{r_N}]$. Assume that each relay has equal average transmit power P_r. Then the power allocation vector $\mathbf{a} = [a_1, ..., a_j, ..., a_N]$ in equation (2) is determined as

$$a_j = \sqrt{\frac{P_r}{E\left[|\min(|\Lambda_{1j}|, |\Lambda_{2j}|)|^2\right]}}. \tag{9}$$

When terminals T_1 and T_2 received $\mathbf{y}_1$ and $\mathbf{y}_2$ respectively at the third time slot, they use the knowledge of self information to decode the data from counterpart. The terminals will then remove the so-called self LLR to get the LLR of the other terminal. The self LLR is defined as follows:

At terminal i, assume that the system is under the ideal circumstance that no noise or interference exists, if we send $x_i = +\sqrt{P_t}$, then the conventional LLR is $\frac{P\left(x_i=+\sqrt{P_t}\right)}{P\left(x_i=-\sqrt{P_t}\right)} = \infty$; likely, if we send $x_i = -\sqrt{P_t}$, then the conventional LLR is $\frac{P\left(x_i=+\sqrt{P_t}\right)}{P\left(x_i=-\sqrt{P_t}\right)} = -\infty$. For a sufficiently large real number η, the self LLR at terminal i is defined as $\Lambda_{T_i} = \pm\eta$.

For system symmetry and presentation simplicity, we take one component of the received signal at terminal 1 as an example to illustrate decoding process.

1) Zero-Forcing: $y'_{1j} = \frac{t_{1j}y_{1j}}{a_j} = x_{r_j} + \frac{t_{1j}v_{r_j}}{a_j}$, where v_{r_j} is the j-th entry of the noise vector $\mathbf{V}_r$;
2) Remove Self LLR Λ_{T_i} to get the LLR of x_2 transmitted by the j-th relay:

$$\begin{aligned}\Lambda'_{2j} &= \text{sign}\left(y'_{1j}\right)\text{sign}\left(\Lambda_{T_i}\right)\min\left(|y'_{1j}|, |\Lambda_{T_i}|\right)\\ &= \text{sign}\left(y'_{1j}\right)\text{sign}\left(\Lambda_{T_i}\right)|y'_{1j}|,\end{aligned} \tag{10}$$

where the second equal condition is valid since $|y'_{1j}|$ is always less or equal to $|\Lambda_{T_i}|$;
3) Decision computation:
   - If $P\left(x_2^j = +1\right) >= 0$, then $x_2^j = +1$;
   - Else if $P\left(x_2^j = +1\right) < 0$, then $x_2^j = -1$;

Then the value of x_2 is determined by

$$\text{If } \sum_{j=1}^{N} P\left(x_2^j = 1\right) > \frac{N}{2}, \;\; x_2 = 1; \;\text{ else }\; x_2 = -1. \tag{11}$$

III. Performance Analysis of Soft Information Cooperation

In this section, we first review the concept of the generalized Signal-to-Noise Ratio (GSNR), originally proposed in [4] as a measure for the quality of memoryless relay channels. Then we analyze the GSNR performance of the proposed soft

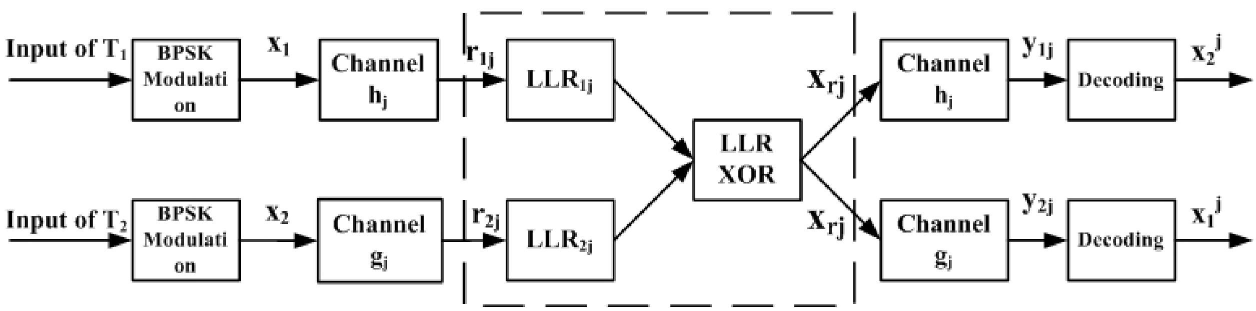

Fig. 2. Block Diagram of Soft Information Cooperation

information relaying strategy at relay and terminal nodes. One-relay and multi-relay networks are studied successively.

A. GSNR Analysis of Soft Information Cooperation

According to the definition in [4], GSNR can be viewed as a decomposition of an observation into a component along the desired signal space and its orthogonal signal (uncorrelated space), i.e., GSNR is the fraction of the power of the signal to the power of uncorrelated noise. In the soft information cooperation, we first analyze the relationship between the power of LLR and the power of uncorrelated noise at relay nodes, that is

$$\begin{aligned} E\left[|\Lambda_{ij}|^2|\text{without noise}\right] &= \eta^2, i=1,2 \text{ and } j=1,2,...,N, \\ MSUE_{ij} &= E\left[|\Lambda_{ij}-\eta|^2\right], \\ GSNR_{ij} &= \frac{\eta^2}{E\left[|\Lambda_{ij}-\eta|^2\right]}, \end{aligned} \tag{12}$$

where $MSUE_{ij}$ is the mean square of the uncorrelated noise from terminal i at relay node j. Since $|\Lambda_{ij}|<\eta$, $GSNR_{ij}$ increases when $|\Lambda_{ij}|$ increases. Specially, when $\Lambda_{ij}=\eta$, $GSNR_{ij}=\infty$.

Now we consider the GSNR at two terminals.

1) Single-relay bi-directional networks: Similar as in the previous discussion, we take y_{1j} as an example. According to (8), $y'_{1j}=x_{r_j}+\frac{t_{1j}v_{r_j}}{a_j}$ can also be written as

$$\begin{aligned} y'_{1j} &= \text{sign}\left(\Lambda_{1j}\right)\text{sign}\left(\Lambda_{2j}\right)\min\left(|\Lambda_{1j}|,|\Lambda_{2j}|\right)+\frac{t_{1j}v_{r_j}}{a_j} \\ &= \begin{cases} \text{sign}\left(\Lambda_{1j}\right)\text{sign}\left(\Lambda_{2j}\right)|\Lambda_{1j}|+\frac{t_{1j}v_{r_j}}{a_j}, & |\Lambda_{1j}|\le|\Lambda_{2j}|, \\ \text{sign}\left(\Lambda_{1j}\right)\text{sign}\left(\Lambda_{2j}\right)|\Lambda_{2j}|+\frac{t_{1j}v_{r_j}}{a_j}, & |\Lambda_{1j}|>|\Lambda_{2j}|. \end{cases} \end{aligned} \tag{13}$$

Since $|y'_{1j}|\le|\Lambda_{T_i}|=\eta$, the LLR of x_2 at terminal 1 can be further written as

$$\Lambda'_{2j}=\text{sign}\left(y'_{1j}\right)\text{sign}\left(\Lambda_{T_i}\right)|y'_{1j}|. \tag{14}$$

Then the uncorrelated noise from relay j at terminal 1 is defined as

$$e_u^{T_1,j}=|y'_{1j}-\eta x_1|. \tag{15}$$

Therefore

$$\begin{aligned} MSUE_{T_1,j} &= E\left[|e_u^{T_1,j}|^2\right] \\ &= E\left[|y'_{1j}-\eta x_1|^2\right]P\left(|\Lambda_{1j}|\le|\Lambda_{2j}|\right) \\ &+ E\left[|y'_{1j}-\eta x_1|^2\right]P\left(|\Lambda_{1j}|>|\Lambda_{2j}|\right) \\ &= E\left[\left||\Lambda_{1j}|+\frac{t_{1j}v_{r_j}}{a_j}-\eta x_1\right|^2\right]P\left(|\Lambda_{1j}|\le|\Lambda_{2j}|\right) \\ &+ E\left[\left||\Lambda_{2j}|+\frac{t_{1j}v_{r_j}}{a_j}-\eta x_1\right|^2\right]P\left(|\Lambda_{1j}|>|\Lambda_{2j}|\right), \end{aligned} \tag{16}$$

where $P\left(|\Lambda_{1j}|\le|\Lambda_{2j}|\right)=P\left(|\Lambda_{1j}|>|\Lambda_{2j}|\right)=\frac{1}{2}$.

Notice that

$$E\left[|\Lambda_{1j}|+\frac{t_{1j}v_{r_j}}{a_j}-\eta x_1|^2\right]=E|\Lambda_{1j}|^2+E|\frac{t_{1j}v_{r_j}}{a_j}|^2+E|\eta x_1|^2, \tag{17}$$

and $E(|\eta x_1|^2)=\eta^2$. To find out the other two mean squares in equation (17), we first introduce a lemma on Λ_{ij}.

Lemma 1: The probability density function of Λ_{ij} is

$$f_{\Lambda_{ij}}(x)=\frac{\alpha}{2\sqrt{2\pi}\delta_\Lambda}\left[e^{-\frac{(x-\sqrt{P_t})^2}{2\delta_\Lambda^2}}+e^{-\frac{(x+\sqrt{P_t})^2}{2\delta_\Lambda^2}}\right], \tag{18}$$

where $\alpha=\frac{2\sqrt{p_t}}{|t_{ij}|^2\delta^2}$, $\delta_\Lambda=\frac{|t_{ij}|^2\delta^2}{2}$ and $-\infty<x<\infty$. Therefore,

$$f_{|X|}(x)=\frac{\alpha}{\sqrt{2\pi}\delta_\Lambda}\left[e^{-\frac{(x-\sqrt{P_t})^2}{2\delta_\Lambda^2}}+e^{-\frac{(x+\sqrt{P_t})^2}{2\delta_\Lambda^2}}\right], \tag{19}$$

where $x\ge0$, and $|X|$ follows the folded Gaussian distribution with mean

$$E(|x|)=\delta_\Lambda\sqrt{2/\pi}\exp-P_t/2\delta_\Lambda^2+\sqrt{P_t}[1-2\Phi(-\sqrt{P_t}/\delta_\Lambda)], \tag{20}$$

and variance

$$Var(|x|)=P_t+\delta_\Lambda^2-[E(|x|)]^2. \tag{21}$$

With the mean and variance of $|x|$, we have

$$\begin{aligned} E[|\Lambda_{ij}|^2] &= [E(|x|)]^2+Var(|x|), \\ E[|a_j|^2] &= \frac{P_r}{E[|\Lambda_{ij}|^2]}. \end{aligned} \tag{22}$$

Since $t_{1j}v_{r_j}$ is a zero-mean complex Gaussian variable, then

$$E[|t_{1j}v_{r_j}|^2]=2|t_{1j}|^2\delta^2. \tag{23}$$

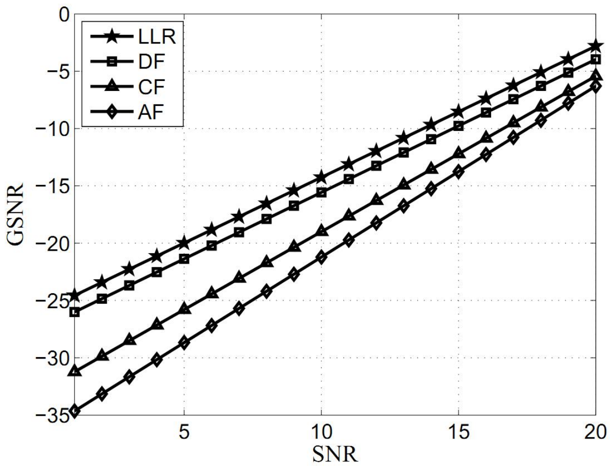

Fig. 3. GSNR comparison of AF, DF, and LLR with multi-relay

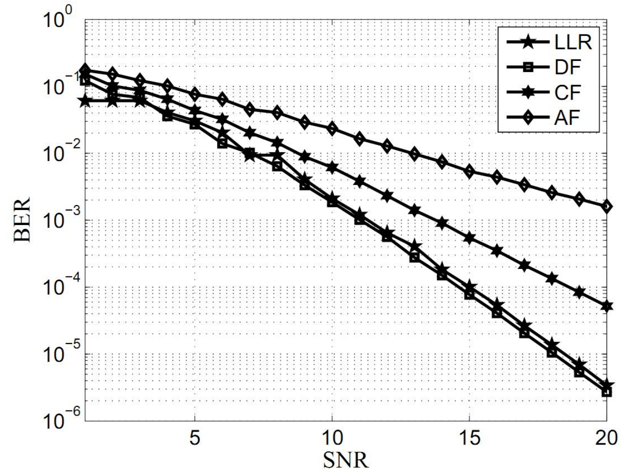

Fig. 4. BER comparison of AF, DF, CF and LLR with multi-relay

With the above discussion, $MSUE_{T_1,j}$ is determined.

Thus the $GSNR_{T_1,j}$ at terminal 1 is

$$GSNR_{T_1,j} = \frac{\Lambda_{T_1}}{E\left[|e_u^{1j}|^2\right]} = \frac{\eta^2}{MSUE_{T_1,j}}. \quad (24)$$

We can obtain the similar result for $GSNR_{T_2,j}$ at terminal 2.

2) Multi-relay bi-directional networks: In the multi-relay bi-directional networks, since $\alpha_j = 1$, the overall $GSNR_i$ at terminal i for $i = 1, 2$ can be written as

$$\begin{aligned} GSNR_i &= \frac{N^2\eta^2}{\sum_{j=1}^{N} MSUE_{T_i,j} + \sum_{j=1}^{N}\sum_{j=1,j\neq k}^{N} C_{jk} + \delta^2} \\ &= \frac{N^2\eta^2}{\sum_{j=1}^{N} MSUE_{T_i,j} + \delta^2}, \end{aligned} \quad (25)$$

where $MSUE_{T_i,j}$ is shown in (16), and the error correlation C_{jk} is assumed to be zero since the relays are independent with each other.

IV. Simulation Results

In this section, we first compare the GSNR performance of soft information cooperation with AF, DF and CF protocols with multi-relay at terminals. Then we use Monte-Carlo simulation to compare the soft information cooperation's BER performance with AF, DF and CF scheme.

Comparing AF, DF, CF and soft information relaying in the GSNR metric with multi-relay nodes at terminals, soft information relaying outperform AF and DF scheme in the low-SNR regime (see Fig. 3).

In the BER performance comparison of AF, DF, CF and soft information protocols with multi-relays, see Fig. 4, we note that soft information cooperation is always better than AF and CF strategies from 1dB to 20dB for its well *protection* of information from both terminals. In the first 3dB, soft information cooperation works better than DF scheme; from 4dB to 20dB, soft information strategy is comparable with DF scheme. However, we have pointed out that DF schemes requires more complicated hardware design, thus we can concluded that, with multiple relays, our proposed soft information cooperation is the optimal scheme.

Compare the GSNR figures with BER figures, we notice that the scheme with higher GSNR always have better performance in BER metric. When the closed-form BER expression is hard to derive, we can use GSNR metric to represent the relationship of different relay strategies.

V. Conclusion

We propose the soft information cooperation scheme in the bi-directional relaying networks. We first compute the LLR of relay received signal, and implement an efficient LLR-based network coding. Then, we analyze the system performance with GSNR for one-relay and multi-relay cases. The simulation results show that soft information cooperation scheme outperform traditional AF, DF and CF protocols in GSNR and BER performance in low-SNR regime with multiple relays. While with only one relay, the performance of soft information cooperation scheme is comparable with the other relaying schemes.

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