

# Resource Trading for a Small-Cell Caching System: A Contract-Theory Based Approach

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**Abstract**—Evidences indicate that wireless video traffic has played an important role in cellular networks. Caching mechanisms which store popular contents into local small-cell base stations (SBSs) in cellular networks are proposed to further reduce transmission delay and release the traffic pressure over backhaul channels. In this paper, we consider a commercialized small-cell caching system consisting of a network service provider (NSP), several video retailers (VRs) and multiple mobile users (MUs). The NSP as a network facility monopoly releases its resources to the VRs in order to maximize its own profits. The distribution of VR's type is known to the NSP, while the actual type of a given VR is not known. We research on such an information asymmetric market within the framework of contract theory, formulated as an adverse selection problem. The MUs and SBSs are modeled as two independent Poisson point processes, and the directly downloading probability from the adjacent SBS is derived via stochastic geometry theory. Based on the probability, we formulate the utility functions of the NSP and the VRs. Then, the optimal contract problem is constructed. Also, we provide the feasibility of the contract, and the optimal contract is proposed when VR's popularity parameter  $\gamma$  takes different values. Numerical results are provided to show the optimal quality and the optimal price designed for each VR.

## I. INTRODUCTION

With the growing needs for entertainment and social connections, wireless video traffic has played an important role in cellular networks. It is predicted by Cisco that wireless video data will reach 75% of the total amount of network data in 2020 [1]. Usually, there are numerous repetitive requests of popular videos from users, leading to heavy traffic pressure on the backhaul channel. The redundancy of video transmissions can be reduced by storing popular videos into the local caches of intermediate network nodes. The authors in [2] propose a femto-caching scheme in which data placement at the small-cell base stations (SBSs) is optimized in a centralized manner in order to reduce transmission delay. In [3], an optimal caching scheme is proposed to place the popular video contents in device-to-device networks where the cluster size is optimized to reduce the transmission delay. The authors in [4] present a many-to-many matching game algorithm for proactive caching in social networks to significantly reduce the transmission latency.

## A. Commercial Schemes and Contract Theory

Besides the data placement issue for reducing transmission delay in the small-cell caching system, it should be coupled with many other issues. For example, we can consider the small-cell caching system from the commercial perspective. In a commercialized small-cell caching system, there consists of a network service provider (NSP), some video retailers (VRs) and multiple mobile users (MUs). In such a commercial caching system, NSP can use its monopoly on the network facilities to make profits by renting its resources. VRs are willing to buy some caches to provide better services for their users. In this sense, both NSP and VRs can make profits from the caching system. However, both sides are selfish and want to maximize their own profits, inducing an interest's conflict problem.

Game theory is treated as an effective way to solve this kind of interests' conflict problem [5, 6]. In an information symmetric environment, the Stackelberg game is provided as a useful approach to determine the equilibrium between two sides [7–11]. Specifically, in [10], authors investigate the commercial video-caching system using the Stackelberg game by treating the SBSs as a specific type of resources within the environment of one NSP and multiple VRs. Furthermore, the authors extend their researches to the environment with one VR and multiple NSPs [11].

When trading is modeled in an information asymmetric environment, contract theory is provided as an effective mechanism to solve this kind of problem [12]. Most of the existing works on contract theory are grossly classified into two categories: one is adverse selection which often refers to the problems of hidden information [13–17], and the other is moral hazard which is related to the problems of hidden actions [18–20]. Specifically, authors in [13] introduce contract theory into the quality discrimination spectrum trading in cognitive radio networks, where the buyer's types are the hidden information. In [14], the authors propose a contract-theoretic framework for the broker-based TV white space market. A third-party geo-location database is treated as a broker to reserve spectrum from licensees and resell it to the secondary users. The difference between the database's and the

white space devices's knowledge is regarded as information asymmetry. [15] addresses the hybrid access models employing either opportunistic or exclusive access of free frequency bands for cognitive radio networks. By offering bandwidth-price contracts to the secondary users whose service class is the private information, the primary spectrum owner gains additional revenue. [20] designs the incentive contract between a mobile network operator (MNO) and the content providers (CP) in a fixed network. The proposed model exploits the strategic interdependence between the CPs, and the optimization problem is defined to maximize the utility of the CPs while the MNO is ensured to be budget balanced.

## B. Contributions

As mentioned above, we find that the existing contract-based researches on resource trading are usually modeled in the asymmetric information environment, where the seller doesn't know the specific type of each buyer. In the small-cell caching system, the distribution of the VR's type is known to the NSP, while the actual type of a given VR is unknown. To this end, we are inspired to investigate the contract-based resource trading mechanism in this asymmetric information system. The proposed scheme, together with the derived solutions, can offer the proper economical incentives for the NSP. The main contributions of this paper are summarized as follows:

- 1) Contract theory is proposed to exploit the trading issues between a monopolist NSP and multiple VRs in a commercialized small-cell caching system. VRs are classified into different 'types' according to their popularity, while the NSP divides the SBSs into different fractions with the notation of 'quality'. The optimal contract is constructed in maximizing the utility of the NSP when information asymmetry is considered.
- 2) The system model is based on the stochastic network structure. The probability of MUs to direct downloading from SBSs is provided via the stochastic geometry theory. Based on the direct downloading probability, the profits of the NSP and VRs are developed.
- 3) The necessary and sufficient conditions of the feasible contract are proved. Furthermore, we reduce the constraints in solving the optimal contract. We investigate the concavity and convexity of the target function in two cases. The optimal contract entries which are the combinations of the optimal quality and price are given in the closed forms when VR's popularity parameter takes different values.

The rest of this paper is organized as follows: the system model is presented in section II. The contract-based service model is elaborated in section III. The optimal contract design and solutions are derived in section IV. Numerical results are presented in section V, and conclusions are drawn in section VI.

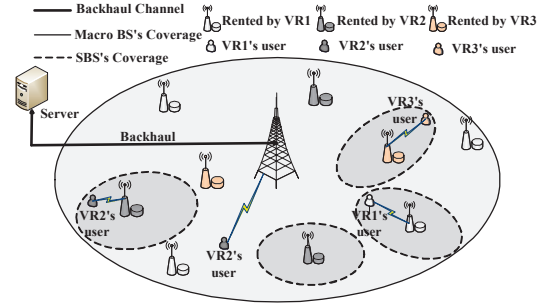


Fig. 1. System model with one NSP and 3 VRs, while VR1 rents 4 SBSs, VR2 rents 3 SBSs and VR3 rents 2 SBSs.

## II. SYSTEM MODEL

We focus on a commercial small-cell caching system with one NSP,  $N$  VRs and multiple MUs. In this system, by viewing the SBSs as a kind of resources of the NSP, each VR purchases a certain fraction of the SBSs from the NSP for placing its popular videos. The corresponding MU affiliated with a VR may directly download videos from its nearby SBSs that have been rented by this VR. Otherwise, the MU has to acquire its videos from the macro-cell base station (MBS). An example system is depicted in Fig. 1, there are 3 VRs who rent 4 SBSs, 3 SBSs and 2 SBSs in this system, respectively. MUs who are within the coverage of SBSs storing the requested videos will download the videos directly from the local SBSs. A user of VR2 who is not covered by any SBS is communicating with the MBS.

### A. Network Model

Let's consider a small cell network consisting of multiple SBSs owned by the monopolist NSP, and we denote the set of  $N$  VRs by  $\mathcal{V} = \{\mathcal{V}_1, \dots, \mathcal{V}_v, \dots, \mathcal{V}_N\}$  and the NSP by  $\mathcal{L}$ . SBSs have the uniform transmission power  $P$  and the same caching size of  $Q$  video files, are spatially distributed as a homogeneous Poisson point process (PPP)  $\Phi$  with density  $\lambda$ . The distribution of MUs is modeled as an independent homogeneous PPP  $\Psi$  with density  $\zeta$ . SBSs transmit on the orthogonal channels to the MBSs, and hence we do not consider interferences induced by the MBSs.

We consider a typical MU located at the origin and an SBS located at  $x$ . The path-loss between an SBS and the typical MU is denoted by  $\|x\|^{-\alpha}$ , where  $\alpha$  is the path-loss exponent. The channel power of the Rayleigh fading between an SBS and the typical MU is denoted by  $h_x$ , where  $h_x \sim \exp(1)$ . The noise is modeled as the additive white gaussian noise (AWGN) with zero mean and  $\sigma^2$  variance.

A saturated network is considered in this model, where all SBSs are powered on and keep transmission for their subscribers. Hence, the signal to interference and noise ratio (SINR) at the typical MU from an SBS whose location is  $x$  can be expressed as

$$\rho(x) = \frac{Ph_x\|x\|^{-\alpha}}{\sum_{x' \in \Phi \setminus x} Ph_{x'}\|x'\|^{-\alpha} + \sigma^2}. \quad (1)$$

If  $\rho(x)$  is larger than the predefined threshold  $\delta$ , an MU can be covered by the SBS located at  $x$ . The predefined threshold  $\delta$  defines the highest delay of downloading a video files.

### B. Preference and Popularity

Let's denote the file set by  $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_F\}$  consisting of  $F$  video files. The popularity of the videos are denoted by  $\mathbf{p} = \{p_1, \dots, p_i, \dots, p_F\}$ . Generally,  $\mathbf{p}$  can be modeled by the Zipf distribution [21] as

$$p_i = \frac{1/i^\beta}{\sum_{f=1}^F 1/f^\beta}, \quad \forall i, \quad (2)$$

where the positive value  $\beta$  represents the video's popularity. A higher  $\beta$  means a higher content reuse, where the most popular videos account for the majority of download requests. Note that each SBS can store at most  $Q$  video files, and we assumed that  $Q < F$ .

Usually, MUs have different preferences to  $N$   $\mathcal{V}$ s, combining with many factors including personal favor, Quality of Service (QoS), the charging standards, etc. We use the popularity representing the MUs' preferences of  $\mathcal{V}$  which are denoted by  $\Theta = \{\theta_1, \dots, \theta_v, \dots, \theta_N\}$ , in which  $\theta_v$  is modeled by the Zipf distribution [22] as

$$\theta_v = \frac{1/v^\gamma}{\sum_{j=1}^N 1/j^\gamma}, \quad \forall v, \quad (3)$$

where  $\gamma$  is a positive value and determines the distribution of  $\mathcal{V}$ 's popularity.

### C. Caching Procedure

In this subsection, we describe the caching procedures in a small-cell caching system in detail. There are three stages in our system. In the first stage,  $\mathcal{V}$  rents a certain fraction of the SBSs from  $\mathcal{L}$  for placing its videos. We denote the fraction vector as  $\mathbf{\Gamma} = \{\tau_1, \tau_2, \dots, \tau_N\}$ , in which  $\tau_v$  represents the fraction assigned to  $\mathcal{V}_v$ . In general, if a  $\mathcal{V}$  doesn't rent any fraction from  $\mathcal{L}$ , it can be regarded as renting a fraction of  $\tau_v = 0$ . Obviously, the fraction of the SBS cannot be negative or infinity, i.e.,  $\sum_{v=1}^N \tau_v = 1$  and  $\tau_v \geq 0$ . We assume that the SBSs rented by each  $\mathcal{V}_v$  are uniformly distributed and can be modeled as a homogeneous PPP  $\Phi_v$  of density  $\tau_v \lambda$ .

In the second stage, the data placements commence during off-peak time after each  $\mathcal{V}$  obtains the access to the SBSs. Each SBS will be placed with  $Q$  most popular video files, limited to its caching size. Because each SBS assigned to  $\mathcal{V}_v$  caches the same set of  $Q$  files, the file  $\mathcal{F}_i, i \leq Q$  cached in the rented fraction  $\tau_v$  can be further modeled as a homogeneous PPP  $\Phi_{v,i}$  of density  $\tau_v \lambda, \forall i \leq Q$ .

In the third stage, an MU of  $\mathcal{V}_v$  requests a video file  $\mathcal{F}_i$ . It first searches the SBSs in  $\Phi_{v,i}$  and connects to the nearest SBS that covers it. If such an SBS exist, the subscriber will obtain this video directly from the caching of this SBS which event is defined by  $\mathcal{E}_{v,i}$ . Otherwise, the MU will trigger transmission via the backhaul channel from the central server to the serving SBS, leading to an extra cost on  $\mathcal{L}$ .

Similar to [22], the probability  $\Pr(\mathcal{E}_{v,i})$  of the event  $\mathcal{E}_{v,i}$  can be derived as follows,

$$\begin{aligned} \Pr(\mathcal{E}_{v,i}) &= \frac{\tau_v}{\tau_v A(\delta, \alpha) + \sum_{j=1, j \neq v}^N \tau_j C(\delta, \alpha) + \tau_v} \\ &= \frac{\tau_v}{\tau_v A(\delta, \alpha) + (1 - \tau_v) C(\delta, \alpha) + \tau_v}, \forall i \leq Q, \quad (4) \end{aligned}$$

in which,  $A(\delta, \alpha) = \frac{2\delta}{\alpha-2} {}_2F_1(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta)$ ,  $C(\delta, \alpha) = \frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}) \cdot {}_2F_1(\cdot)$  in the function of  $A(\delta, \alpha)$  is the hypergeometric function and  $B(\cdot, \cdot)$  is the beta function in  $C(\delta, \alpha)$ . From (4), we find that the probability of  $\Pr(\mathcal{E}_{v,i})$  is independent of the transmission power  $P$ , the intensity  $\lambda$  of the SBSs and also the caching size  $Q$ . For the clearance of notation, we use  $\Pr(\tau_v)$  to denote  $\Pr(\mathcal{E}_{v,i})$  when  $i \leq Q$  as,

$$\Pr(\mathcal{E}_{v,i}) = \begin{cases} \Pr(\tau_v), & \forall i \leq Q; \\ 0, & \forall i > Q. \end{cases} \quad (5)$$

## III. CONTRACT-BASED SERVICE MODEL

We now focus on modeling the contract-based issues in the small-cell caching system which including a seller  $\mathcal{L}$  and  $N$  buyer  $\mathcal{V}_v$ . Since  $\mathcal{L}$  owns the network facilities and dominates the trading process, we model the SBS trading as a monopoly market. Instead of offering the same contract to  $\mathcal{V}$ ,  $\mathcal{L}$  will offer different contract entries.  $\mathcal{V}$  is free to accept or decline any contracts with  $\mathcal{L}$ . Incorporating the caching and data transmission process, we elaborate the contract issues in a small-cell caching system.

### A. NSP's Model

$\mathcal{L}$  who is the monopolist in this market sets the contract entries  $\{\mathbf{\Gamma}, \mathbf{\Pi}\}$ , which are the combinations of quality and price for its resources, i.e., the SBSs. The SBSs are divided into different sizes of fractions, regarded as different qualities. A set of price is called  $\mathbf{\Pi} = \{\pi_1, \pi_2, \dots, \pi_N\}$ . Each quality  $\tau_v$  in  $\mathbf{\Gamma}$  corresponds to a price  $\pi_v$ .  $\mathcal{V}$  is free to decide whether or which quality to buy. These are regarded as the contract construction and commitment which take place before video caching procedures.

The profit of the NSP mainly depends on the rent of its SBSs. Meanwhile, the NSP needs to pay for the transmission cost which is denoted by  $c$  per unit power. Then, the utility of  $\mathcal{L}$  can be expressed as

$$S^{\text{NSP}} = \sum_{v=1}^N (\pi_v - cP\lambda\tau_v). \quad (6)$$

### B. VR's Model

If  $\mathcal{V}_v$  signs a contract with  $\{\tau_v, \pi_v\}$ , and there are averagely  $K$  video demands from each MU within a unit period, and the average backhaul cost for a video transmission is  $s^{\text{BH}}$ . Based on (4), we can get the saved cost  $S_v^{\text{BH}}$  on backhaul channel for  $\mathcal{V}_v$  as

$$S_v^{\text{BH}} = \sum_{f=1}^Q p_f \theta_v \zeta K s^{\text{BH}} \Pr(\tau_v). \quad (7)$$

According to different popularity among  $\mathcal{V}$ , we classify  $\mathcal{V}$  into different types.

**Definition 1:  $\mathcal{V}$ 's type.** We use popularity defined in (3) to represent the types of  $\mathcal{V}$ , which are sorted in an descending order as,

$$\theta_1 > \dots > \theta_v > \dots > \theta_N, v \in \{1, 2, \dots, N\}. \quad (8)$$

A higher type implies more popularity. Please note that it is the asymmetric information environment where the exact values of the VR types are private information. The NSP doesn't know the specific type of each  $\mathcal{V}$ . The NSP only has the distribution information about the VR types, which is modeled by the Zipf distribution in this system.

Furthermore, we define  $M \triangleq \sum_{f=1}^Q p_f \zeta K S^{\text{BH}}$ , which is a constant and irrelevant to  $\theta_v$  and  $\tau_v$ . We have  $S_v^{\text{BH}} = M \theta_v \text{Pr}(\tau_v)$ . For the clearance of discussion, we define the valuation of quality  $\tau_v$  by a type- $\theta_v$   $\mathcal{V}$  as

$$V(\theta_v, \tau_v) \triangleq M \theta_v \text{Pr}(\tau_v), \quad (9)$$

which is increasing with the type  $\theta_v$  and is also a strictly increasing concave function of  $\tau_v$ , with,  $V'(\tau_v) > 0$ ,  $V''(\tau_v) < 0$ .  $V(\theta_v, \tau_v)$  represents the benefits a type  $\theta_v$  received by employing the quality  $\tau_v$ .

The utility of  $\mathcal{V}_v$  can be expressed as

$$U_v(\theta_v, \tau_v, \pi_v) = V(\theta_v, \tau_v) - \pi_v. \quad (10)$$

Obviously, a rational  $\mathcal{V}$  will not accept a negative utility, and thus  $V(\theta_v, \tau_v) \geq \pi_v$ .

#### IV. OPTIMAL CONTRACT DESIGN

$\mathcal{L}$  designs the contracts and each  $\mathcal{V}$  chooses the appropriate entry to buy. The goal of  $\mathcal{L}$  is to maximize  $S^{\text{NSP}}$  by offering the optimal contract entries  $\{\tau_v^*, \pi_v^*\}$ ,  $v = 1, 2, \dots, N$ .

##### A. Contract Formulation

The optimal contract must comply with the feasibility constraints which are the individual rationality and incentive compatibility for all  $\mathcal{V}$  types [12].

**Definition 2: Individual Rationality (IR):** It's defined in contract theory that each  $\mathcal{V}$  is rational. It will not accept a contract when it receives a negative utility for its type. This can be expressed as

$$V(\theta_v, \tau_v) - \pi_v \geq 0, \quad \forall v. \quad (11)$$

**Definition 3: Individual Compatibility (IC):** The IC constraint means that a  $\mathcal{V}$  cannot gain more utility by accepting a contract entry which is not designed for its type. It can be written as

$$V(\theta_v, \tau_v) - \pi_v \geq V(\theta_v, \tau_{\tilde{v}}) - \pi_{\tilde{v}}, \quad \forall v \neq \tilde{v}. \quad (12)$$

Based on the IC and IR constraints, the optimal contract can be formulated as

$$(\tau_v^*, \pi_v^*) = \arg \max_{\tau_v} \sum_{v=1}^N (\pi_v - cP\lambda\tau_v), \quad (13)$$

$$\text{s.t. IR(11), IC(12), } \sum_{v=1}^N \tau_v = 1, \tau_v \geq 0.$$

##### B. Feasibility of Contract

In order to solve the problem in (13), we need to simplify the constraints firstly.

**Lemma 1:** For the optimal solution, given that the IC constraint is satisfied, the IR constraint for the lowest type  $\theta_N$  is a binding, i.e.,

$$V(\theta_N, \tau_N) - \pi_N = 0, \quad (14)$$

other IR constraints can be ignored.

Due to page limitation, all proofs are omitted.

The IR constrains are reduced by Lemma 1 which indicates that the lowest type  $\theta_N$  gains a zero profit. A similar conclusion is also provided in [12, 13, 19]. Other  $\mathcal{V}$ 's profits are larger than the binding one's. We also find that the price  $\pi_N$  for the lowest type should be equal to the valuation of quality  $\tau_N$ . Next, we will prove that the IC constraints can be reduced in the following lemmas.

**Lemma 2:** If the contract is feasible, the following condition holds true: given  $\tau_i > \tau_j$ , if and only if  $\pi_i > \pi_j$ .

Lemma 2 presents an important property for a feasible contract: a higher quality corresponding to a higher price and vice versa.

**Proposition 1:** If  $\mathcal{V}$ 's utility function satisfies the Spence-Mirrlees Condition (SMC) [12], for any  $\mathcal{V}$  type  $\theta_m > \theta_n$  and  $\tau_i > \tau_j$ , the saved cost of each  $\mathcal{V}$  satisfies the following inequation,

$$(V(\theta_m, \tau_i) - V(\theta_m, \tau_j)) \geq (V(\theta_n, \tau_i) - V(\theta_n, \tau_j)). \quad (15)$$

**Lemma 3:** Given the IC constraint satisfied, the quality of SBSs  $\tau_i$  in the contract monotonically increases with  $\mathcal{V}$ 's type  $\theta_i$ , i.e. if  $\theta_i > \theta_j$ , then  $\tau_i > \tau_j$ . This is a necessary condition for the IC constraint.

This implies that the quality  $\tau_v$  is monotonically increasing with type  $\theta_v$  when the contract satisfies the IC constraints. That means the quality assigned to a higher type must be larger than that to a lower one. Let's review the type which is defined as the MU's preference over  $\mathcal{V}$ , it indicates the requesting proportion of the MUs. And the quality is the fraction of the SBSs assigned to  $\theta_v$ . In a feasible contract,  $\mathcal{V}_v$  with higher-type  $\theta_v$  buying a larger quality  $\tau_v$ , will be allocated with more fractions of the SBSs, and hence with more coverage and larger downloading probability.

**Proposition 2:** If  $\mathcal{V}$ 's utility function satisfies the SMC, the Local Downward Incentive Constraint (LDIC) and the Local Upward Incentive Constraint (LUIC), the IC constraint will be satisfied, i.e.,

$$V(\theta_i, \tau_i) - \pi_i \geq V(\theta_i, \tau_j) - \pi_j. \quad (16)$$

**Corollary 1:** If the contract is at the optimal, the IC constraint can be replace by

$$V(\theta_i, \tau_i) - \pi_i = V(\theta_i, \tau_{i+1}) - \pi_{i+1}. \quad (17)$$

### C. Optimality of Contract

Using the above lemmas and corollary, the optimization problem in (13) can be reduced as

$$\begin{aligned} (\tau_v^*, \pi_v^*) &= \operatorname{argmax} \sum_{v=1}^N (\pi_v - cP\lambda\tau_v), \\ \text{s.t. (14), (17), } &\sum_{v=1}^N \tau_v = 1, \tau_v \geq 0. \end{aligned} \quad (18)$$

In order to solve (18), we iterate the constraints in (18), then we get

$$\begin{aligned} \tau_v^* &= \operatorname{argmax} \sum_{v=1}^N (vV(\theta_v, \tau_v) - (v-1)V(\theta_{v-1}, \tau_v) - cP\lambda\tau_v), \\ \text{s.t. } &\sum_{v=1}^N \tau_v = 1, \tau_v \geq 0. \end{aligned} \quad (19)$$

We define

$$R_v \triangleq vV(\theta_v, \tau_v) - (v-1)V(\theta_{v-1}, \tau_v) - cP\lambda\tau_v, \quad (20)$$

and find that the optimal  $R_v$  is only associated with quality  $\tau_v$  and independent of other qualities  $\tau_{v'}, v' \neq v$ . So  $\tau_v^* = \operatorname{argmax} R_v$ .

**Lemma 4:** When  $\gamma > 1$ ,  $R_v$  is convex to  $\tau_v$ ; When  $0 < \gamma < 1$ ,  $R_v$  is concave to  $\tau_v$ .

1) *Remarks on  $\gamma > 1$ :* When  $\gamma > 1$ ,  $R_v$  is convex. In order to get the maximum value, we need to check two points with  $\tau_v = 0$  or  $\tau_v = 1$ . Considering the constraint  $\sum_{v=1}^N \tau_v = 1$ , the rational value of  $\tau_v = 0$  is that  $\tau_1 = 1$ . Because when  $v > 1$ ,  $(v\theta_v - (v-1)\theta_{v-1}) < 0$  as proved, we have  $R_v < 0$ , it doesn't satisfy the IR constraint. When  $v = 1$ , i.e.,  $\tau_1 = 1$ ,  $R_1 = V(\theta_1, \tau_1) - cP\lambda\tau_1 > 0$  satisfies the IR constraint. So, it's better to assign  $\theta_1$  with the total SBSs when  $\gamma > 1$ . So the optimal contract for  $\gamma > 1$  is

$$\tau_v^* = \begin{cases} 0, & v > 1; \\ 1, & v = 1, \end{cases} \quad (21)$$

and

$$\pi_v^* = \begin{cases} 0, & v > 1; \\ V(\theta_1, \tau_1^*), & v = 1, \end{cases} \quad (22)$$

2) *Remarks on  $0 < \gamma < 1$ :* When  $0 < \gamma < 1$ ,  $R_v$  is concave to  $\tau_v$ . Use the standard Lagrangian method, we have

$$\tau_v^* = \begin{cases} 0, & \varepsilon > \frac{(v\theta_v - (v-1)\theta_{v-1})M}{C(\delta, \alpha)} - cP\lambda; \\ \sqrt{\frac{(v\theta_v - (v-1)\theta_{v-1})MC(\delta, \alpha)}{cP\lambda + \varepsilon} - C(\delta, \alpha)}, & \text{otherwise.} \end{cases} \quad (23)$$

Substituting (23) into (20), we get the optimal price as

$$\pi_v^* = V(\theta_N, \tau_N^*) + \sum_{i=v}^N w_i^*, \quad (24)$$

in which

$$w_v^* = \begin{cases} 0, & v = N; \\ V(\theta_v, \tau_v^*) - V(\theta_v, \tau_{v+1}^*), & v = 1, \dots, N-1. \end{cases} \quad (25)$$

## V. NUMERICAL RESULTS

In this section, we conduct numerical simulations to illustrate the performance of contract theory in a commercialized caching system.

For the physical layer, we set path-loss parameter  $\alpha = 4$ , the transmit power  $P = 10\text{W}$  and the SINR threshold  $\delta = 0.01$ . The density of SBSs is set to  $\lambda = 20/km^2$  and MU's density is set to  $\zeta = 80/km^2$ . File number is  $F = 100$ , the maximum number of files cached in each SBS is set to  $Q = 40$ . The number of VRs is  $N = 5$ . For the commercial system, the cost for  $\mathcal{L}$  to transmit a unit power is  $c = 1$ . The cost for transmitting data via backhaul is  $s^{\text{BH}} = 1$ . The number of videos requested from one MU within a unit period is set to  $K = 50$ .

In Fig. 2 we set  $\mathcal{V}$ 's popularity parameter  $\gamma$  to 0.4 and there are 5 VRs in this network. From Fig. 2(a) we can see that the types of VRs which vary from 0.2856 to 0.15 are decreasing as their orders increase. In Fig. 2(b), there are the same trend in the values of optimal quality. When a  $\mathcal{V}$  has a larger type, it tends to rent more fractions to offer popular videos to its subscribers. Fig. 2(c) shows how optimal prices change with  $\mathcal{V}$ 's types. When  $\mathcal{V}$ 's type  $\theta_v$  is high, the optimal price charged by  $\mathcal{L}$  is high. The optimal price declines as the  $\mathcal{V}$ 's type decreases.

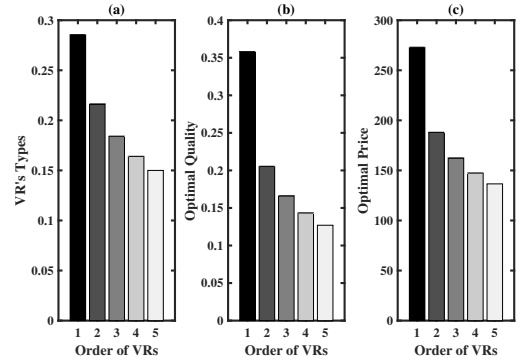


Fig. 2. VRs' types, the optimal quality and the optimal prices when  $\gamma = 0.4$ .

In Fig. 3, we plot the profits gained by the NSP as  $\gamma$  varies. It can be seen that the curve of the NSP's profits is a combination of a convex curve when  $0 < \gamma < 1$  and a concave curve when  $\gamma > 1$ . When  $\gamma > 1$  the profits of NSP is associated with  $\theta_1$  which is increased with  $\gamma$ . The convex curve of  $0 < \gamma < 1$  is a combination which can be seen from Fig.4, in which, the profits gained from VR1 is convex and dominated.

## VI. CONCLUSIONS

In this paper, we propose a contract based trading mechanism for a commercialized small-cell caching system. We treat the system as a monopoly market where  $\mathcal{L}$  owns the network facilities, i.e., SBSs with caches, and conducts the contracts in the purpose of maximizing its own profits. The feasibility of the contract is presented and the optimal price and quality

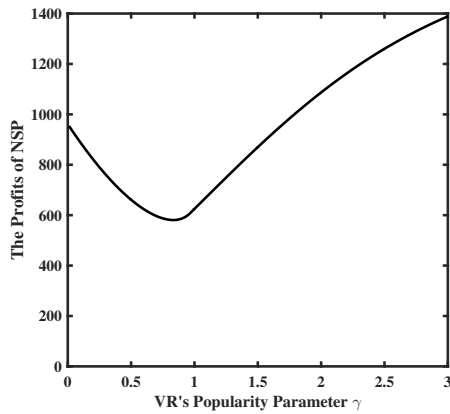


Fig. 3. The profits of  $\mathcal{L}$  using the proposed scheme.  $\gamma$  varies from 0.01 to 3

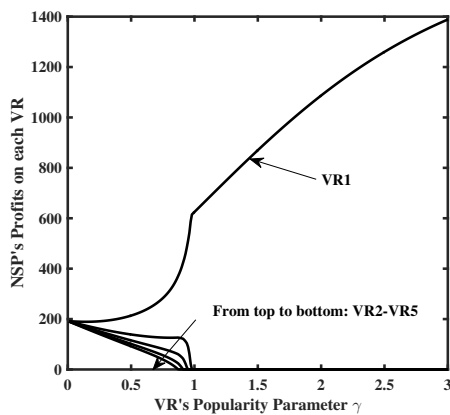


Fig. 4.  $\mathcal{L}$ 's profits on each  $\mathcal{V}$

pairs are derived in two different conditions according to the VR's popularity parameter  $\gamma$ . Simulation results are provided to show the optimal qualities and prices. The profits of the NSP are presented as  $\gamma$  varies.

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