Wireless Powered Communications With Initial Energy: QoS Guaranteed Energy-Efficient Resource Allocation

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***Abstract*—This letter considers a wireless powered communication network (WPCN) where users first harvest energy in the downlink (DL) and then use the energy to transmit information signals in the uplink (UL). Furthermore, each user is also equipped with some initial energy, which generalizes the previous works on energy transfer and initial energy. Our goal is to maximize the system energy efficiency while guaranteeing the user's quality of service via joint time allocation and power control in both DL and UL. We find that the power station always transmits with the maximum power in DL while the transmit power of users in UL follows the waterfilling structure. In addition, the time allocation between DL and UL can be simplified into a linear programming problem. Finally, we propose an efficient algorithm to obtain the optimal solution. Simulation results demonstrate the system EE analysis for the WPCN as well as the special cases.**

***Index Terms*—Energy efficiency, power transfer, QoS, resource allocation.**

I. Introduction

Wireless energy transfer (WET) has drawn significant attention because of its capability of prolonging the lifetime of wireless networks [1], [2]. The authors in [1] propose a "harvest and then transmit" protocol for wireless powered communication networks (WPCNs) where users first harvest energy in downlink (DL) WET stage and then transmit information signals in uplink (UL) wireless information transmission (WIT) stage. Then, the closed-form expressions of the time allocation are derived for maximizing the spectral efficiency (SE). Meanwhile, due to the the explosive growth of high-data-rate applications, energy efficiency (EE), measured by bits per joule, has been accepted as an important indicator for wireless communications [3]–[5]. In [3], the EE of a WPCN is studied for a single-user scenario, but it can not be extended to the multiuser case due to the coupling of the time allocation and power control. The authors in [4] reveal that without considering the user quality of service (QoS), the EE maximization problem of a multiuser WPCN can be reduced to two sub-problems. To our best knowledge, the QoS guaranteed energy-efficient resource allocation in the WPCN has not been investigated in the literature.

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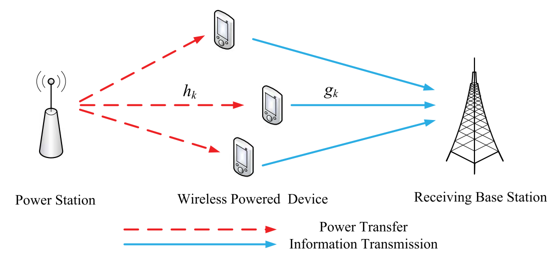

Fig. 1. The system model of a wireless powered communication network.

In this letter, we study the EE maximization problem by taking into account the individual user QoS. Furthermore, each user is also equipped with some initial energy, which may be the energy left from the previous transmission blocks or harvested from other sources, such as solar and wind. These setups enable users to have higher flexibility in utilizing energy and also make the considered problem more general than the previous works [1], [3], [4].

II. Preliminaries

A. System Description

We consider a WPCN, which consists of one power station, K wireless-powered users, and one information receiving base station that is not necessarily co-located with the power station, as illustrated in Fig. 1. The "harvest and then transmit" protocol is adopted for this network [1]. We also assume that perfect channel state information (CSI) is obtained in order to explore the EE upper bound of the WPCN [6]. The DL channel gain between the power station and user terminal k, and the UL channel gain between user terminal k and the receiving base station are denoted as h_k and g_k, respectively.

During DL WET, the power station broadcasts the energy signal with power P_0 and time τ_0. The energy harvested from the noise and the received UL WIT signals from other users is assumed to be negligible, since the noise power and the user transmit powers are both much smaller than the transmit power of the power station in practice [1]. Thus, the amount of energy harvested at user k can be expressed as

$$E_k^h = \eta \tau_0 P_0 h_k, \tag{1}$$

where $\eta \in (0, 1]$ is the energy conversion efficiency of the receiver. During UL WIT, each user k transmits its information signal to the receiving station with transmit power p_k and time τ_k. Then, the achievable throughput of user k is given by

$$B_k = \tau_k W \log_2 \left(1 + \frac{p_k g_k}{\sigma^2}\right), \tag{2}$$

where W is the system bandwidth. In the sequel, we use $\gamma_k = \frac{g_k}{\sigma^2}$ to denote the the normalized channel gain in UL WIT. Thus, the system throughput of the WPCN, denote as B_{tot}, is given by $B_{\text{tot}} = \sum_{k=1}^{K} B_k = \sum_{k=1}^{K} \tau_k W \log_2(1 + p_k \gamma_k)$.

B. Power Consumption Model

The total energy consumption of the WPCN consists of two parts. During DL WET, the system energy consumption, denoted as $E_{\rm WET}$, is modeled as

$$E_{\rm WET} = P_0\tau_0 - \sum_{k=1}^{K} E_k^h + P_c\tau_0, \tag{3}$$

where P_c is the circuit power of the power station. Note that $P_0\tau_0 - \sum_{k=1}^{K} E_k^h$ is the energy loss due to wireless channel propagation in DL. In practice, it is always positive due to the law of energy conservation, and $0 < \eta \le 1$. During UL WIT, each user independently transmit its own signal with transmit power p_k and time τ_k. Thus, the energy consumed by user k can be modeled as

$$E_k = p_k\tau_k + p_c\tau_k, \tag{4}$$

where p_c is the circuit power of the user terminal. Therefore, the total energy consumption of the system, denoted as $E_{\rm tot}$, is given by $E_{\rm tot} = E_{\rm WET} + \sum_{k=1}^{K} E_k$.

C. Problem Formulation

The system EE is defined as the ratio of the achievable system throughput to the consumed system energy, i.e., $EE = \frac{B_{\rm tot}}{E_{\rm tot}}$. Our goal is to maximize the system EE while guaranteeing the individual user QoS. Thus, the EE maximization problem in the WPCN is formulated as

$$\max_{\substack{\tau_0,\{\tau_k\},\\ P_0,\{p_k\}}} \frac{\sum_{k=1}^{K}\tau_k W\log_2(1+p_k\gamma_k)}{P_0\tau_0\left(1-\sum_{k=1}^{K}\eta h_k\right)+P_c\tau_0+\sum_{k=1}^{K}(p_k\tau_k+p_c\tau_k)}$$
$$\begin{aligned}
\text{s.t.}\quad & \text{C1}: P_0 \le P_{\max}, \quad \text{C2}: \tau_0 + \sum_{k=1}^{K}\tau_k \le T_{\max},\\
& \text{C3}: p_k\tau_k + p_c\tau_k \le \eta P_0\tau_0 h_k + Q_k, \quad \forall k,\\
& \text{C4}: \tau_k W\log_2(1+p_k\gamma_k) \ge B_{\min}^k, \quad \forall k,\\
& \text{C5}: \tau_0 \ge 0, \tau_k \ge 0, \quad \forall k,\\
& \text{C6}: P_0 \ge 0, p_k \ge 0, \quad \forall k.
\end{aligned} \tag{5}$$

In (5), C1 limits the maximum allowed transmit power of the power station as $P_{\max}$ in DL WET. In C2, the total available transmission time is denoted as $T_{\max}$. C3 guarantees that the energy consumed by user k in UL WIT does not exceed the total available energy which includes both the harvested energy $\eta P_0\tau_0 h_k$ and the initial energy Q_k. In C4, $B_{\min}^k$ is the minimum throughput requirement of user k. C5 and C6 are non-negative constraints on the optimization variables.

Remark 2.1: It is worth noting that by setting $Q_k = 0$, $\forall k$, problem (5) can be regarded as the EE maximization of the *pure WPCN* (PWPCN) where users depend only on the harvested energy to support communications as in [1], [3]. In contrast, by setting $\tau_0 = 0$, problem (5) is reduced to the EE maximization of the conventional *initial energy limited communication network* (IELCN) where the power transfer is not employed as in [7]. Therefore, our considered problem is a generalized one and the result also presents an unified optimization framework for these two kinds of networks.

III. Energy-Efficient Transmission for the WPCN

We first investigate the transmit power of the power station.

Theorem 3.1: For problem (5), the maximum system EE can always be achieved at $P_0 = P_{\max}$.

Proof: As the energy transfer may not be activated due to the initial energy of users, we discuss the following two cases. First, if $\tau_0^* > 0$ and $P_0^* < P_{\max}$ hold in the optimal solution $\{P_0^*, \{p_k^*\}, \tau_0^*, \{\tau_k^*\}\}$, then we can construct another feasible solution $\{\widetilde{P}_0, \{\widetilde{p}_k\}, \widetilde{\tau}_0, \{\widetilde{\tau}_k\}\}$ with $\widetilde{P}_0 = P_{\max}$, $\widetilde{\tau}_0\widetilde{P}_0 = \tau_0^* P_0^*$, $\widetilde{\tau}_k = \tau_k^*$, and $\widetilde{p}_k = p_k^*$, which guarantees that C1–C6 still hold. Thus, it follows that $\widetilde{\tau}_0 < \tau_0^*$ and thus $\widetilde{P}_0\widetilde{\tau}_0\left(1-\sum_{k=1}^{K}\eta h_k\right) + P_c\widetilde{\tau}_0 < P_0^*\tau_0^*\left(1-\sum_{k=1}^{K}\eta h_k\right) + P_c\tau_0^*$. Then, we can easily check that $\widetilde{EE} > EE^*$, and thus $P_0 = P_{\max}$ always holds. Second, if $\tau_0^* = 0$ holds, then the value of the transmit power P_0^* does not affect the maximum system EE, and thus $P_0 = P_{\max}$ is also optimal. □

Considering Theorem 3.2, we only have to optimize τ_0, p_k, and τ_k, $\forall k$, for solving problem (5). In the next, we shall employ fractional programming theory to transform the original problem into a convex one that can be solved efficiently. According to [6], for a problem of the form,

$$q^* = \max_{\tau_0,\{p_k\},\{\tau_k\}\in\mathcal{F}} \frac{B_{\rm tot}(p_k,\tau_k)}{E_{\rm tot}(\tau_0,p_k,\tau_k)}, \tag{6}$$

where $\mathcal{F}$ is the feasible set, there exists an equivalent problem in subtractive form, which satisfies

$$\max_{\tau_0,\{p_k\},\{\tau_k\}\in\mathcal{F}} \left\{B_{\rm tot}(p_k,\tau_k) - q^* E_{\rm tot}(\tau_0,p_k,\tau_k)\right\} = 0. \tag{7}$$

The equivalence of (6) and (7) can be easily verified at the optimal point $\left(\tau_0^*, p_k^*, \tau_k^*\right)$ with the maximum value q^* which is also the maximum system EE to be achieved. Dinkelbach provides an iterative method in [6] to obtain q^*. In each iteration, a subtractive-form maximization problem (7) is solved for a given q. The value of q is updated and problem (7) is solved again in the next iteration until convergence is achieved.

However, due to the products of optimization variables in C3 and C4, the transformed problem (7) is still non-convex. Hence, we further introduce a set of auxiliary variables, i.e., $\widetilde{E}_k = p_k\tau_k$, $\forall k$, which can be interpreted as the actual energy consumed on information signals by user k. Replacing p_k with $\frac{\widetilde{E}_k}{\tau_k}$, $\forall k$, problem (7) can be written as

$$\max_{\substack{\tau_0,\{\tau_k\},\\ \{\widetilde{E}_k\}}} \sum_{k=1}^{K}\tau_k W\log_2\left(1+\frac{\widetilde{E}_k}{\tau_k}\gamma_k\right) - q\left(\sum_{k=1}^{K}(\widetilde{E}_k + p_c\tau_k) + P_{\max}\tau_0\left(1-\sum_{k=1}^{K}\eta h_k\right) + P_c\tau_0\right)$$
$$\begin{aligned}
\text{s.t.}\quad & \text{C2, C5, C6}: \widetilde{E}_k \ge 0, \quad \forall k,\\
& \text{C3}: \widetilde{E}_k + p_c\tau_k \le \eta P_{\max}\tau_0 h_k + Q_k, \quad \forall k,\\
& \text{C4}: \tau_k W\log_2\left(1+\frac{\widetilde{E}_k}{\tau_k}\gamma_k\right) \ge B_{\min}^k, \quad \forall k.
\end{aligned} \tag{8}$$

Theorem 3.2: The objective function of problem (8) is jointly concave over $(\tau_0, \{\tau_k\}, \widetilde{E}_k)$.

Proof: Since $\tau_k W\log_2\left(1+\frac{\widetilde{E}_k}{\tau_k}\gamma_k\right)$ is jointly concave over $(\widetilde{E}_k, \tau_k)$ and the sum operation preserves the concavity [8], Theorem 3.2 holds. □

Note that all constraints in problem (8) result in a convex set. Thus, problem (8) is a standard concave optimization problem that can be solved by the Lagrangian duality approach [8]. The partial Lagrangian function of problem (8) can be written as

$$\mathcal{L}\left(\tau_0, \widetilde{E}_k, \{\tau_k\}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \delta\right) = \delta\left(T_{\max} - \tau_0 - \sum_{k=1}^{K}\tau_k\right) + (1+\mu_k)\sum_{k=1}^{K}\tau_k W\log_2\left(1+\frac{\widetilde{E}_k}{\tau_k}\gamma_k\right) - \sum_{k=1}^{K}\mu_k B_{\min}^k - q\left(\sum_{k=1}^{K}(\widetilde{E}_k + p_c\tau_k) + P_{\max}\tau_0\left(1-\sum_{k=1}^{K}\eta h_k\right) + P_c\tau_0\right) + \sum_{k=1}^{K}\lambda_k(Q_k + \eta P_{\max}\tau_0 h_k - \widetilde{E}_k - p_c\tau_k), \quad (9)$$

where δ, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$ are non-negative Lagrange multipliers associated with C2, C3, and C4, respectively. Note that the boundary constraints C5 and C6 will be absorbed into the optimal solution in the following.

Theorem 3.3: Given $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and δ, the optimal transmit power of maximizing $\mathcal{L}\left(\tau_0, \widetilde{E}_k, \{\tau_k\}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \delta\right)$ is given by

$$p_k = \frac{\widetilde{E}_k}{\tau_k} = \left[\frac{W(1+\mu_k)}{(q+\lambda_k)\ln 2} - \frac{1}{\gamma_k}\right]^+, \quad \forall k, \quad (10)$$

where $[x]^+ \triangleq \max\{x, 0\}$. Moreover, the optimal transmission time τ_0 and τ_k, $\forall k$, is given by solving the following linear programming problem

$$\max_{\tau_0, \{\tau_k\}} \sum_{k=1}^{K}\tau_k\left(W\log_2(1+p_k\gamma_k) - q(p_k+p_c)\right) - \tau_0 q\left(P_{\max}\left(1-\sum_{k=1}^{K}\eta h_k\right) + p_c\right)$$
$$\text{s.t.} \quad \text{C2, C5,}$$
$$\text{C3}: \tau_k(p_k+p_c) \le \tau_0\eta P_{\max}h_k + Q_k, \quad \forall k,$$
$$\text{C4}: \tau_k W\log_2(1+p_k\gamma_k) \ge B_{\min}^k, \quad \forall k. \quad (11)$$

Proof: Please refer to Appendix A. □

From (10), we observe that the optimal transmit power of users, p_k, still follows the conventional multi-level water-filling structure [6]. In addition, since problem (11) is a linear programming problem with respect to τ_0 and τ_k, linear optimization techniques, such as the simplex method [8], can be employed to obtain the optimal solution efficiently. Substituting τ_k back to (10), $\widetilde{E}_k$ is then obtained immediately.

After computing primary variables $(\tau_0, \widetilde{E}_k, \tau_k)$, the commonly used subgradient method can be employed to update Lagrange multipliers $(\boldsymbol{\lambda}, \boldsymbol{\mu}, \delta)$ towards the optimal soltuion [8]. The subgradients required are given by

$$\begin{cases} \Delta\lambda_k = \eta P_{\max}\tau_0 h_k + Q_k - E_k, & \forall k, \\ \Delta\mu_k = B_k - B_{\min}^k, & \forall k, \\ \Delta\delta = T_{\max} - \tau_0 - \sum_{k=1}^{K}\tau_k, & \end{cases} \quad (12)$$

Due to the concavity of problem (8), the iterative optimization between $(\tau_0, \widetilde{E}_k, \tau_k)$ and $(\boldsymbol{\lambda}, \boldsymbol{\mu}, \delta)$ is guaranteed to converge to the optimal solution of (8) [8]. The detailed procedures are summarized in Algorithm 1.

Algorithm 1 Energy-Efficient Resource Allocation Algorithm

1: **Initialize** the maximum accuracy ϵ and set $q = 1$;
2: **repeat**
3: Initialize $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and δ;
4: **repeat**
5: Obtain p_k from (10);
6: Obtain τ_0 and τ_k by solving problem (11);
7: Update the dual variables $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and δ from (12);
8: **until** $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and δ converge;
9: Update q from (6);
10: **until** $(B_{\text{tot}}(p_k, \tau_k) - qE_{\text{tot}}(\tau_0, p_k, \tau_k)) \le \epsilon$.

The complexity of Algorithm 1 is analyzed as follows. First, the complexity of obtaining τ_0, p_k, and τ_k, $\forall k$, linearly increases with the number of users, K. Second, since there are $2K+1$ dual variables, the complexity of the subgradient method is thus $\mathcal{O}((2K+1)^2)$ [8]. Finally, the Dinkelbach method for updating q is regardless of K [6]. Therefore, the total complexity of the proposed algorithm is $\mathcal{O}(K(2K+1)^2)$.

IV. Discussion on Finite Capacity Battery Case

In this section, we consider finite capacities batteries for users by adding new constraints:

$$\text{C7}: \eta P_0\tau_0 h_k + Q_k \le C_k, \quad \forall k, \quad (13)$$

where C_k is the battery capacity of user k. C7 suggests that the energy stored in the battery can not exceed C_k.

However, these new constraints do not affect the conclusion in Theorem 3.1. This can be explained as follows. In order to show $P_0 = P_{\max}$ still holds even under the assumption of finite capacity batteries, we only need to show that the constructed solution in the proof of Theorem 3.1, i.e., $\{\widetilde{P}_0, \{\widetilde{p}_k\}, \widetilde{\tau}_0, \{\widetilde{\tau}_k\}\}$ also satisfies new constraints C7. Substituting the constructed solution into C7 yields

$$\eta\widetilde{P}_0\widetilde{\tau}_0 h_k + Q_k = \eta\tau_0^* P_0^* h_k + Q_k \le C_k, \quad \forall k. \quad (14)$$

This suggests that the constructed solution is indeed a feasible solution as $\{P_0^*, \{p_k^*\}, \tau_0^*, \{\tau_k^*\}\}$ is a feasible solution. Similarly, we can conclude that the constructed solution still achieves a higher system EE, i.e., $\widetilde{EE} > EE^*$. Therefore, even though the finite battery capacities are considered for users, Theorem 3.1 still holds true. Furthermore, under Theorem 3.1, constraints C7 are reduced to

$$\text{C7}: \tau_0 \le \frac{C_k - Q_k}{\eta P_{\max}h_k}, \quad \forall k. \quad (15)$$

which are just to put maximum limits for the transmission time τ_0 in order to avoid energy overflow. Since these are linear constraints, the concavity of transformed problem (8) still holds, i.e., C7 do not affect the algorithm design and can be easily addressed at the expense of simple derivations.

V. Numerical Results

In this section, we present simulation results to demonstrate the proposed resource allocation. Four users are randomly and uniformly distributed on the right side of the power station with the reference distance of 2 meters and the maximum service distance of 10 meters, respectively. The receiving BS is located 150 meters away from the power station. The path loss exponent for large scale fading is 2.4. The small scale fading for WET

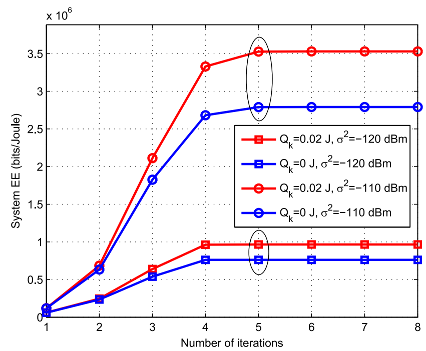

Fig. 2. Convergence of Algorithm 1.

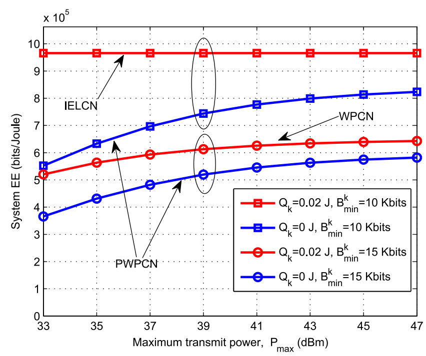

Fig. 3. EE for different B_k and Q_k.

is the Rician fading with Rician factor 7 dB while that of for WIT is the normalized Rayleigh fading. Other parameters are set as $W = 20$ kHz, $\eta = 0.9$, $P_c = 500$ mW, $p_c = 10$ mW, $\sigma^2 = -110$ dBm, $B_{\min}^k = 10$ Kbits, $P_{\max} = 40$ dBm and $T_{\max} = 1$ s, unless specified otherwise.

Fig. 2 depicts the system EE achieved by the proposed algorithm versus the number of iterations of the Dinkelbach method. We can observe that at most five iterations are needed to reach the optimal solution for all cases. Furthermore, the performance gain becomes larger for lower noise power. This is because for lower σ^2, the same amount of energy is able to support more bits transmitted, which thereby results in larger EE gain. Fig. 3 illustrates the system EE versus different settings of Q_k and $B_{\min}^k$, $\forall k$. Without loss of generality, we assume that all users have the same values of Q_k and $B_{\min}^k$, respectively. For the blue curve where $Q_k = 0$ Joule (J), $\forall k$, we know from Remark 2.1 that the system is in the PWCPN mode, i.e., all users only depend on WET. For the red curve where $Q_k = 0.02$ J, $\forall k$, we know that system is either in the IELCN mode or in the WPCN mode, which depends on the throughput requirements B_k, $\forall k$. It is observed that when increasing $B_{\min}^k$ from 10 Kbits to 15 Kbits, the system EE decreases for both scenarios, which is due to the fundamental trade-off between EE and spectral efficiency (SE). In particular, when $B_{\min}^k = 10$ Kbits, the corresponding EE is regardless of $P_{\max}$, which means that the initial energy $Q_k = 0.02$ J is able to support $B_{\min}^k$ and WET is thus not used, i.e., the system is in the IELCN mode. While when $Q_k = 0$ J, users have to depend on WET and the corresponding EE thereby increases with $P_{\max}$, i.e., the system is in the PWPCN mode. *Thus, the performance gap between IELCN and PWPCN is in fact the cost of enjoying the convenience of WET, but it is alleviated by improving* $P_{\max}$. This is because large $P_{\max}$ reduces the time needed for WET, which thereby reduces the circuit energy consumption and leaves more time for WIT. In addition, when increasing $B_{\min}^k$ from 10 Kbits to 15 Kbits, the initial energy is not able to support $B_{\min}^k$. Then, WET is further used to meet throughput requirements, which results in that the corresponding EE increases with $P_{\max}$, i.e., the system is in the WPCN mode.

VI. CONCLUSION

In this letter, we propose an unified optimization framework to energy-efficient resource allocation in the WPCN while guaranteeing the QoS of each user. Simulation results demonstrate the convergence of the proposed algorithm and verify our analysis for the WPCN as well as the special cases. It is found that the system EE performance loss caused by enjoying WET can be compensated by improving the transmit power of the power station.

APPENDIX A
PROOF OF THEOREM 3.3

Taking the partial derivative of $\mathcal{L}$ with respect to τ_0, $\widetilde{E}_k$, and τ_k, respectively, we obtain

$$\frac{\partial \mathcal{L}}{\partial \tau_0} = P_{\max}\left(\sum_{k=1}^{K}\lambda_k h_k - q\left(1-\sum_{k=1}^{K}\eta h_k\right)\right) - qP_c - \delta, \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial \widetilde{E}_k} = \frac{W(1+\mu_k)\tau_k\gamma_k}{(\tau_k+\widetilde{E}_k\gamma_k)\ln 2} - (q+\lambda_k), \quad \forall k, \quad (17)$$

Setting $\frac{\partial \mathcal{L}}{\partial \widetilde{E}_k} = 0$, the relationship of $\widetilde{E}_k$ and τ_k can be obtained as (10). Substituting (10) into $\frac{\partial \mathcal{L}}{\partial \tau_k}$ yields

$$\frac{\partial \mathcal{L}}{\partial \tau_k} = (1+\mu_k)W\log_2\left(\frac{W(1+\mu_k)\gamma_k}{(q+\lambda_k)\ln 2}\right) - \delta - (q+\lambda_k)\left(\frac{W(1+\mu_k)}{(q+\lambda_k)\ln 2} - \frac{1}{\gamma_k} + p_c\right), \quad \forall k. \quad (18)$$

From (16) and (18), we observe that the Lagrangian function $\mathcal{L}$ is a *linear function* of both τ_0 and τ_k, $\forall k$. This means that the optimal solution of τ_0 and τ_k, $\forall k$, can always be found at the vertices of the feasible region [8]. Therefore, in order to obtain τ_0 and τ_k, $\forall k$, substituting (10) into (8) yields the linear programming problem (11).

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