

# High-Performance Beamformer and Low-Complexity Detector for DF-Based Full-Duplex MIMO Relaying Networks

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**Abstract:** In this paper, we consider a full-duplex multiple-input multiple-output (MIMO) relaying network with the decode-and-forward (DF) protocol. Due to the full-duplex transmissions, the self-interference from the relay transmitter to the relay receiver degrades the system performance. We thus propose an iterative beamforming structure (IBS) to mitigate the self-interference. In this method, the receive beamforming at the relay is optimized to maximize the signal-to-interference-plus-noise-ratio (Max-SINR), while the transmit beamforming at the relay is optimized to maximize the signal-to-leakage-plus-noise-ratio (Max-SLNR). To further improve the performance, the receive and transmit beamforming matrices are optimized between Max-SINR and Max-SLNR in an iterative manner. Furthermore, in the presence of the residual self-interference, a low-complexity whitening-filter (WF) maximum likelihood (ML) detector is proposed. In this detector, a WF is designed to transform a colored interference-plus-noise to a white noise, while the singular value decomposition is used to convert coupled spatial subchannels to parallel

independent ones. From simulations, we find that the proposed IBS performs much better than the existing schemes. Also, the proposed low-complexity detector significantly reduces the complexity of the conventional ML (CML) detector from exponential time (an exponential function of the number of the source transmit antennas) to polynomial one while achieving a slightly better BER performance than the CML due to interference whitening.

**Keywords:** decoding-and-forward; MIMO relay; full-duplex; self-interference; iterative beamforming

## I. INTRODUCTION

Recently, full-duplex relaying systems have been intensively investigated due to a wide coverage area and high spectral efficiency [1]. The full duplex operation by supporting concurrent transmission and reception in a single time/frequency channel is expected to be implemented in future generation networks [2,3]. The promise of nearly doubled channel capacity and spectral efficiency compared to conventional half-duplex communications is

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In this paper, the authors proposed a high-rate beamforming design and a low-complexity ML detector at the relay in a DF-based full-duplex MIMO relay network.

the most attractive feature in full-duplex communications [4,5]. However, in full-duplex systems, self-interference at the relay, due to the signal leakage from its transmitter to the receiver, will lead to a serious degradation of the system performance. Therefore, one of the key challenges in full-duplex relaying systems is how to manage and control the effect of self-interference successfully and effectively.

There are several strategies proposed for mitigating self-interference in full-duplex relaying systems [6-14]. In particular, in multiple-input multiple-output (MIMO) based on full-duplex relaying systems, beamforming schemes can be optimized for spatially suppressing the self-interference by utilizing the degree of freedom provided by multiple antennas [10-14]. In [10], the authors consider an amplify-and-forward (AF) based MIMO relaying system with full-duplex transmissions. To deal with the self-interference at the relay, the authors maximize the signal-to-interference ratio at both the input and output of the relays. However, this work neglects the effect of noise. In [11], the authors adopt a two-way full-duplex MIMO relaying system with AF protocol. Instead of purely suppressing the self-interference, the authors improve the system achievable rate by jointly optimizing the beamforming matrix at the relay and the power allocation at the two sources.

For full-duplex MIMO relay systems with the decoded-and-forward (DF) protocol, several typical beamforming schemes, such as minimum mean square error (MMSE) and null-space projection (NSP) are investigated [12-14]. The basic idea of the null-space projection is that it forces the self-interference to be zero by designing the receive and transmit beamforming matrices. However, these schemes are not designed to achieve the optimality of the system in terms of achievable rate.

In this paper, we focus on the optimization of beamforming coefficients in a full-duplex MIMO relaying system consisting of one source, one DF-based relay and one destination. We propose an iterative beamforming structure (IBS) at the relay for enhancing the

system achievable rate by alternately maximizing the average signal-to-interference-plus noise ratio (SINR) at receiver and average signal to-leakage-plus-noise ratio (SLNR) at transmitter. Although the proposed structure achieves an improved performance, there still exists residual self-interference and noise. It is a challenge to design an optimal low-complexity detector in the presence of residual self-interference which is not white. Therefore, we then develop a low-complexity maximum likelihood (ML) detector at the relay with an improved decoding performance by using whitening-filter, singular-value decomposition (SVD), and eigenvalue decomposition (EVD).

Our contributions in this paper are as follows. We first maximize the SINR, namely, Max-SINR, at the input of the relay by optimizing its receive antenna beamforming coefficients. At the same time, we maximize the SLNR, namely, Max-SLNR, at the output of the relay by optimizing its transmit beamforming coefficients. Then an iterative beamforming structure is designed to optimize the receive and transmit beamforming in an iterative manner. For the detector, we first design an interference-and-noise whitening filter (WF) for the colored interference-plus-noise. Then a beamforming scheme based on SVD is employed to transform the spatially coupled source-to-relay channels to independent parallel ones. Simulations show that the proposed IBS achieves much better rate and error performance within a few iterations compared with its counterparts. Also, the bit rate error performance of our low-complexity ML detector is better than the conventional ML detector with a much reduced complexity.

This paper is organized as follows. The system model is described in Section II. Section III presents the design of the IBS. Section IV describes the low complexity ML detector. The simulation results are shown and discussed in section V, and Section VI concludes this entire paper.

Notations: Throughout the paper, matrices and vectors are denoted by letters of bold upper case and bold lower case, respectively.

Signs  $(\bullet)^H$ ,  $(\bullet)^{-1}$ ,  $\text{tr}(\bullet)$ ,  $\|\bullet\|_F$  and  $\det(\bullet)$  denote matrix conjugate transpose, inverse, trace, Frobenius norm, and determinant, respectively. The notation  $\mathcal{E}\{\bullet\}$  refers to the expectation operation. The symbol  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix. We use  $\mathcal{C}$  and  $\mathcal{C}_{MQAM}$  to denote the complex field and set of signal constellation points of MQAM, respectively.

## II. SYSTEM MODEL

As shown in Fig. 1, we consider a two-hop link between a multi-antenna source node and a multi-antenna destination node with a full-duplex DF relay connecting them. The source and destination are equipped with  $N_s$  and  $N_d$  antennas, respectively. The relay has  $N_{RR}$  receive and  $N_{RT}$  transmit antennas. The source transmits data towards the relay, while upon receiving the signal from the source, the relay regenerates the source's information, and transmits it towards the destination using the same frequency band. Here, it is assumed that the direct source-to-destination link is blocked, for example, due to shadow fading and path loss. Let  $\mathbf{H}_{SR} \in \mathcal{C}^{N_{RR} \times N_s}$  and  $\mathbf{H}_{RD} \in \mathcal{C}^{N_d \times N_{RT}}$  represent the source-to-relay and relay-to-destination channel matrices, respectively. The self-interference channel matrix is denoted by  $\mathbf{H}_{RR} \in \mathcal{C}^{N_{RR} \times N_{RT}}$ . All the channels are assumed to experience Rayleigh fading and the perfect CSI's are available at the relay. The signal vector  $\mathbf{x}_s \in \mathcal{C}^{N_s \times 1}$  transmitted

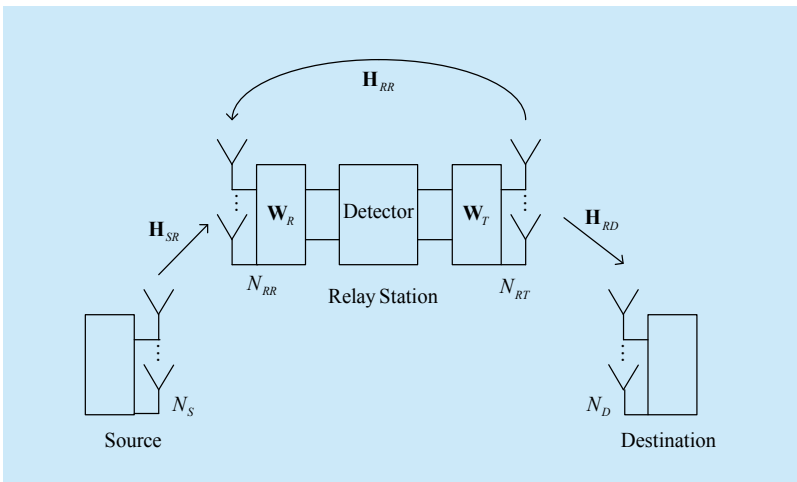


Fig.1 System model consisting of one source, one relay, and one destination

from the source is assumed to be taken from an M-QAM constellation with the power constraint  $\mathcal{E}(\text{tr}\{\mathbf{x}_s \mathbf{x}_s^H\}) = P_s$ , where  $P_s$  is the transmit power at the source.

The received signal vector with receive beamforming at the relay is represented as

$$\mathbf{y}_R = \mathbf{W}_r (\mathbf{H}_{SR} \mathbf{x}_s + \mathbf{H}_{RR} \mathbf{x}_R + \mathbf{n}_R) \quad (1)$$

where  $\mathbf{W}_r \in \mathcal{C}^{N_{RR} \times N_{RR}}$  stands for the receive beamforming at the relay with  $\|\mathbf{W}_r\|_F^2 = 1$ ,  $\mathbf{n}_R \in \mathcal{C}^{N_{RR} \times 1}$  denotes the circular symmetric additive white Gaussian noise (CSAWGN) vector with covariance matrix  $\sigma_R^2 \mathbf{I}_{N_{RR}}$ , and  $\mathbf{x}_R$  denotes the transmit signal from the relay to destination.

In order to further suppress interference and improve the error performance, we introduce the transmit beamforming  $\mathbf{W}_t \in \mathcal{C}^{N_{RT} \times N_s}$  at the relay, and then  $\mathbf{x}_R$  is given by

$$\mathbf{x}_R = \beta \mathbf{W}_t \hat{\mathbf{x}}_s \quad (2)$$

where  $\hat{\mathbf{x}}_s$  denotes the detected signal of  $\mathbf{x}_s$  at the relay, and  $\beta$  denotes the power normalization factor, which can be written as

$$\beta = \sqrt{N_s P_R [\mathcal{E}\{\text{tr}(\mathbf{W}_t \mathbf{W}_t^H)\}]^{-1}} \quad (3)$$

where  $P_R$  is the transmit power constraint at the relay, i.e.,  $\mathcal{E}(\text{tr}\{\mathbf{x}_R \mathbf{x}_R^H\}) = P_R$ .

At the destination, the received signal can be written as

$$\mathbf{y}_D = \mathbf{H}_{RD} \mathbf{x}_R + \mathbf{n}_D \quad (4)$$

where  $\mathbf{n}_D \in \mathcal{C}^{N_d \times 1}$  is the CSAWGN vector with covariance matrix  $\sigma_D^2 \mathbf{I}_{N_d}$  at the destination.

Since we consider the DF protocol, the end-to-end achievable rate can be formulated as [15]

$$R = \min(R_{SR}, R_{RD}) \quad (5)$$

where  $R_{SR}$  and  $R_{RD}$  represent the achievable rates of the source-to-relay and relay-to-destination channels, respectively.

We have

$$R_{SR} = \log_2 \det \left( \mathbf{I}_{N_s} + \frac{P_s}{N_s} \mathbf{W}_r \mathbf{H}_{SR} (\mathbf{W}_r \mathbf{H}_{SR})^H \mathbf{W}_r^{-1} \right) \quad (6)$$

where the matrix  $\mathbf{W} \in \mathcal{C}^{N_s \times N_s}$  is defined as

$$\mathbf{W} = P_s \beta^2 N_s^{-1} (\mathbf{W}_r \mathbf{H}_{RR} \mathbf{W}_t) (\mathbf{W}_r \mathbf{H}_{RR} \mathbf{W}_t)^H + \sigma_R^2 \mathbf{W}_r \mathbf{W}_r^H \quad (7)$$

Regarding the rate  $R_{RD}$ , we have

$$R_{RD} = \log_2 \det (\mathbf{I}_{N_d} + P_s N_s^{-1} \beta^2 \mathbf{H}_{RD} \mathbf{W}_t (\mathbf{H}_{RD} \mathbf{W}_t)^H) \quad (8)$$

In the following section, we will enhance the achievable system rate  $R$  in Eq. (5) by optimizing the design of the beamforming matrices  $\mathbf{W}_r$  and  $\mathbf{W}_i$  at the relay.

### III. ITERATIVE BEAMFORMING STRUCTURE AT THE RELAY

In this section, we propose a novel beamforming structure at the relay for improving the system achievable rate  $R$ . Since  $R$  is determined by  $R_{SR}$  and  $R_{RD}$ , we will alternatively focus on maximizing both the SINR at the relay's input and the SLNR at the relay's output.

First, based on Eqs. (1~3), the average SINR at the relay receiver can be written as  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i) =$

$$\frac{\frac{P_s}{N_s} \text{tr}\{\mathbf{W}_r \mathbf{H}_{SR} \mathbf{H}_{SR}^H \mathbf{W}_r^H\}}{\text{tr}\left\{\mathbf{W}_r \left(\sigma_R^2 \mathbf{I}_{N_{RR}} + \frac{P_s \beta^2}{N_s} \mathbf{H}_{RR} \mathbf{W}_i \mathbf{W}_i^H \mathbf{H}_{RR}^H\right) \mathbf{W}_r^H\right\}} \quad (9)$$

In general, a higher  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i)$  means a higher achievable rate  $R_{SR}$ .

From (1), the average leakage of the transmit signal  $\mathbf{x}_r$  to the relay receiver is of the form  $\mathbf{W}_r \mathbf{H}_{RR} \mathbf{x}_r$ . Therefore, the SLNR at the relay transmitter can be written as [16,17]  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i) =$

$$\frac{\beta^2 \text{tr}\{\mathbf{W}_i^H \mathbf{H}_{RD}^H \mathbf{H}_{RD} \mathbf{W}_i\}}{\text{tr}\left\{\mathbf{W}_i^H \left(\sigma_D^2 \mathbf{I}_{N_{DR}} + \frac{P_s \beta^2}{N_s} \mathbf{H}_{RR}^H \mathbf{W}_r^H \mathbf{W}_r \mathbf{H}_{RR}\right) \mathbf{W}_i\right\}} \quad (10)$$

A higher  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i)$  means that the destination receiver should collect a larger percentage of the transmit signal power of the relay, thereby resulting in a higher received SINR at the destination and a larger rate  $R_{RD}$ . We now focus on solving the maximization of  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i)$  and  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i)$  by optimizing the beamforming matrices  $\mathbf{W}_r$  and  $\mathbf{W}_i$ . Obviously, from Eqs. (9) and (10), both  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i)$  and  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i)$  are the functions of  $\mathbf{W}_r$  and  $\mathbf{W}_i$ . However, simultaneous maximization of  $\text{SINR}_R$  and  $\text{SLNR}_R$  by jointly optimizing  $\mathbf{W}_r$  and  $\mathbf{W}_i$  would be very complex and there are no closed-form expression for  $\mathbf{W}_r$  and  $\mathbf{W}_i$ . In the following, we propose an iterative structure for solving  $\mathbf{W}_r$  and  $\mathbf{W}_i$  separately, with a reduced complexity compared to the joint one.

#### 3.1 Maximization of $\text{SINR}_R$ : Fix $\mathbf{W}_i$ , Optimize $\mathbf{W}_r$

To maximize  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i)$  in (9), for a given  $\mathbf{W}_i$ , the optimal  $\mathbf{W}_r$  consists of the conjugate transpose of the  $N_s$  eigenvectors associated with the largest  $N_s$  eigenvalues of the matrix

$$(\sigma_R^2 \mathbf{I}_{N_{RR}} + P_s \beta^2 N_s^{-1} \mathbf{H}_{RR} \mathbf{W}_i \mathbf{W}_i^H \mathbf{H}_{RR}^H)^{-1} \mathbf{H}_{SR} \mathbf{H}_{SR}^H \quad (11)$$

For more details of the optimization process please see **Appendix A**. By optimizing the receive beamforming matrix  $\mathbf{W}_r$ , we jointly suppress the self-interference and noise to enhance the useful signal.

#### 3.2 Maximization of $\text{SLNR}_R$ : Fix $\mathbf{W}_r$ , Optimize $\mathbf{W}_i$

For a given  $\mathbf{W}_r$ , to maximize  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i)$  in (10), the  $N_s$  columns of  $\mathbf{W}_i$  should be composed of the  $N_s$  eigenvectors corresponding to the largest  $N_s$  eigenvalues of the matrix

$$(\sigma_D^2 \mathbf{I}_{N_{DR}} + P_s \beta^2 N_s^{-1} \mathbf{H}_{RR}^H \mathbf{W}_r^H \mathbf{W}_r \mathbf{H}_{RR})^{-1} \mathbf{H}_{RD}^H \mathbf{H}_{RD} \quad (12)$$

A detailed optimization process follows a similar deriving process in **Appendix A**.

By optimizing the transmit beamforming matrix  $\mathbf{W}_i$ , we reduce the leakage part of the transmit signal at the relay and improve the useful relay-to-destination part of the transmitted signal.

#### 3.3 Iterative beamforming structure

Based on the optimization processes in the previous two subsections, we thereby propose an iterative beamforming structure (IBS) at the relay as indicated in Fig. 2 by optimizing  $\mathbf{W}_r$  and  $\mathbf{W}_i$  in an alternating iterative manner. The IBS is described as follows. We first utilize the initial value of  $\mathbf{W}_i$  to obtain the optimal  $\mathbf{W}_r$  by solving the maximization of  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_i)$ . Then this optimized  $\mathbf{W}_r$  is utilized to obtain  $\mathbf{W}_i$  by solving the maximization of  $\text{SLNR}_R(\mathbf{W}_r, \mathbf{W}_i)$ . By doing this process iteratively, we finally arrive at our solutions of  $\mathbf{W}_r$  and  $\mathbf{W}_i$ .

Additionally, in order to have a good initial value of  $\mathbf{W}_i$ , we define

$$\mathbf{W}_r^H \mathbf{W}_r = N_{RR}^{-1} \mathbf{I}_{N_{RR}} \quad (13)$$

Then we substitute (13) into (12) to obtain

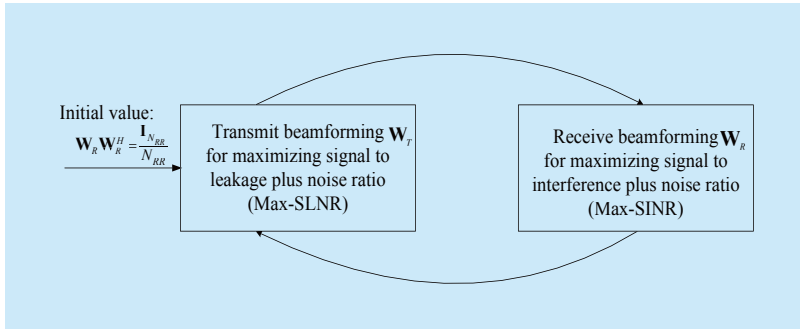


Fig.2 Proposed iterative beamforming structure at the relay

**Algorithm 1** Proposed IBS by Iterative Optimization between Max-SINR and Max-SLNR

**Input:**  $\mathbf{H}_{RR}, \mathbf{H}_{SR}, \mathbf{H}_{RD}, P_S, P_R, \sigma_D^2, \sigma_R^2$  and  $\delta$ .

**Output:**  $\mathbf{W}_r$  and  $\mathbf{W}_t$

**Initialization:** Set  $n = 1$ , where  $n$  is the index of iterations, compute the initial value of  $\mathbf{W}_t^1$  by using (14). Then based on  $\mathbf{W}_t^1$ , we calculate  $\mathbf{W}_r^1$  based on (11), and obtain system rate  $R^1$  in (5) by using  $\mathbf{W}_r^1$  and  $\mathbf{W}_t^1$ .

**Steps:**

- 1)  $n = n + 1$
- 2) Update  $\mathbf{W}_t^n$  based on Eq. (12).
- 3) Update  $\mathbf{W}_r^n$  based on Eq.(11)
- 4) Compute the system rate  $R^n$  in (5) by using  $\mathbf{W}_r^n$  and  $\mathbf{W}_t^n$ .
- 5) If  $|R^n - R^{n-1}| > \delta$ , go to **Step 1**); otherwise, quit.

the initial value of  $\mathbf{W}_t$ , which consists of the eigenvectors corresponding to the largest  $N_s$  eigenvalues of the matrix

$$\left( \sigma_D^2 \mathbf{I}_{N_{st}} + \frac{P_S \beta^2 \mathbf{H}_{RR}^H \mathbf{H}_{RR}}{N_{RR} N_S} \right)^{-1} \mathbf{H}_{RD}^H \mathbf{H}_{RD} \quad (14)$$

To make clear, the IBS operations are summarized as an algorithm below.

## VI. INTERFERENCE-AND-NOISE-WHITENING BEAMFORMING FOR ML DETECTION

In this section, we will propose an interference-and-noise-whitening beamforming method for reducing the complexity of the conventional ML detection at the relay. We rewrite Eq. (1) as

$$\mathbf{y}_R = \mathbf{H}'_{SR} \mathbf{x}_S + \mathbf{n}'_R \quad (15)$$

where  $\mathbf{H}'_{SR} = \mathbf{W}_r \mathbf{H}_{SR}$ , and  $\mathbf{n}'_R = \mathbf{W}_r \mathbf{H}_{RR} \mathbf{x}_R + \mathbf{W}_r \mathbf{n}_R$ . Due to the presence of the residual interference  $\mathbf{W}_r \mathbf{H}_{RR} \mathbf{x}_R$ ,  $\mathbf{n}'_R$  may be viewed as a colored noise vector with the

covariance matrix

$$\begin{aligned} \mathbf{R}_{n'_R} &= \mathcal{E} \left( (\mathbf{n}'_R)^H \mathbf{n}'_R \right) \\ &= P_S \beta^2 N_S^{-1} \mathbf{W}_r \mathbf{H}_{RR} \mathbf{W}_r \mathbf{W}_r^H \mathbf{H}_{RR}^H \mathbf{W}_r + \sigma_n^2 \mathbf{W}_r \mathbf{W}_r^H \end{aligned} \quad (16)$$

### 4.1 Conventional ML detector

For the conventional ML detection in a white Gaussian channel, the detecting problem of source transmitted signal at the relay  $\mathbf{x}_S$  can be casted as

$$\hat{\mathbf{x}}_{S,ML} = \arg \min_{\mathbf{x}_S \in \mathcal{C}_{M_{QAM}}^{N_S}} \|\mathbf{y}_R - \mathbf{H}'_{SR} \mathbf{x}_S\|^2 \quad (17)$$

which has an exponential complexity in the number of the source transmit antennas  $N_s$ , i.e.,  $O(M^{N_s})$  float point operations (FLOPs), where  $M$  is the number of points in the signal constellation. Nevertheless, the high-complexity detector cannot even achieve an optimal detection performance, due to the colored interference-plus-noise in (15). In the following, we design an optimal detector with a reduced complexity.

### 4.2 Proposed whitening-filter-based ML detector

From (15), the vector  $\mathbf{n}'_R$  of interference-plus-noise is colored and can be converted to a vector of white Gaussian noise vector. By making use of the covariance matrix in (16), we construct the following whitening filter (WF) as follows

$$\mathbf{W}_{WF} = (\mathbf{Q} \mathbf{\Lambda} \frac{1}{2})^{-1} \quad (18)$$

where the unitary matrix  $\mathbf{Q}$  and the diagonal matrix  $\mathbf{\Lambda}$  are computed by the following eigenvalue decomposition as

$$\mathbf{R}_{n'_R} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^H \quad (19)$$

since  $\mathbf{R}_{n'_R}$  is a positive definite Hermitian matrix in accordance with (16). Multiplying both sides of (15) by  $\mathbf{W}_{WF}$ , we obtain the following equation with a white complex Gaussian noise vector

$$\mathbf{y}'_R = \mathbf{W}_{WF} \mathbf{y}_R = \mathbf{W}_{WF} \mathbf{H}'_{SR} \mathbf{x}_S + \mathbf{W}_{WF} \mathbf{n}'_R \quad (20)$$

with

$$\mathcal{E} \left( \mathbf{W}_{WF} \mathbf{n}'_R (\mathbf{n}'_R)^H \mathbf{W}_{WF}^H \right) = \mathbf{I}_{N_s} \quad (21)$$

Based on the new model in (20), we get the optimal ML detector, namely, whitening filter ML (WFML), as follows



$$\widehat{\mathbf{x}}_{S,WFML} = \arg \min_{\mathbf{x}_S \in C_{MQAM}^{N_S}} \|\mathbf{y}'_R - \mathbf{W}_{WF} \mathbf{H}'_{SR} \mathbf{x}_S\|^2 \quad (22)$$

where  $C_{MQAM}^{N_S}$  is a  $N_S$ -dimensional space with each dimension being chosen from the set of MQAM constellation points.

**Theorem 1:** The conventional detector in Eq. (17) is equivalent to our WFML detector in Eq. (22), under the condition  $\mathbf{W}_{WF}^H \mathbf{W}_{WF} = a \mathbf{I}_{N_S}$ , where  $a$  is a positive value. Additionally, this condition approximately holds for the large-scale MIMO cases.

**Proof:** Please see **Appendix B**.

Although the proposed WFML detector in (22) has the optimal detection performance, it still has an exponential complexity of  $O(M^{N_S})$  FLOPs. In what follows, we will present an equivalent low-complexity form of our WFML detector from the beamforming viewpoint.

### 4.3 Proposed low-complexity WFML detector

Firstly, the matrix  $\mathbf{W}_{WF} \mathbf{H}'_{SR}$  in (20) can be decomposed by SVD as follows

$$\mathbf{W}_{WF} \mathbf{H}'_{SR} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H \quad (23)$$

where  $\mathbf{U}$ ,  $\mathbf{\Sigma}$ , and  $\mathbf{V}^H$  are the results of the SVD decomposition. We use  $\mathbf{V}$  as the precoding matrix of the source, and  $\mathbf{U}^H$  as the receive beamforming at the relay. After applying precoding and receive beamforming, Eq. (20) can be converted to

$$\mathbf{y}''_R = \mathbf{U}^H \mathbf{W}_{WF} \mathbf{H}'_{SR} \mathbf{V} \mathbf{x}_S + \mathbf{U}^H \mathbf{W}_{WF} \mathbf{n}'_R = \mathbf{\Sigma} \mathbf{x}_S + \mathbf{n}''_R \quad (24)$$

Considering that  $\mathbf{U}^H$  is an unitary matrix, based on Eq. (21), all components of the noise vector  $\mathbf{n}''_R$  are still mutually independent with matrix  $\mathbf{I}_{N_S}$ . This means that the above process of WF-plus-SVD converts  $N_S$  coupled spatial subchannels in (20) into  $N_S$  independent parallel spatial subchannels in (24). Then, each substream can be individually detected or decoded. In a mathematical language, since both  $\mathbf{U}^H$  and  $\mathbf{V}$  are unitary matrices, the optimization problem in (22) can be formulated as the following problem

$$\widehat{\mathbf{x}}_{S,WF-SVD} = \arg \min_{\mathbf{x}_S \in C_{MQAM}^{N_S}} \|\mathbf{y}''_R - \mathbf{\Sigma} \mathbf{x}_S\|^2 \quad (25)$$

where  $\lambda_i$  is the  $(i, i)$  diagonal element of matrix  $\mathbf{\Sigma}$ , and  $\mathbf{y}''_R(i)$  and  $\mathbf{x}_S(i)$  are the  $i$ th element of

$\mathbf{y}''_R$  and  $\mathbf{x}_S$ , respectively. The above problem is divided into  $N_S$  sub-optimization problems

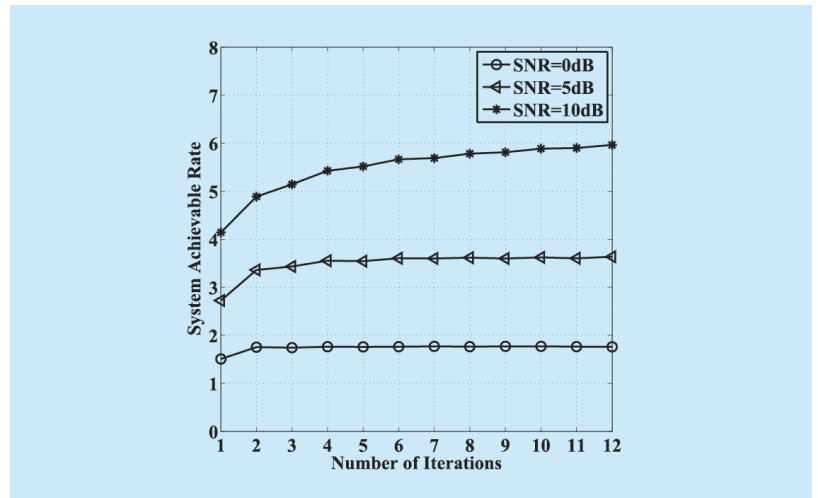
$$\widehat{\mathbf{x}}_{S,WF-SVD}(i) = \arg \min_{\mathbf{x}_S(i) \in C_{MQAM}} \|\mathbf{y}''_R(i) - \lambda_i \mathbf{x}_S(i)\|^2 \quad (26)$$

for  $\forall i \in \{1, 2, \dots, N_S\}$ . In terms of the above simplification process, the complexity of the ML detector using exhaustive search is significantly reduced from  $O(M^{N_S})$  FLOPs to  $O(N_S^3 + MN_S)$  FLOPs, where  $O(N_S^3)$  comes from the SVD decomposition and  $O(MN_S)$  is the detection complexity. With the increase of  $N_S$ , the complexity reduction becomes more attractive due to the use of WF-plus-SVD.

## V. SIMULATIONS

In this section, we evaluate the performance of the proposed schemes and compare them with the conventional ones. The system parameters in our simulation are set as follows:  $N_{RT} = 4$ ,  $N_{RR} = 4$ , and  $N_S = N_D = 2$ . All the channels are modeled by Rayleigh block-fading.

To observe the convergence of the proposed IBS in Section III, Fig. 3 demonstrates the curves of achievable rates  $R$  versus the number of iterations. We consider three different SNR values, i.e., SNR = 0 dB, 5dB, and 10dB. For all the three SNR values, the proposed IBS converges rapidly, i.e., in about four iterations. With the increment of SNR, the required number of iterations will also become larger.



**Fig.3** The system achievable rate  $R$  versus the iteration numbers for the proposed IBS with different SNR values

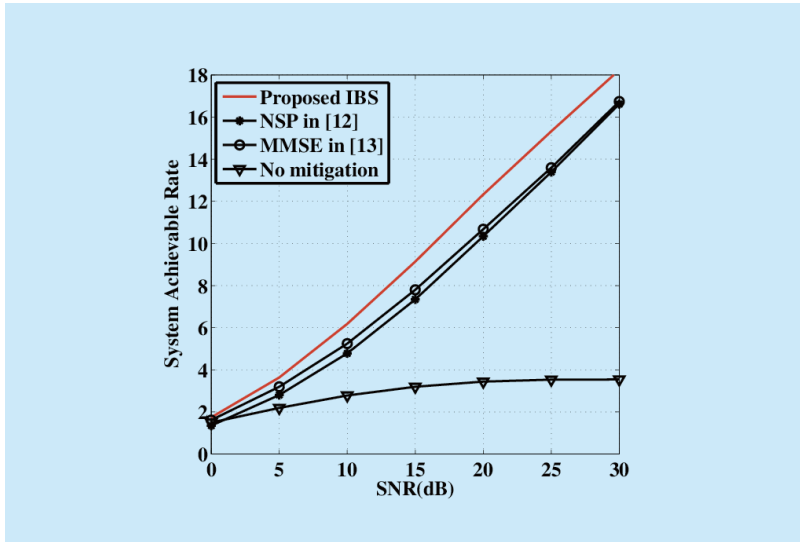


Fig.4 Comparisons of the system achievable rate under different beamforming schemes

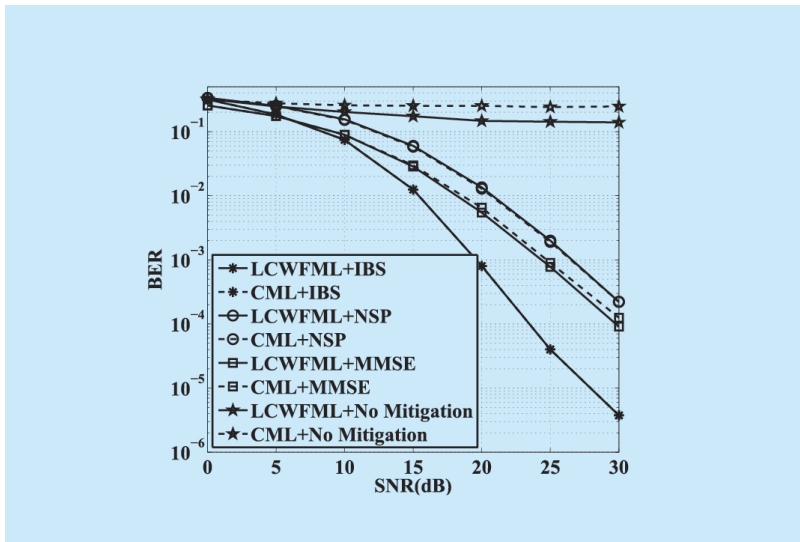


Fig.5 BER performance under different combination schemes for 16QAM

Fig.4 compares the achievable rate  $R$  of the proposed IBS with the ones of two other beamforming schemes and the case of no mitigation. The first benchmark is the null-space projection (NSP) scheme in [12], where the self-interference at the relay is forced to zero via the null-space projection. The second one is the MMSE scheme in [13], where the receive beamforming  $\mathbf{W}_r$  at the relay is designed based on the MMSE criterion. Additionally,

we consider the scheme without the mitigation of interference, i.e., where both  $\mathbf{W}_t$  and  $\mathbf{W}_r$  are identity matrices. It can be observed that the proposed IBS performs much better than the other three schemes in all SNR regions. And the curve of no mitigation tends to be smoothly convergent with a lowest system achievable rate. It has about a 2.5dB gain compared with the second best scheme, i.e., MMSE for the given achievable rate above 15bits/bit/Hz. The main reason for its superiority is that the proposed IBS fully takes both the self-interference and noise into account. Suppose the proposed IBS requires  $I$  iterations, then the complexity of the proposed IBS is  $I$  times those of NSP and MMSE considering the three schemes use matrix decomposition to compute the beamforming coefficients ( $O(N_{RR}^2 N_s)$  FLOPs per matrix decomposition).

We further evaluate the bit error rate (BER) performance of our low-complexity WFML detector proposed in Section IV, namely, LCWFML. We compare our LCWFML with the conventional ML detector for Gaussian channels, namely, CML. We will investigate the BER performance for these two detectors combined with the four aforementioned beamforming schemes at the relay. As such, we have the following combination schemes: LCWFML+IBS, CML+IBS, LCWFML+NSP, CML+NSP, LCWFML+MMSE, CML+MMSE, LCWFML+No Mitigation, and CML+No Mitigation.

Fig.5 and Fig. 6 show the BER performance versus SNR for different combination schemes for 16QAM and 64QAM, respectively. From the two figures, it is obvious that the BER performance of the proposed LCWFML+IBS as well as CML+IBS is much better than the other schemes. For a given detector, the proposed IBS is better than MMSE, NSP, and No Mitigation. In particular, the IBS has a 45 dB gain at the  $\text{BER} = 10^{-2}$  for 16QAM and 4dB gain at the  $\text{BER} 10^{-3}$  for 16QAM, compared with the second best scheme MMSE. Additionally, for a given beamforming scheme, e.g., IBS, MMSE, and NSP, we find that the BER curves of the proposed LCWFML detector are slight-

ly better than those of the CML. On the other hand, in the case of No Mitigation, the BER performance of our LCWFML detector is better than that of the CML with an obvious gain. The reason behind this is as follows. When using IBS, MMSE, or NSP at the relay, the residual self-interference is very small compared to the useful signal and can be omitted. For No Mitigation, the self-interference at the relay is much stronger and colored, and thus the whitening filter enables that the LCWFML detector in (22) outperforms the CML detector.

## VI. CONCLUSION

In this paper, we proposed a high-rate beamforming design and a low-complexity ML detector at the relay in a DF-based full-duplex MIMO relay network. The proposed IBS achieves a high-performance rate by alternately optimizing the transmit and receive beamforming. Additionally, a low-complexity ML detector combining the WF and SVD was designed. From our simulations, we found that the proposed IBS converges within only four iterations, and performs much better than the existing beamforming schemes such as NSP and MMSE in terms of both achievable rate and BER performance. The IBS shows about 5dB gain over the MMSE at BER=  $10^{-3}$  for 16QAM. Additionally, for a given beamforming scheme, the BER performance of the proposed detector is better than that of the conventional with a lower-complexity.

### Appendix A: Derivation of the optimal solution of Max-SINR

For the convenience of proof below, we first consider the case  $N_s = 1$ . Given a fixed  $\mathbf{W}_r$ , maximizing  $\text{SINR}_R(\mathbf{W}_r, \mathbf{W}_t)$  over  $\mathbf{W}_t$  in (9) can be formulated as

$$\text{maximize } \frac{\text{tr}(\mathbf{w}\mathbf{A}\mathbf{w}^H)}{\text{tr}(\mathbf{w}\mathbf{B}\mathbf{w}^H)} \quad (27)$$

$$\text{subject to } \mathbf{w}\mathbf{w}^H = 1$$

where  $\mathbf{w}$  is a row vector, and

$$\begin{aligned} \mathbf{w} &= \mathbf{W}_r \mathbf{A} = \mathbf{H}_{SR} \mathbf{H}_{SR}^H \\ \mathbf{B} &= \sigma_R^2 \mathbf{I}_{N_{SR}} + P_S \beta^2 N_S^{-1} \mathbf{H}_{RR} \mathbf{W}_t \mathbf{W}_t^H \mathbf{H}_{RR}^H \end{aligned} \quad (28)$$

Although we can view the optimization problem in (27) as a generalized Rayleigh-

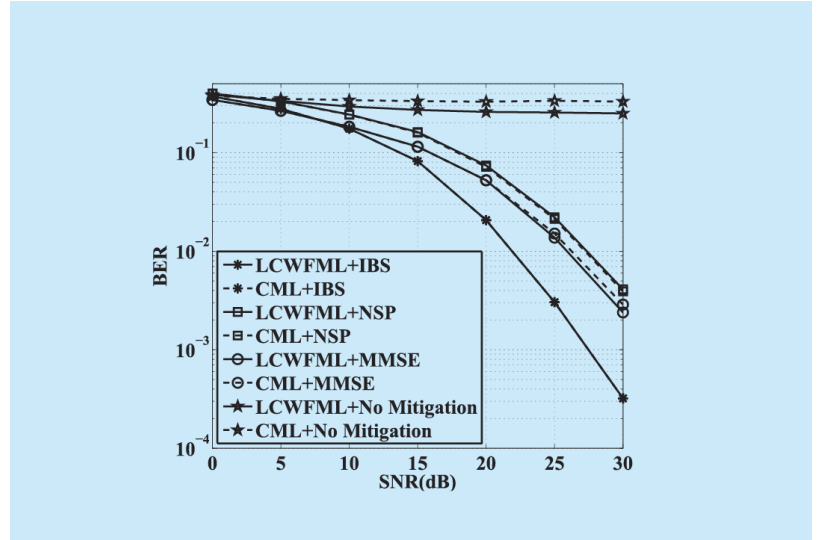


Fig.6 BER performance under different combination schemes for 64QAM

Ritz ratio and directly give the optimal solution similar to the Max-SLNR in [16]. A strict proof need to be presented to guarantee that the optimal solution holds. Thus, in what follows, we will propose a detailed rigorous mathematical derivation for the optimal solution to the maximization problem in (27).

Considering that scaling  $\mathbf{w}$  by any scalar does not change the value of the objective function of the problem (27), the problem (27) may be simplified to the following unconstrained maximization problem

$$\text{maximize } \frac{\text{tr}(\mathbf{w}\mathbf{A}\mathbf{w}^H)}{\text{tr}(\mathbf{w}\mathbf{B}\mathbf{w}^H)} \quad (29)$$

To solve the above problem, let us first define  $\lambda = \frac{\text{tr}(\mathbf{w}\mathbf{A}\mathbf{w}^H)}{\text{tr}(\mathbf{w}\mathbf{B}\mathbf{w}^H)}$ . Then, the optimization problem (29) can be formulated as

$$\begin{aligned} &\text{maximize } \lambda \\ &\text{subject to } \text{tr}(\mathbf{w}\mathbf{A}\mathbf{w}^H) - \lambda \text{tr}(\mathbf{w}\mathbf{B}\mathbf{w}^H) = 0 \end{aligned} \quad (30)$$

which can be explained as the problem of finding the largest one from all  $\lambda$ 's satisfying the following constraint

$$f(\lambda, \mathbf{w}^H) = \text{tr}(\mathbf{w}\mathbf{A}\mathbf{w}^H) - \lambda \text{tr}(\mathbf{w}\mathbf{B}\mathbf{w}^H) = 0 \quad (31)$$

which implies that the partial derivative of the function  $f(\lambda, \mathbf{w}^H)$  with respect to  $\mathbf{w}$  satisfies

$$\frac{\partial f(\lambda, \mathbf{w}^H)}{\partial \mathbf{w}^H} = 2\mathbf{A}\mathbf{w}^H - 2\lambda\mathbf{B}\mathbf{w}^H = 0 \quad (32)$$

This means that the problem of finding the optimal  $\lambda$  and  $\mathbf{w}$  can be found by solving the following equation



$\mathbf{A}\mathbf{w}^H = \lambda\mathbf{B}\mathbf{w}^H \Rightarrow \mathbf{B}^{-1}\mathbf{A}\mathbf{w}^H = \lambda\mathbf{w}^H$  (33)  
since  $\mathbf{B}$  is full rank in terms of (28). The above equation is just the definition of the eigenvalue-eigenvector equation. Obviously, when the optimal  $\mathbf{w}^H$  is the normalized eigenvector associated with the largest eigenvalue of the matrix  $\mathbf{B}^{-1}\mathbf{A}$ , which is a unique optimal feasible solution to the problem in (27) since  $\mathbf{w}\mathbf{w}^H = 1$ ,  $\lambda$  is maximized and equal to the largest eigenvalue of the matrix  $\mathbf{B}^{-1}\mathbf{A}$ . Using this idea, we can extend the case  $N_s=1$  to any scenario with  $N_s \geq 1$ .

#### Appendix B: Proof of Theorem 1

For the convenience of the proof below, let us first define  $\Delta\mathbf{y}'_R = \mathbf{y}_R - \mathbf{H}'_{SR}\mathbf{x}_S$ . Then we have

$$\begin{aligned} & \|\mathbf{y}'_R - \mathbf{W}_{WF}\mathbf{H}'_{SR}\mathbf{x}_S\|^2 \\ &= \text{tr}\left\{(\Delta\mathbf{y}'_R)^H\mathbf{W}_{WF}^H\mathbf{W}_{WF}\Delta\mathbf{y}'_R\right\} \\ &= \text{tr}\left\{\Delta\mathbf{y}'_R(\Delta\mathbf{y}'_R)^H\mathbf{W}_{WF}^H\mathbf{W}_{WF}\right\} \end{aligned} \quad (34)$$

Obviously, when  $\mathbf{W}_{WF}^H\mathbf{W}_{WF} = \alpha\mathbf{I}_{N_s}$ , where  $\alpha$  is any positive value

$$\begin{aligned} & \|\mathbf{y}'_R - \mathbf{W}_{WF}\mathbf{H}'_{SR}\mathbf{x}_S\|^2 = \alpha\text{tr}\left\{\Delta\mathbf{y}'_R(\Delta\mathbf{y}'_R)^H\right\} \\ &= \alpha\|\mathbf{y}_R - \mathbf{H}'_{SR}\mathbf{x}_S\|^2, \end{aligned} \quad (35)$$

which means that the optimization problem in (22) achieves the same optimal value as the optimization problem in (17). This is only a sufficient condition for the equivalence between (17) and (22).

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