Contract-based Trading on Parallel Computing Resources for Cellular Networks with Virtualized Base Stations

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***Abstract*—As a promising wireless network virtualization technology, virtualized base station (BS) has been proposed to tackle the problem of low-efficient utilization of BS's computing resources, e.g., baseband processing units (BPU). In this paper, we design a novel scheme to achieve the efficient BPU allocation based on a contract-theoretic approach. To achieve this, we consider the BPUs as a kind of trading resources. We establish a monopoly market, where the infrastructure provider (InP) is the monopolist owning all the BPUs, and multiple mobile network operators (MNOs) intend to rent BPUs from the InP for processing their baseband signals. In such a market, the InP offers a set of quantity-price contract items to the MNOs based on statistical information of their types, and at the same time, the MNOs are stimulated to accept the offers for the purpose of making profit. We propose the optimal contract design to maximize the InP's profit, as well as develop an incentive mechanism to guarantee each MNO choosing a proper contract item. Numerical results validate the effectiveness of our incentive mechanism for BPU resource allocation.**

***Index Terms*—Virtualized base station, cellular networks, contract theory, resource trading, incentive mechanism**

I. Introduction

With the rapid proliferation of mobile devices, the amount of cellular traffic has dramatically increased. Conventional solutions, such as increasing the density of base station deployment, inevitably incur lower efficiency of resource utilization. To tackle this issue, wireless network virtualization technology has been proposed as a promising approach [1], [2]. The basic idea of wireless network virtualization is to regard the wireless infrastructures as common resources which can be shared by various mobile network operators (MNO).

The wireless infrastructure sharing is a new research topic that has attracted lots of attentions recently, and virtualized base station (VBS) technology is a typical example of it [3]–[5]. In conventional cellular networks, baseband processing units (BPU) resources of base stations are separated. By contrast, in the VBS, BPU resources are congregated in a BPU pool, and then shared by multiple VBSs, thereby providing an efficient usage of these resources [6]. There are many studies conducted on VBS design. In [6], the authors evaluated the statistical multiplexing gain of BPU resources based on a multi-dimensional Markov model. In [7], the authors studied the BPU allocation in a VBS system to minimize the energy consumption, and the problem was solved by using integer linear programming.

While the above works focus on system performance or energy consumption optimization, none of them have studied the VBS design from an economic aspect. Inspired by this, in this paper, we optimize BPU allocations in the context of a market trading process. In such a market, the owner of the BPUs, i.e., the infrastructure provider (InP), leases its resources to the MNOs to optimize its profit, where contract theory can be adopted for optimizing the trading process.

Contract theory is an efficient economic tool to design contract items and incentive mechanisms between monopolist and consumers in monopoly markets. It has been introduced to the wireless communications for resource allocations. In [8], the authors utilized contract theory to design an interference coordination framework for mitigating the interference among remote radio heads (RRHs).

In this paper, we pay our attention to a BPU trading market, where the InP owns the BPU resources and leases them to multiple MNOs. We classify these MNOs into multiple types according to the number of their RRHs, while the InP only knows the statistical information about MNOs' types. Under this assumption, we aim at proposing a contract-theoretic incentive mechanism for helping the InP achieve its best interest. The main contributions of this article are listed as follows. 1) We introduce contract theory into the computing resource trading between InP and MNOs for the VBS architecture; 2) We design the contract to maximize the InP's profit as well as to motivate MNOs to participate in the trading; 3) We propose algorithms for solving the optimal contract.

The rest of this paper is organized as follows. In Section II, we present the system model. We formulate the contract and derive the optimal contract for continuous-MNO-type model in Section III. In Section IV, we present numerical results.

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Finally, we draw our conclusions in Section V.

II. System Model

In this section, the details of BPU resource trading of the VBS architecture are illustrated. We first describe the VBS service model in general. Then we formulate some needed functions of the InP and MNOs in the trading process.

A. Service Model of VBS

We consider there is a BPU pool that is composed of a number of BPUs, and the BPU trading is between one InP (leaser) and multiple MNOs (leasees). The MNOs provide wireless coverage for mobile users with their RRHs, while they are not equipped with the BPUs. Thus, they have to obtain BPUs from the InP to conduct baseband signal processing. The quantity demands of the BPUs vary among the MNOs according to the scale of their RRHs deployed. Thus, we assume first that the MNOs have multiple types according to their RRH scales. Specifically, the type of an MNO is defined by $\theta \in \mathbf{\Theta}$, where θ represents the number of the RRH, and $\mathbf{\Theta}$ is the set of all the MNO types. Then, the fraction of the BPU resources that an MNO rents is based on this MNO's type, which is denoted by $p(\theta) \in [0,1]$. We further define $\varepsilon(p(\theta))$ as the price for renting the BPU resource quota $p(\theta)$. For simplification, we use $\varepsilon(\theta)$ instead of $\varepsilon(p(\theta))$ in the following derivations. The InP provides a set of contract items $\big(p(\theta), \varepsilon(\theta)\big)$ to the MNOs, while each MNO decides whether to accept it. Our contract-based framework motives each MNO to rent a certain fraction of the BPUs.

B. InP Model

Once a fraction of BPUs are assigned to a designated MNO, the InP needs to pay for the energy consumed by the BPUs. To capture the cost of the InP, we need to build the power consumption model. According to [9], the power consumption of a BPU can be modeled as $c = P_{Bm} + \frac{P_{BM}-P_{Bm}}{s_0^{\beta}}\phi s^{\beta}$, where P_{BM} and P_{Bm} are the maximum and minimum power consumption of each BPU, ϕ is the BPU's load, s is the BPU's speed, s_0 is the reference BPU's speed, and β is the exponential coefficient of the BPU's speed. The total power consumption is linear with the proportion of rented BPUs, which can be expressed as $t\left(p(\theta)\right) = cp(\theta)N$, where N is the overall number of BPUs in the BPU pool.

Inspired by the price model adopted in [10], the cost of the consumed power can be quantified by using the polynomial function $C\left(p(\theta)\right) = a\big(t\left(p\left(\theta\right)\right)\big)^{b}$, where a, b are constants, with $a > 0$ and $b \geq 2$.

For the InP, the utility (or revenue) of leasing the BPU resources with the fraction $p(\theta)$ to an MNO of type θ, denoted by $U\left(\theta, p\right)$, can be defined as the difference between the rental income and the power consumption cost of the resources, i.e.,

$$U\left(\theta, p\right) = \varepsilon\left(\theta\right) - C\left(p(\theta)\right). \tag{1}$$

Obviously, a rational InP will not accept a negative utility from the contract, thus it will always set $\varepsilon(\theta) > C(p(\theta)),\ \forall p(\theta) \in [0,1]$.

C. MNO Model

In this paper, we assume that the MNOs will conduct parallel tasks on the rented BPUs in order to make efficient usages of BPUs and achieve high performance. Consequently, the performance gain will be affected by the ratio of the program that cannot be processed parallelly. To capture this effect, we adopt Amdahl's law to depict the maximum improvement achieved by applying parallel computing. Therefore, the profit gained from this improvement is formulated as $\Gamma(\theta, p) = \frac{\kappa}{\eta + \frac{1-\eta}{p(\theta)N}}$, where η indicates the ratio of task that can only be processed serially, and κ denotes the profit gained from a unit improvement due to parallel computing. Without loss of generality, we suppose $\kappa = 1$. Then the profit that a MNO can obtain from renting the BPU resources can be calculated as $V\left(\theta, p\right) = \theta\Gamma(\theta, p)$.

In the following sections, our objective is to implement the contract-theoretic model and maximize the InP's utility.

III. Contract for Continuous-MNO-Type

In this section, we investigate the optimal contract design for the continuous-MNO-type model, which is a general model. Our main objective is to optimize a set of quantity-price contract items $\big(p(\theta), \varepsilon(\theta)\big)$ to maximize the InP's profit.

A. Contract Formulation

We assume that the InP does not know the exact MNO type θ and only has the knowledge of its distribution, which is determined by the probability density function, denoted by $f(\theta)$, on an interval $[\theta_l, \theta_u]$. Then, based on (1), the expected profit of the InP in continuous-MNO-type model can be written as

$$U_C = \int_{\theta_l}^{\theta_u} \big(\varepsilon(\theta) - C\left(p(\theta)\right)\big) f(\theta) \mathrm{d}\theta. \tag{2}$$

The incentive for a MNO to rent BPU resources to conduct its task is that its utility is nonnegative. The utility of a type-θ MNO renting BPU resources with the proportion $p(\theta)$ can be derived by using the MNO's benefit obtained from BPU renting minus the BPU's price, i.e., $V\left(\theta, p\right) - \varepsilon\left(\theta\right)$. Thus, the *individual rationality* (IR) constraint can be expressed as

$$V\left(\theta, p\left(\theta\right)\right) - \varepsilon\left(\theta\right) \geq 0. \tag{3}$$

Meanwhile, the MNO should find its best interest through renting the BPU resources assigned to its own type, namely, *incentive compatible* (IC) constraint, which is given by

$$V\left(\theta, p\left(\theta\right)\right) - \varepsilon\left(\theta\right) \geq V\left(\theta, p\left(\theta'\right)\right) - \varepsilon\left(\theta'\right), \tag{4}$$

where $\theta' \in \mathbf{\Theta}$ and $\theta' \neq \theta$.

The IR and IC constraints are the basic conditions to ensure the feasibility of a contract. Under these conditions, the optimal contract can be formulated as InP's profit maximization problem, i.e.,

$$\begin{aligned} \max_{p(\theta),\varepsilon(\theta)} \quad & U_C \\ \text{s.t.} \quad & V\left(\theta, p\right) - \varepsilon(\theta) \geq 0, \\ & \theta\Gamma(\theta) - \varepsilon(\theta) \geq \theta\Gamma(\theta') - \varepsilon(\theta'), \theta, \theta^{'} \in [\theta_l, \theta_u], \\ & \int_{\theta_l}^{\theta_u} p(\theta)\mathrm{d}\theta \leq 1, \quad 0 \leq p(\theta) \leq 1. \end{aligned} \tag{5}$$

The profit maximization problem in (5) is nontrivial to solve, since it involves the optimization over a schedule $(p(\theta), \varepsilon(\theta))$ under the constraints, where other conflicting optimization problems are involved in themselves.

In the following, in order to solve problem (5), we need to simplify the IC and IR constraints.

B. Contract Optimization

Lemma 1: As for the optimal contract under the asymmetric information scenario, the IR constraint can be replaced by $V(\theta_l, p) - \varepsilon(\theta_l) \geq 0$, given that the IC constraint holds.

Lemma 2: If the MNO's utility function satisfies $\frac{\partial}{\partial\theta}\left[-\frac{\partial V/\partial p}{\partial V/\partial \varepsilon}\right] > 0$, then the IC constraint is equivalent to the following two constraints:

Monotonicity: $\frac{\partial p(\theta)}{\partial \theta} \geq 0$,

Local Incentive Compatibility:

$$\frac{\theta(1-\eta)N}{(\eta p(\theta)N + 1 - \eta)^2}\frac{\partial p(\theta)}{\partial \theta} = \frac{\partial \varepsilon(\theta)}{\partial \theta}. \tag{6}$$

Based on the above two lemmas, the InP's profit maximization problem (5) can be rewritten as

$$\begin{aligned} \max_{p(\theta),\varepsilon(\theta),} \quad & U_C \\ \text{s.t.} \quad & V(\theta_l, p) - \varepsilon(\theta_l) \geq 0, \quad \frac{\partial p(\theta)}{\partial \theta} \geq 0, \\ & \frac{\theta(1-\eta)N}{(\eta p(\theta)N + 1 - \eta)^2}\frac{\partial p(\theta)}{\partial \theta} = \frac{\partial \varepsilon(\theta)}{\partial \theta}, \\ & \int_{\theta_l}^{\theta_u} p(\theta)\mathrm{d}\theta \leq 1, \quad 0 \leq p(\theta) \leq 1. \end{aligned} \tag{7}$$

To solve this problem, we first define

$$W(\theta) \triangleq V(\theta, p) - \varepsilon(\theta) = \max_{\hat{\theta}}(\theta\Gamma(\hat{\theta}) - \varepsilon(\hat{\theta})). \tag{8}$$

By applying the envelope theorem, we can obtain

$$\frac{\mathrm{d}W(\theta)}{\mathrm{d}\theta} = \left.\frac{\partial W(\theta)}{\partial \theta}\right|_{\hat{\theta}=\theta} = \frac{1}{\eta + \frac{1-\eta}{p(\theta)N}} = \Gamma(\theta). \tag{9}$$

Integrating both sides of (9), we further have

$$W(\theta) = \int_{\theta_l}^{\theta} \Gamma(\theta)\mathrm{d}\theta + W(\theta_l). \tag{10}$$

When the optimal contract holds, the IR constraint of the lowest type is binding, and thus we obtain $W(\theta_l) = 0$. Therefore, (10) can be rewritten as

$$W(\theta) = \int_{\theta_l}^{\theta} \Gamma(x)\mathrm{d}x. \tag{11}$$

Based on (8) and (11), we can calculate the price of the BPU resource as

$$\varepsilon(\theta) = V(\theta, p) - \int_{\theta_l}^{\theta} \Gamma(x)\mathrm{d}x. \tag{12}$$

Then by substituting (12) into (2), we have

$$\begin{aligned} U_C = & \int_{\theta_l}^{\theta_u} \big(V(\theta, p) - C(p(\theta))\big) f(\theta)\mathrm{d}\theta \\ & - \int_{\theta_l}^{\theta_u}\int_{\theta_l}^{\theta} \Gamma(x) f(\theta)\mathrm{d}x\mathrm{d}\theta. \end{aligned} \tag{13}$$

Integrating the last term of (13) by parts leads to

$$\int_{\theta_l}^{\theta_u}\int_{\theta_l}^{\theta} \Gamma(x) f(\theta)\mathrm{d}x\mathrm{d}\theta = \int_{\theta_l}^{\theta_u} \Gamma(\theta)(1 - F(\theta))\mathrm{d}\theta, \tag{14}$$

where $F(\theta)$ is the cumulate distribution function of type θ. The last line of (14) follows due to $F(\theta_l) = 0$ and $F(\theta_u) = 1$. Substituting (14) into (13), we have

$$U_C = \int_{\theta_l}^{\theta_u} \bar{U}_C \mathrm{d}\theta, \tag{15}$$

where $\bar{U}_C \triangleq \left(V(\theta, p) - C(p(\theta)) - \Gamma(\theta)\frac{(1-F(\theta))}{f(\theta)}\right) f(\theta)$. To maximize U_C, we can maximize the term $\bar{U}_C$ with respect to the function $p(\cdot)$. Then, the relaxed problem of (15) can be represented as $\max_{p(\theta)} \bar{U}_C$, and the relaxed InP's profit maximization problem is given by

$$\begin{aligned} \max_{p(\theta)} \quad & \bar{U}_C \\ \text{s.t.} \quad & \frac{\mathrm{d}p(\theta)}{\mathrm{d}\theta} \geq 0, \quad \int_{\theta_l}^{\theta_u} p(\theta)\mathrm{d}\theta \leq 1, \\ & 0 \leq p(\theta) \leq 1, \quad \theta, \theta^{'} \in [\theta_l, \theta_u]. \end{aligned} \tag{16}$$

To solve the above maximization problem, we first find the optimal solution for its object function, and then we check whether the solution satisfies the constrains.

According to Fermat's theorem, if any local optimal solution of problem (16), denoted by $\hat{p}(\theta)$, exists, the following condition is satisfied, i.e.,

$$\left.\frac{\partial \bar{U}_C}{\partial p}\right|_{p(\theta)=\hat{p}(\theta)} = 0. \tag{17}$$

Furthermore, we calculate the second order condition of $\bar{U}_C$, and find that $\partial^2\bar{U}_C/\partial p^2 \leq 0$ holds all the time, which indicates that the local optimal solution $\hat{p}(\theta)$ to the problem (16) is unique and is actually the global optimal solution.

After obtaining the optimal solution based on (17), we need to check whether it satisfies the monotonicity constraint. Refer to [11], we adopt the "Bunching and Ironing" algorithm. To facilitate the understanding of the essential part of this algorithm, we present the following lemma.

Lemma 3: Assume $P(p(x))$ be concave functions, $\hat{p}(x) = \arg\max_{p(x)} P(p(x))$ and $\mathrm{d}p(x)/\mathrm{d}x \geq 0$. If $\mathrm{d}\hat{p}(x)/\mathrm{d}x \leq 0$, then there exists a $x \in [\underline{x}, \overline{x}]$, which satisfies

$$\hat{p}(x) = \hat{p}(\underline{x}) = \hat{p}(\overline{x}) = \arg\max_{p(x)} \int_{\underline{x}}^{\overline{x}} P(p(x)). \tag{18}$$

We first suppose that $\hat{p}(x)$ is the infeasible solution to problem (7), which violates the monotonicity constraint. The corresponding infeasible region is $[\underline{x}, \overline{x}] \subseteq \mathbf{\Theta}$, which satisfies $\mathrm{d}p(x)/\mathrm{d}x \geq 0$, $\forall x \in [\underline{x}, \overline{x}]$. According to *Lemma 3*, the infeasible region can be "ironed" to be feasible using (18).

The constraint $\int_{\theta_l}^{\theta_u} p(\theta)\mathrm{d}\theta \leq 1$ reflects the fact that the InP may not be able to satisfy all requirements from the MNOs due to the limited BPU resources. In order to meet this constraint while maximizing InP's utility, one strategy for the InP is to selectively serve the MNOs with higher types. Following this, we define the type θ_c as a threshold type, which holds the conditions $\int_{\theta_c}^{\theta_u} p(\theta)\mathrm{d}\theta > 1$ and $\int_{\theta_c+\delta}^{\theta_u} p(\theta)\mathrm{d}\theta \leq 1$, where δ is a small positive "change in". The InP will set the BPU ratio $p(\theta) = 0$ for the MNOs below threshold type. Mathematically, this process can be expressed as

$$\hat{p}(\theta) = \begin{cases} 0 & \forall\theta \le \theta_c, \\ \hat{p}(\theta) & \forall\theta > \theta_c. \end{cases} \tag{19}$$

Substituting (19) into (12), we can obtain the optimal price for the BPU resources as

$$\hat{\varepsilon}(\theta) = \frac{\theta}{\eta + \frac{1-\eta}{\hat{p}(\theta)N}} - \int_{\theta_l}^{\theta} \frac{1}{\eta + \frac{1-\eta}{\hat{p}(x)N}} \mathrm{d}x. \tag{20}$$

We conclude the detailed algorithm as shown in the following algorithm.

Algorithm 1:

initialize $\hat{p}(\theta) = \arg\max_{p(\theta)} \bar{U}_C, \forall\theta \subseteq \boldsymbol{\Theta}$
while $\hat{p}(\theta)$ is not feasible **do**
    allocate an infeasible region $[\underline{\theta}, \overline{\theta}] \subseteq \boldsymbol{\Theta}$
    set $\hat{p}(\theta) = \hat{p}(\underline{\theta}) = \hat{p}(\overline{\theta}) = \arg\max_{p(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \bar{U}_C(p(\theta))$
end while
search for critical type θ_c
set $\hat{p}(\theta) = \begin{cases} 0 & \forall\theta \le \theta_c \\ \hat{p}(\theta) & \forall\theta > \theta_c \end{cases}$
for $\theta \subseteq \boldsymbol{\Theta}$ **do**
    set $\hat{\varepsilon}(\theta) = \frac{\theta}{\eta + \frac{1-\eta}{\hat{p}(\theta)N}} - \int_{\theta_l}^{\theta} \frac{1}{\eta + \frac{1-\eta}{\hat{p}(x)N}} \mathrm{d}x$
end for
    set $U_C = \int_{\theta_l}^{\theta_u} \left(\hat{\varepsilon}(\theta) - C(\hat{p}(\theta))\right) f(\theta) \mathrm{d}\theta$

IV. Numerical Results

In this section, we present the numerical results for the proposed contract design. For simplicity, we present the numerical results for the discrete-MNO-type distributions. We first verify the feasibility and monotonicity property of our proposed contract. Then we analyze the optimal contract through various parameters. The effectiveness of our designed contract is verified via numerical results. Unless otherwise stated in the following, related parameters are shown in Table I.

TABLE I
Related Parameters in Numerical Results

Total BPU number within the pool N	100
Portion of non-prarallel computing η	0.2
BPU's load ϕ	0.9
Maximum power consumption per BPU P_{BM}	20 W
Minimum power consumption per BPU P_{Bm}	5 W
BPU's speed s	2 GHz
Reference BPU's speed s_0	2 GHz
Exponential coefficient of BPU's speed β	2
First constant of the power cost a	2/10000000
Second constant of the power cost b	4
Profit per improvement of parallel computing κ	1

A. Feasibility of Contract

In this subsection, we study the feasibility of the proposed contract. To have a better understanding of the feasibility property, we only consider the one-MNO case, and begin with the IC constraint. Fig. 1 shows the MNO's utilities under various types $\theta = 5, 6, 7, 8$, and 9 with fixed number $N = 100$ of BPUs in the pool and the parameter of parallel computing $\eta = 0.2$. We can see that, the MNO of each type can achieve

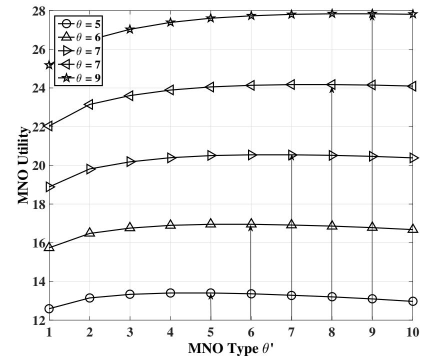

Fig. 1. MNO utility vs. MNO type θ', where $\theta' \in \boldsymbol{\Theta}$ and $\theta' \neq \theta$, for fixed $\eta = 0.2$, fixed $N = 100$, five MNO type values $\theta = 5, 6, 7, 8, 9$.

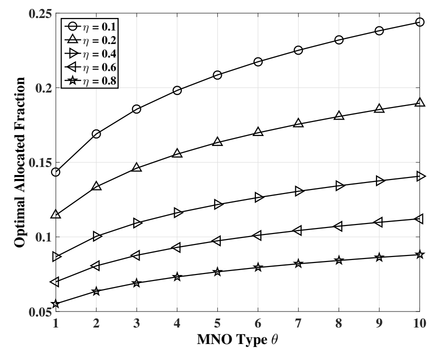

Fig. 2. Optimal allocated fraction $\hat{p}(\theta)$ vs. MNO type θ for fixed $N = 100$, and five values of $\eta = 0.1, 0.2, 0.4, 0.6, 0.8$.

its maximum utility only by selecting the type of contract designed for its own type, which is indicated by the constraint (4). The figure also shows that the MNO with a higher type has a higher utility, which is consistent with *Lemma 2*.

Next, we focus on the monotonicity of the optimal fraction versus the type of MNO in Fig. 2. It shows that the optimal BPU resource allocation increases with MNO type θ, and grows more slowly with a higher θ than that with a smaller one. It matches the monotonicity characteristic in *Lemma 2*.

B. Optimal Contract versus Parallel Parameter η

Fig. 2 also depicts the effect of various η under the optimal contract. The numerical results are conducted for fixed $N = 100$ and uniformly-distributed MNOs. It shows that when η is relatively small, i.e., the portion of parallel computing is high, the InP is willing to allocate more BPU resources to explore the benefits brought by parallel computing. The optimal fraction of BPU resources shrinks rapidly when η is approaching 1, which means that when η becomes larger, the performance gain that an MNO can obtain from parallel computing becomes less, thereby leading to a sharp reduction in the allocated BPU resources.

C. Optimal Contract versus BPU Pool Scale

Fig. 3 depicts the optimal contract versus the scale N of the BPU pool, where the parameter N stands for the total BPU numbers owned by the InP. The numerical results are conducted for fixed $\eta = 0.2$, with the uniformly-distributed MNOs of

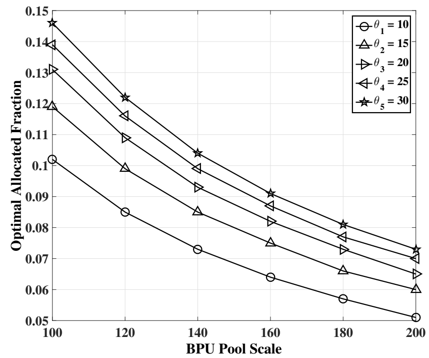

Fig. 3. Optimal allocated fraction $\hat{p}(\theta)$ vs. BPU pool scale N for fixed $\eta = 0.2$, five MNO types of $\theta = 10, 15, 20, 25, 30$.

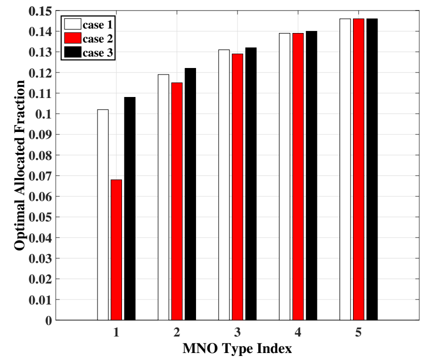

Fig. 4. Optimal allocated fraction $\hat{p}(\theta)$ vs. the index of the MNO types under three cases of distributions $f(\theta_i)$ of the MNO type for fixed $\eta = 0.2$, fixed $N = 100$, and five MNO types of $\theta = 10, 15, 20, 25, 30$.

types $\theta = 10, 15, 20, 25, 30$. It illustrates the optimal resource allocations under different MNO types as N increases. From (1), we can observe that the InP's utility is actually related to the number of BPU allocated to the MNO, which is calculated by the fraction of BPU resources times overall number of the BPUs. Although the overall number of the BPUs increases, the best strategy to achieve optimal profit is to allocate the same amount of BPUs to the MNOs. Thus, we can find that the optimal BPU resource allocation is inversely proportional to N, which can be verified from Fig. 3.

D. Optimal Contract under Type Distributions

Next, we show the performance of the optimal contracts under three different distribution cases of MNO type, i.e., case 1: $f(\theta_i) = \frac{1}{n}$; case 2: $f(\theta_i) = \frac{1}{\delta}\theta_i$; case 3: $f(\theta_i) = \frac{1}{\delta}\theta_{n-i+1}$, where $\delta = \sum_{i=1}^{n} \theta_i$. To be more clear, in case 1, all MNOs follow the uniform distribution; In case 2, the MNOs of a higher type have a greater proportion; In case 3, the MNOs of a lower type have a greater proportion, which is the opposite to case 2. Fig. 4 depicts the optimal BPU resource allocations for different MNO type distributions. It can be seen that if the MNOs of a lower type become scarce, InP may or reduce BPU allocation of lower type MNOs. It is interesting to observe that for a specific MNO type in case 2, the corresponding optimal resource allocation is smallest. The reason is that, case 2 has the greatest fraction of high-type MNOs who can bring in more profits for the InP. Thus, the InP can achieve the maximal utility through serving less clients. On the other hand, resource allocations in case 3 for MNOs of higher types become lower compared with other cases since the MNOs of higher types occupy a less portion according its type distribution.

V. Conclusion

In this paper, we study the computing resource trading in a virtualized cellular network with two kinds of participant, i.e., InP and MNOs. We investigate the trading process through contract-theoretic approach. In such a market, by viewing the BPUs as a kind of resources, the InP acts as a monopolist, setting up a set of resource quantity-price contracts. The MNOs, on the other hand, act as consumers, choosing proper contract items according to their own type. We formulate the contract and designed incentive mechanisms to motivate the InP to lease its BPUs to MNOs to maximize its profit. We develop the optimal contract and design corresponding algorithms to achieve the optimal contracts. Numerical results are performed to validate the feasibility of our proposed contract for giving the optimal BPU resource allocation and price, where we take into account various parameters. These numerical results show the effectiveness of the proposed contract design and analysis.

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References

[1] X. Costa-Perez, J. Swetina, R. Mahindra, and S. Rangarajan, "Radio access network virtualization for future mobile carrier networks," *IEEE Commun. Mag.*, vol. 51, no. 7, pp. 27–35, Jul. 2013.

[2] M. Qian, Y. Wang, Y. Zhou, L. Tian, and J. Shi, "A super base station based centralized network architecture for 5G mobile communication systems," *Digital Communications and Networks*, vol. 1, no. 2, pp. 152–159, 2015.

[3] M. Qian, W. Hardjawana, J. Shi, and B. Vucetic, "Baseband processing units virtualisation for cloud radio access networks," *IEEE Wireless Commun. Lett.*, vol. 4, no. 2, pp. 1–4, Apr. 2015.

[4] M. Gao, H. Chen, Y. Li, Y. Zhou, and J. Shi, "Switching delay aware computing resource allocation in virtualized base station," *China Commun.*, vol. 13, no. 11, pp. 226–233, 2016.

[5] G. Zhai, L. Tian, Y. Zhou, and J. Shi, "Load diversity based optimal processing resource allocation for super base stations in centralized radio access networks," *Science China Information Sciences*, vol. 57, no. 4, pp. 1–12, Apr. 2014.

[6] J. Liu, S. Zhou, J. Gong, Z. Niu, and S. Xu, "Statistical multiplexing gain analysis of heterogeneous virtual base station pools in cloud radio access networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 8, pp. 5681–5694, 2016.

[7] X. Wang, S. Thota, M. Tornatore, H. S. Chung, H. H. Lee, S. Park, and B. Mukherjee, "Energy-efficient virtual base station formation in optical-access-enabled cloud-RAN," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1130–1139, 2016.

[8] M. Peng, X. Xie, Q. Hu, J. Zhang, and H. V. Poor, "Contract-based interference coordination in heterogeneous cloud radio access networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1140–1153, 2015.

[9] T. Zhao, J. Wu, S. Zhou, and Z. Niu, "Energy-delay tradeoffs of virtual base stations with a computational-resource-aware energy consumption model," in *IEEE Int. Conf. on Commun. Syst. (ICCS)*, Nov. 2014, pp. 26–30.

[10] Z. Fan, "A distributed demand response algorithm and its application to PHEV charging in smart grids," *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1280–1290, 2012.

[11] Y. Li, J. Zhang, X. Gan, L. Fu, H. Yu, and X. Wang, "A contract-based incentive mechanism for delayed traffic offloading in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 8, pp. 5314–5327, 2016.