Network Coded Soft Forwarding for Multiple Access Relay Channels with Compressive Sensing

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***Abstract*—In this paper, we propose a novel estimate-and-forward (EF) transmission protocol, and combine it with the essence of compressive sensing (CS) for a network consisting of two correlated sources, one relay and one destination. Compared with the conventional estimate-and-forward (EF) protocol, in our protocol, correlation is exploited in calculating the soft symbols at the relay. Then we transform the network coded soft symbol vector into a sparse vector, which is suitable for compression by using CS. We analyze that the soft symbols in the proposed protocol are more suitable than those in the EF protocol for CS. Simulations show that our protocol can achieve as good bit error rate performance as the uncompressed EF protocol with reduced transmission time at the relay, thus improving the system throughput performance.**

I. Introduction

Two of the relay strategies, namely the amplify-and-forward (AF) protocol [1] and the decode-and-forward (DF) protocol [2], have been widely studied in literature. In the AF protocol, the relay amplifies the incoming signal and forwards it to the destination, which suffers from the noise amplification. In the DF protocol, the relay decodes the received signal, re-encodes it and then forwards it to the destination, and the drawback is that it propagates the erroneous decisions to the destination. To solve these problems, the more advanced relay protocol by forwarding soft information, namely soft information forwarding (SIF), has been raised [3]–[5]. The soft symbol, log-likelihood ratio (LLR), and soft mutual information were respectively used as the soft information in [3], [4] and [5]. In this paper, we draw attention to the soft-symbol based estimate-and-forward (EF) protocol [3] which calculates the minimum mean square errors (MMSE) of the received symbols at the relay.

However, none of these SIF protocols considered multi-source relay channels, in which multiple sources transmit their information to the common destination with the help of a relay. Multiple access relay channels (MARC) are common and basic building blocks in wireless networks, such as wireless sensor networks (WSN) [6]. In WSN, extending the MARC structure to correlated sources is quite practical [7], [8]. However, the current SIF protocols only consider uncorrelated sources.

In this paper, we consider an MARC network with two correlated sources. And we use compressive sensing (CS) [9], [10] to exploit the redundant information at the relay before forwarding the network coded soft information. Specifically, we propose a novel EF based SIF protocol by compressing the soft network-coded symbols at the relay with CS, and refer it as correlated EF with CS (CEF-CS). We first derive the network coded soft symbol expression in the MARC with the correlated sources. For the obtained soft symbols, because of the correlation within the two sources, the majority of them are close to the value of 1. Then we transform the network coded soft symbol vector into a sparse vector, to which CS is applied. Next, we analyze the performance of the CEF-CS protocol by comparing the soft symbol value with that in the conventional EF protocol. Simulations show that our protocol can achieve as good bit error rate (BER) performance as the uncompressed EF protocol with much reduced transmission time at the relay, thus improving the system throughput.

This research work is supported by Australian Research Council under Grant No. $LP110100110$ with ZTE for financial support.

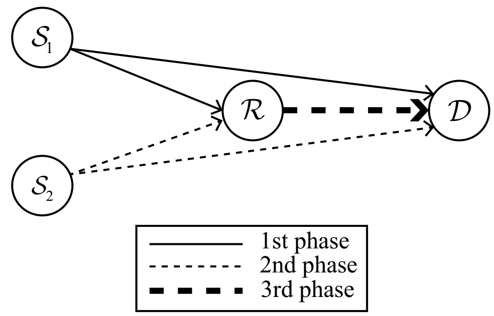

Fig. 1. The orthogonal uplink relay channel with two sources, one relay, and one destination.

II. System Model

We consider the multiple access relay network shown in Fig. 1, where the two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ transmit messages to a single destination $\mathcal{D}$ with the help of a relay $\mathcal{R}$. We assume that the two sources transmit or receive information at different phases. At the first phase, $\mathcal{S}_1$ transmits signals to both the relay and the destination. At the second phase, $\mathcal{S}_2$ transmits signals to both the relay and the destination. At the third phase, the relay processes the network coded information based on the signals from the two sources, which is then forwarded to the destination. At the end of each transmission period, the destination decodes the messages of the two sources based on the signals from the sources and the relay.

We denote by $h_{i\mathcal{R}}$, $i = 1, 2$, $h_{i\mathcal{D}}$, and $h_{\mathcal{R}\mathcal{D}}$ the channel coefficients between $\mathcal{S}_i$ and $\mathcal{R}$, between $\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively, and denote by $d_{i\mathcal{R}}$, $d_{i\mathcal{D}}$, and

$d_{\mathcal{RD}}$ the distances between $\mathcal{S}_i$ and $\mathcal{R}$, between $\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively. We assume that $h_{i\mathcal{R}}$, $h_{i\mathcal{D}}$, and $h_{\mathcal{RD}}$ are independent and identically Rayleigh distributed with the channel gains as $\lambda_{i\mathcal{R}}$, $\lambda_{i\mathcal{D}}$, and $\lambda_{\mathcal{RD}}$, respectively. These channel gains are related to the corresponding distances with the attenuation exponent γ, i.e., $\lambda_{i\mathcal{R}} = 1/(d_{i\mathcal{R}})^\gamma$, $\lambda_{i\mathcal{D}} = 1/(d_{i\mathcal{D}})^\gamma$, and $\lambda_{\mathcal{RD}} = 1/(d_{\mathcal{RD}})^\gamma$. We consider quasi-static fading channels, i.e., the channel coefficients are constant during one transmission period, and change independently from one period to another.

In each transmission period, we assume that each source transmits l independent and identically distributed (*i.i.d.*) binary phase-shift keying (BPSK) symbols. Thus, the symbol vector of $\mathcal{S}_i$ is denoted by $\boldsymbol{x}_i = (x_i^1, \cdots, x_i^l)^T$, $x_i^j \in \{\pm 1\}$ and $j \in \{1, \cdots, l\}$, with the power E_i. The received signals at the relay and at the destination from $\mathcal{S}_i$ are expressed as

$$\boldsymbol{y}_{i\mathcal{R}} = h_{i\mathcal{R}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{R}}, \quad \boldsymbol{y}_{i\mathcal{D}} = h_{i\mathcal{D}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{D}}, \tag{1}$$

respectively, where the vectors $\boldsymbol{y}_{i\mathcal{R}}$ and $\boldsymbol{y}_{i\mathcal{D}}$ consist of l received signals, i.e., $\boldsymbol{y}_{i\mathcal{R}} = (y_{i\mathcal{R}}^1, \cdots, y_{i\mathcal{R}}^l)^T$ and $\boldsymbol{y}_{i\mathcal{D}} = (y_{i\mathcal{D}}^1, \cdots, y_{i\mathcal{D}}^l)^T$, the vector $\boldsymbol{n}_{i\mathcal{R}} = (n_{i\mathcal{R}}^1, \cdots, n_{i\mathcal{R}}^l)^T$ consists of l additive white Gaussian noise (AWGN) samples at the relay, and the vector, $\boldsymbol{n}_{i\mathcal{D}} = (n_{i\mathcal{D}}^1, \cdots, n_{i\mathcal{D}}^l)^T$, consists of l AWGN samples at the destination. We assume that all the noise samples at the relay and destination are *i.i.d.* Gaussian variables with a mean zero and variance σ^2, and the sources' power satisfies $E_1 = E_2 = E_{\mathcal{S}}$. We define the SNR as $\rho \triangleq E_{\mathcal{S}}/\sigma^2$.

After receiving $\boldsymbol{y}_{i\mathcal{R}}$, the relay detects $\boldsymbol{x}_i$ and obtains the network coded symbol vector as $\boldsymbol{x}_{\mathcal{R}} = (x_{\mathcal{R}}^1, \cdots, x_{\mathcal{R}}^l)^T$, $x_{\mathcal{R}}^j \in \{\pm 1\}$. A network coded symbol $x_{\mathcal{R}}^j$ in $\boldsymbol{x}_{\mathcal{R}}$ can be calculated as $x_{\mathcal{R}}^j = x_1^j x_2^j$. However, in our CEF-CS protocol, the relay does not forward $\boldsymbol{x}_{\mathcal{R}}$ directly. Instead, it first obtains the soft symbol of $x_{\mathcal{R}}^j$ in $\boldsymbol{x}_{\mathcal{R}}$, denoted by $\tilde{x}_{\mathcal{R}}^j$. Note that the soft symbol of $x_{\mathcal{R}}^j$ is equivalent to the expectation of $x_{\mathcal{R}}^j$ given the received signals, i.e., $\tilde{x}_{\mathcal{R}}^j = \mathcal{E}[x_{\mathcal{R}}^j | y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j]$ [3]. Then the relay compresses the soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}} = (\tilde{x}_{\mathcal{R}}^1, \cdots, \tilde{x}_{\mathcal{R}}^l)^T$ by exploiting the correlation between the two sources, which will be discussed in detail in the next section. We denote by $\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}})$ the power-normalized vector after compression. The length of $\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}})$, denoted by m, $(m < l)$, varies according to different compression rates. Then the received signal vector at the destination in the third phase can be expressed as

$$\boldsymbol{y}_{\mathcal{RD}} = h_{\mathcal{RD}}\sqrt{E_{\mathcal{R}}}\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}}) + \boldsymbol{n}_{\mathcal{RD}}, \tag{2}$$

where $E_{\mathcal{R}}$ is the power at the relay, $\boldsymbol{y}_{\mathcal{RD}} = (y_{\mathcal{RD}}^1, \cdots, y_{\mathcal{RD}}^m)^T$ is the received signal vector, and $\boldsymbol{n}_{\mathcal{RD}} = (n_{\mathcal{RD}}^1, \cdots, n_{\mathcal{RD}}^m)^T$ is the AWGN vector at the destination.

At the end of the third phase, the destination first recovers $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ from $\boldsymbol{y}_{\mathcal{RD}}$, which are then combined with the signals from the source-to-destination channels to make hard decisions.

III. Correlated EF with Compressive Sensing

In this section, we introduce our CEF-CS protocol. First, we derive the network coded soft symbols based on the correlated messages from the two sources. We note that the correlated messages from the two sources lead to redundancy in the soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ at the relay. Then we compress $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ by using compressive sensing.

A. Network Coded Soft Symbols with Correlated Sources

The correlation between the sources is modeled as follows [11]. Let Z_j, $j = 1, \cdots, l$, be an *i.i.d.* binary random variable (r.v.) with $\Pr(Z_j = 0) = \tau$, where $\tau \in [0, 1]$. Without a loss of generality, we focus on the j-th symbol x_i^j in $\boldsymbol{x}_i$. We define the correlation between x_1^j and x_2^j as follows.

$$(x_1^j, x_2^j) = \begin{cases} \text{independent Bernoulli} - \frac{1}{2} \text{ r.v.}, & \text{if } Z_j = 0, \\ \text{same Bernoulli} - \frac{1}{2} \text{ r.v.}, & \text{if } Z_j = 1. \end{cases} \tag{3}$$

To simplify the derivation of the network coded soft symbol, we further define the correlation coefficient between x_1^j and x_2^j by $\zeta(x_1^j, x_2^j)$, which can be expressed as

$$\zeta(x_1^j, x_2^j) = \begin{cases} \frac{2-\tau}{4}, & \text{if } x_1^j = 1, x_2^j = 1, \\ \frac{\tau}{4}, & \text{if } x_1^j = -1, x_2^j = 1, \\ \frac{\tau}{4}, & \text{if } x_1^j = 1, x_2^j = -1, \\ \frac{2-\tau}{4}, & \text{if } x_1^j = -1, x_2^j = -1, \end{cases} \tag{4}$$

Based on $\zeta(x_1^j, x_2^j)$, we now derive the soft symbol $\tilde{x}_{\mathcal{R}}^j = \mathcal{E}[x_{\mathcal{R}}^j | y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j]$, and we have

$$\begin{aligned} \mathcal{E}[x_{\mathcal{R}}^j | y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j] &= \sum_{x_1^j=\pm1, x_2^j=\pm1} x_1^j x_2^j p\left(x_1^j, x_2^j | y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j\right) \\ &= \sum_{x_1^j=\pm1, x_2^j=\pm1} x_1^j x_2^j \frac{p\left(x_1^j, x_2^j, y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j\right)}{p\left(y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j\right)} \\ &= \sum_{x_1^j=\pm1, x_2^j=\pm1} x_1^j x_2^j \frac{p\left(y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j | x_1^j, x_2^j\right)\zeta\left(x_1^j, x_2^j\right)}{\sum\limits_{u=\pm1, v=\pm1} p\left(y_{1\mathcal{R}}^j, y_{2\mathcal{R}}^j | u, v\right)\zeta(u, v)} \\ &= \sum_{x_1^j=\pm1, x_2^j=\pm1} x_1^j x_2^j \frac{p\left(y_{1\mathcal{R}}^j | x_1^j\right) p\left(y_{2\mathcal{R}}^j | x_2^j\right)\zeta\left(x_1^j, x_2^j\right)}{\sum\limits_{u=\pm1, v=\pm1} p\left(y_{1\mathcal{R}}^j | u\right) p\left(y_{2\mathcal{R}}^j | v\right)\zeta(u, v)}, \end{aligned} \tag{5}$$

where $p(\bullet)$ denotes the probability density function (PDF) of $\bullet$, and $p\left(y_{i\mathcal{R}}^j | x_i^j\right)$ is conditionally Gaussian distributed, which is expressed as

$$p\left(y_{i\mathcal{R}}^j | x_i^j\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(y_{i\mathcal{R}}^j - h_{i\mathcal{R}} x_i^j\right)^2}{2\sigma^2}\right). \tag{6}$$

B. Compressive Sensing on Soft Symbol Vector

The correlation between the two sources leads to redundancy in the soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$. By taking advantage of this redundancy, we can convert $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ to a sparse vector. To show this sparsity, we first investigate the PDF of a network coded soft symbol $\tilde{x}_{\mathcal{R}}^j$. Fig. 2 plots the histogram results of a soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ with τ in (4) equal to 0.2, vector length $l = 10,000$, $h_{1\mathcal{R}} = 0.5$, and $h_{2\mathcal{R}} = 1$. We focus on the curves for the CEF-CS protocol, and consider the SNR $\rho = 0$ dB, 10 dB, and 25 dB (the curve 'EF, SNR=0dB' will be discussed

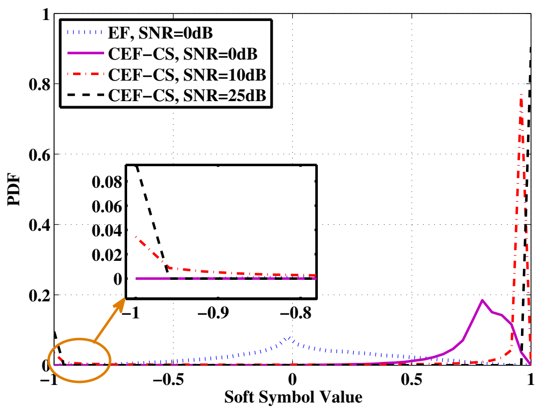

Fig. 2. The PDF of a soft symbol vector with $\tau = 0.2$, $l = 10,000$, $h_{1\mathcal{R}} = 0.5$, and $h_{2\mathcal{R}} = 1$.

later). We can see that the soft symbols are within the range from -1 to 1. When the SNR is large, the soft symbols are polarized around the areas close to -1 and 1. Furthermore, due to the correlation between the two sources, the majority of the soft symbols are close to 1.

Then we obtain another length-l vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ by calculating $\tilde{\boldsymbol{x}}'_{\mathcal{R}} = \tilde{\boldsymbol{x}}_{\mathcal{R}} - 1$. Obviously, the elements in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ are polarized around the areas close to -2 and 0, and the majority of the elements are close to 0. Therefore, the vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ can be viewed as a sparse vector, which can be compressed by using compressive sensing [10]. We now connect the sparsity in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ with τ. We can obtain that when ρ is large enough, there are $(1-\frac{\tau}{2})l$ elements in $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ around zeros and $\frac{\tau l}{2}$ elements around -2. Therefore, the sparsity of the vector $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ is $\frac{\tau}{2}$.

Based on the sparsity of $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, we encode $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ with an $m \times l$ $(m < l)$ sparse matrix $\mathbf{\Phi}$ composed only of the entries $\{1, -1, 0\}$. Specifically, we select $\mathbf{\Phi}$ as a Rademacher encoding matrix according to [10]. For each element ϕ in $\mathbf{\Phi}$, we have

$$\phi = \begin{cases} +1, & \text{with probability } \frac{2}{\tau l} \\ 0, & \text{with probability } 1 - \frac{4}{\tau l} \\ -1, & \text{with probability } \frac{2}{\tau l} \end{cases} . \quad (7)$$

The encoded vector $\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, or called measurement vector, is of length m. Thus, by transmitting $\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ rather than $\tilde{\boldsymbol{x}}_{\mathcal{R}}$, the relay can save $l - m$ time slots. The vector $\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}})$ in (2) is written as

$$\mathcal{C}(\tilde{\boldsymbol{x}}_{\mathcal{R}}) = \frac{\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}}{\sqrt{\mathcal{E}\left[\left|\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}\right|^2\right]}}\sqrt{\frac{l}{m}}. \quad (8)$$

Note that the term $\sqrt{l/m}$ in (8) is included to keep relay's transmission power constant during the third phase, since the relay consumes less time slots by compressing $\tilde{\boldsymbol{x}}_{\mathcal{R}}$.

From Fig. 2, we can approximate the distribution of $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ as a mixture of two Gaussian variables with two different mean values and the same variance. This approximation can be equivalently treated as a two-state Gaussian mixture model described in [10] which requires $m = O\left(\frac{l\tau}{2}\log(l)\right)$ to guarantee the reconstruction. Thus, we need $O\left(\frac{l\tau}{2}\log(l)\right)$ measurements to recover $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$.

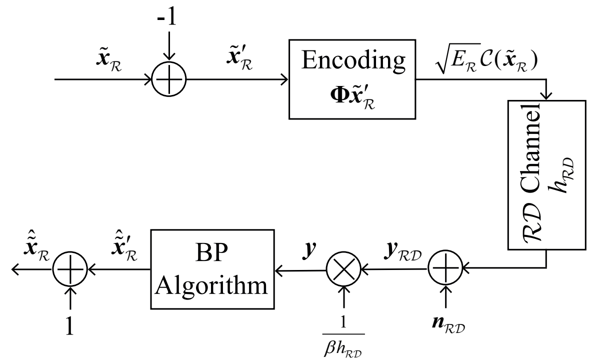

Fig. 3. The compression and reconstruction process of $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ by using compressive sensing.

C. Reconstruction via Belief Propagation

At the destination, the belief propagation (BP) algorithm is applied to recovering $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, which is of length l. We assume the PDFs of the soft symbol vector at different SNRs are known to the destination, which are used as the prior knowledge for BP [10]. We denote by $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$ the recovered $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ from BP algorithm at the destination. It is easy to recover the network coded soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$, denoted by $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$, from $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$ by calculating $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}} = \hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}} + 1$. Then the recovered soft symbol vector $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$ will be combined with the signals from the source-to-destination channels to make final decisions on the sources' messages. In the following, we focus on how to obtain $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$ from the received signal $\boldsymbol{y}_{\mathcal{RD}}$ by using BP.

To make the received signal $\boldsymbol{y}_{\mathcal{RD}}$ suitable for BP process, we first define

$$\beta = \sqrt{\frac{lE_{\mathcal{R}}}{m\mathcal{E}\left[\left|\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}\right|^2\right]}}, \quad (9)$$

and we have $\boldsymbol{y}_{\mathcal{RD}} = \beta h_{\mathcal{RD}}\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{RD}}$. Then we obtain a new vector $\boldsymbol{y}$ by calculating $\boldsymbol{y} = \boldsymbol{y}_{\mathcal{RD}}/(\beta h_{\mathcal{RD}})$, i.e., $\boldsymbol{y} = \mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}} + \boldsymbol{n}$, where $\boldsymbol{n} = \boldsymbol{n}_{\mathcal{RD}}/(\beta h_{\mathcal{RD}})$. The vector $\boldsymbol{y}$ can be interpreted as the encoded vector $\mathbf{\Phi}\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ plus Gaussian noise. We use $\boldsymbol{y}$ as the input of the BP algorithm to recover $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ as in [10]. After we obtain $\hat{\tilde{\boldsymbol{x}}}'_{\mathcal{R}}$, we can thus obtain $\hat{\tilde{\boldsymbol{x}}}_{\mathcal{R}}$. The compression and reconstruction process of the network coded soft symbol vector $\tilde{\boldsymbol{x}}_{\mathcal{R}}$ is shown in Fig. 3.

IV. Comparison with the EF protocol

We analyze the soft symbol values in our CEF-CS protocol, and compare them with the conventional EF protocol [3].

Recall that the soft symbol $\tilde{x}^j_{\mathcal{R}}$ of our CEF-CS protocol is derived in (5). However, in the conventional EF protocol, the soft symbol is calculated as [3]

$$\tilde{x}^j_{\mathcal{R}} = \tanh\left(\frac{LLR_{x^j_1}}{2}\right)\tanh\left(\frac{LLR_{x^j_2}}{2}\right), \quad (10)$$

where $LLR_{x_i^j}$ represents the log-likelihood ratio (LLR) of the symbol x_i^j at the relay, which is calculated as

$$LLR_{x_i^j} = \ln \frac{p\left(y_{i\mathcal{R}}^j | x_i^j = 1\right)}{p\left(y_{i\mathcal{R}}^j | x_i^j = -1\right)} = \frac{2\sqrt{E_S} h_{i\mathcal{R}} y_{i\mathcal{R}}^j}{\sigma^2}. \quad (11)$$

In fact, the calculation of $\tilde{x}_{\mathcal{R}}^j$ in (10) does not consider the correlation between the two sources.

By comparing the values of $\tilde{x}_{\mathcal{R}}^j$ calculated from (5) and (10), we have the following theorem.

Theorem 1: The soft symbol calculated by the CEF-CS protocol in (5) has a higher reliability than that calculated by the conventional EF protocol in (10).

Proof: Please refer to Appendix A.

From the proof of *Theorem 1*, we can see that the CEF-CS protocol considers the correlation of the two sources when calculating $\tilde{x}_{\mathcal{R}}^j$, thus having a higher reliability. Therefore, the CEF-CS protocol always has a better performance than the conventional EF protocol.

In Fig. 2, we compare the probability distributions of the network coded soft symbols in the CEF-CS protocol and the EF protocol when $\rho = 0$ dB. It can be seen that with the correlation knowledge exploited by the CEF-CS protocol at the relay, more soft symbols than those in the EF protocol approach the value of 1. In fact, due to the correlation between the two sources, the majority of the network coded symbols are 1. Therefore, the soft symbols calculated in the CEF-CS protocol have higher reliability compared to the conventional EF protocol. In addition, since the soft symbols in the CEF-CS protocol are more polarized than those in the EF protocol (as seen from Fig. 2), it is more suitable to implement compressive sensing in our CEF-CS protocol.

V. Simulation Results

In the simulations, we consider Rayleigh fading channels. During each transmission period, each source transmits one block which contains 200 binary symbols. We focus on symmetric channels in this work, where d_{S_iR} and d_{RD} are equal to 0.5, d_{S_iD} is equal to 1 and the two sources, the relay and the destination are assumed to be aligned on the same horizon. The attenuation exponent γ is set to be 2, and the sources and the relay have unit transmission power, namely, $E_{\mathcal{S}} = E_{\mathcal{R}} = 1$. Besides, because $O\left(\frac{l\tau}{2}\log(l)\right)$ measurements are needed to recover $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$, and the measurement vector length should be smaller than the uncompressed vector length, then for $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$ of length 200, the source correlation τ should be smaller than the value of $2/(c\log(200))$, where c is a real positive number. So in the simulation, we choose τ to be 0.2. We call the scenario where the CEF-CS protocol forwards directly the soft symbols in (5) instead of implementing CS on it as the CEF protocol.

We compare the BER and throughput performance of the CEF-CS protocol with different transmission rates r, the CEF protocol and the EF protocol. The different rates, namely $2/10, 3/10, 1/2, 7/10$, of the CEF-CS are defined by setting the encoding matrices to be of size $40\times200, 60\times200, 100\times200$ and 140×200, respectively, which means the number of measurements transmitted from relay is $40, 60, 100, 140$, respectively. We choose the row weight L of the encoding matrix to be constantly $L = 4/\tau$, and the column weight C to be $C = rL$ [10]. When sampling the PDF in the BP process, we use messages of length 525, which can provide fast fourier transform computation. To ensure the feasibility of convolving noise item, we uniformly sample the PDF in a sufficient rage of $[-84, 84]$, and this indicates that only 7 effective samples of the PDF locate in the range $(-2, 0)$ of $\tilde{\boldsymbol{x}}'_{\mathcal{R}}$. However, 7 samples are enough to represent the reliability levels of soft symbols, as results in [12] have implied that even 4 quantization levels can achieve full diversity in relay networks, as long as enough information is conveyed from the relay to the destination.

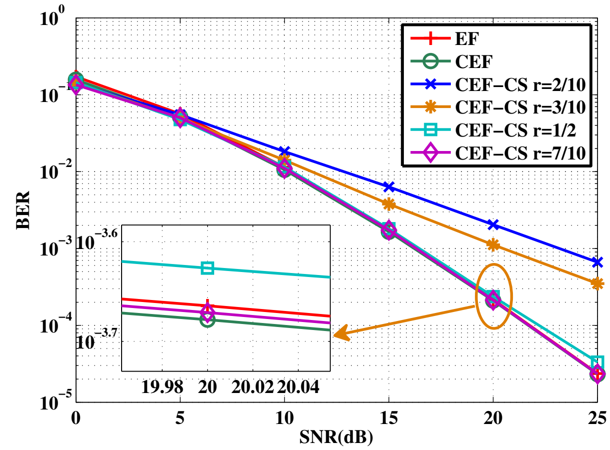

Fig. 4. BER performance of different protocols over fading channels.

First we investigate the BER performance of these protocols. The system BER is taken as the average value of BERs of the two sources' signal at the destination. As can be seen from Fig. 4, except that the CEF-CS protocols with rate $2/10$ and $3/10$ can only achieve diversity gain of 1, the CEF-CS protocols with rate $1/2$ and $7/10$, the CEF protocol and the EF protocol can all achieve diversity gain of 2. The CEF protocol outperforms the EF protocol all the time, which verifies our conclusion in Section IV. Besides, with the increase of the CS transmission rate at the relay, the information loss from the relay to the destination channel becomes less; consequently, the system BER performance becomes better.

Then we investigate the throughput performance. We define the time duration for transmitting an length-l signal as one time slot, and define the throughput as the length of the correctly received signal out of the entire signal length per time slot, which can be expressed as $R = \frac{l-e}{l\cdot T}$, where e denotes the average number of errors of the decoded source messages at the destination, and T is the number of time slots for the total transmission in an MARC network. For the EF and CEF, T is constantly equal to 3. For the CEF-CS, due to compression,

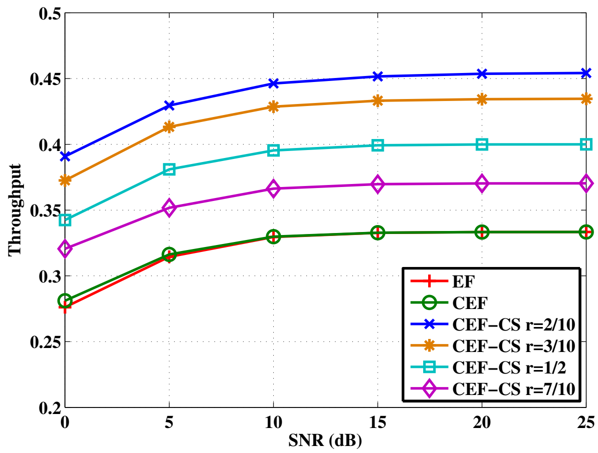

Fig. 5. System throughput comparison between different protocols over fading channels.

we have $T = 2 + \frac{m}{l}$. As shown in Fig 5, the compressed CEF-CS protocols all outperform the uncompressed CEF and the EF protocols due to much less time slots the compressed protocols utilize. For the CEF-CS protocol with different rates, we can see there is a trend that with a lower transmission rate, the throughput performance is better. Combined with the BER performance in Fig 4, we virtually cannot sacrifice the BER performance in order to obtain better throughput. Instead, we have to achieve both good BER performance and good throughput performance. Then in this scenario, we come to a conclusion that a rate-1/2 CEF-CS protocol can achieve good BER performance as well as throughput performance compared with the other protocols.

VI. Conclusion

This paper has presented an EF technique for correlated sources in an MARC and has exploited the sources' correlation at the relay by implementing CS on the soft symbol vector. We first exploit the source correlation by deriving the expression of the soft symbol of the CEF-CS protocol and then we transform the network coded soft symbol vector into a sparse vector, which is suitable for compression by using CS. We prove that the CEF-CS protocol outperforms the conventional EF protocol with more reliable soft symbols. Simulations show that our protocol can achieve as good BER performance as the uncompressed EF protocol while reducing transmission time of the relay, thus improving the system throughput performance. By taking both the BER and the throughput performance into account, we draw a conclusion from our simulations that a rate-$1/2$ CEF-CS protocol is the best.

Appendix A
Proof of *Theorem 1*

We further develop the soft symbol $\tilde{x}_{\mathcal{R}}^j$ in (5) as

$$\tilde{x}_{\mathcal{R}}^j = \frac{\alpha\left(2-\tau\right)-\theta\tau}{\alpha\left(2-\tau\right)+\theta\tau}, \quad (12)$$

where

$$\alpha = \sum_{x_1^j x_2^j = 1} p\left(y_{1\mathcal{R}}^j | x_1^j\right) p\left(y_{2\mathcal{R}}^j | x_2^j\right),$$
$$\theta = \sum_{x_1^j x_2^j = -1} p\left(y_{1\mathcal{R}}^j | x_1^j\right) p\left(y_{2\mathcal{R}}^j | x_2^j\right). \quad (13)$$

Obviously, we have $\alpha > 0$ and $\theta > 0$.

The conventional EF derives $\tilde{x}_{\mathcal{R}}^j$ in (10) by assuming that the two sources are independent, i.e., (10) can be obtained by considering $\tau = 1$ in (12). In the CEF-CS protocol, we derive $\tilde{x}_{\mathcal{R}}^j$ by considering the real τ. In correlated sources, we always have $\tau < 1$. Given α and θ, $\tilde{x}_{\mathcal{R}}^j$ can be regarded as a function of τ, which is denoted by $\delta\left(\tau\right)$. To investigate the relationship between $\delta\left(\tau\right)$ and τ, we derive the first-order derivative of $\delta\left(\tau\right)$ as follows.

$$\frac{\mathrm{d}\delta\left(\tau\right)}{\mathrm{d}\tau} = \frac{-8\alpha\theta}{\left(\alpha(2-\tau)+\theta\tau\right)^2}. \quad (14)$$

Since $\alpha > 0$ and $\theta > 0$, we always have $\frac{\mathrm{d}\delta(\tau)}{\mathrm{d}\tau} < 0$, which meas that $\delta\left(\tau\right)$ is a decreasing function in terms of τ. When correlation exists between the sources, we have $\tau < 1$. Therefore, the soft symbol values in the CEF protocol are always larger than that in the conventional EF protocol. As the majority of the soft symbols $\tilde{x}_{\mathcal{R}}^j$ approach the value of 1, the soft symbols in the CEF protocol are more reliable than those of the EF protocol.

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