PAPER

Physical Layer Network Coding for Wireless Cooperative Multicast Flows*

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SUMMARY It has been proved that wireless network coding can increase the throughput of multi-access system [2] and bi-directional system [5] by taking the advantage of the broadcast nature of electromagnetic waves. In this paper, we introduce the wireless network coding to cooperative multicast system. We establish a basic 2-source and 2-destination cooperative system model with arbitrary number of relays ($2-N-2$ system). Then two regenerative network coding (RNC) protocols are designed to execute the basic idea of network coding in complex field (RCNC) and Galois field (RGNC) respectively. We illuminate how network coding can enhance the throughput distinctly in cooperative multicast system. Power allocation schemes as well as precoder design are also carefully studied to improve the system performance in terms of system frame error probability (SFEP).

key words: *wireless network coding, multicast systems, system frame error probability, power allocation, precoder design*

1. Introduction

Recently, how to leverage network coding into wireless networks for system capacity improvement has drawn increasing interest [1]–[7]. Different from robust wired networks, wireless communications occurring in the air must unfortunately face many disadvantageous factors such as channel fading, noises and broadcast interferences. Though it has been proved that wireless network coding can enhance the throughput of multi-access system [2] and bi-directional system [5], how to make a full and effective use of network coding in wireless multicast system remains many unresolved issues.

We consider, as examples, the two sources, one relay and two destinations ($2-1-2$) cooperative multicast network [10] for example, where all nodes are in half-duplex mode. In the multicast network shown as Fig. 1, we suppose that s_1 as well as s_2 broadcast their information to the two destinations d_1 and d_2 simultaneously. From Fig. 1, we can see d_1 (or d_2) is out of the transmission range of s_2 (or s_1). The shared relay can help s_1 and s_2 reach their destinations.

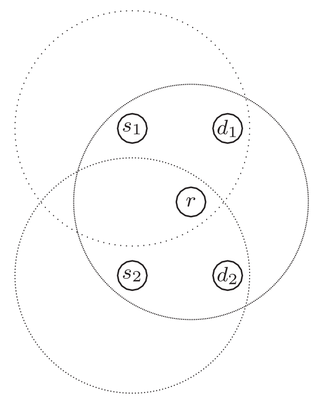

Fig. 1 A wireless $2-1-2$ multicast system where the circles denote the transmission ranges. Due to the power constraint, s_1 (s_2) should borrow r to transmit messages to d_2 (d_1).

There are two transmission schemes. The first scheme is through the traditional way without network coding, which occupies four time slots:

1. $s_1 \to \{r,\ d_1\}$ with information I_{s_1};
2. $r \to d_2$ with information I_{s_1};
3. $s_2 \to \{r,\ d_2\}$ with information I_{s_2};
4. $r \to d_1$ with information I_{s_2}.

The second one is by network coding method with two time slots:

1. $s_1 \to \{r,\ d_1\}$ with information I_{s_1},
   $s_2 \to \{r,\ d_2\}$ with information I_{s_2};
2. $r \to \{d_1,\ d_2\}$ with signal $f(I_{s_1}, I_{s_2})$.

For the transmission scheme with network coding, because d_1 (or d_2) has already detected the information I_{s_1} (or I_{s_2}) in the priori time slot, it can obviously pick up the remained information I_{s_2} (or I_{s_1}) by the combined signals from r. Note that function $f(\cdot)$ is certain mapping mechanism. In non-regenerative network coding (NRNC) protocol, signals from two sources mixed in the air are not decoded at relays before being retransmitted [8], [9]. So $f(\cdot)$ in NRNC is a linear function. While in regenerative network coding (RNC) protocol, mixed signals are jointly decoded into symbols at the relay and then superposed in either the complex filed (RCNC) or Galois field (RGNC) before retransmission.

In this paper, we study the two RNC protocols, i.e., RCNC and RGNC based on the $2-N-2$ cooperative multicast system where we follow the second scheduling scheme in [11], and N relays are arranged in round-robin way. Our contributions are as follows:

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1. **Performance analysis on the protocols.** We define the system frame error probability (SFEP) as the metric of the system performance. Then we successfully derive the SFEP, which will be used as the objective functions to optimize the power allocation schemes.
2. **Power allocation schemes and precoder design.** Based on the SFEP of the two protocols, we conclude the optimal power allocation schemes to improve the system performance. Since the multicast system can not achieve the full diversity gain by power allocation only, the precoders are designed and applied to the two RNC protocols respectively to further obtain higher diversity gains.

The notations used in this paper are as follows. Bold upper- and lower-case letters denote matrices and column vectors respectively, with $(\cdot)^T$ and $(\cdot)^H$ denoting their transpose and conjugate-transpose. $\hat{x}$ represents the decoded symbol of a symbol x. $\hat{\mathbf{x}}$ and $\hat{\mathbf{X}}$ represent the decoded vector and decoded matrix. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix and $\mathbf{\Sigma_v}$ denotes the auto-covariance matrix of the vector $\mathbf{v}$. $\mathcal{E}(\cdot)$ is the expectation operation. Notation $z(x) \triangleq O(y(x))$, $y(x) > 0$ denotes that there is a positive constants c such that $|z(x)| \leq cy(x)$ when x is large.

2. System Model

Figure 2 displays the transmission scheme in $2-N-2$ multicast system. Note that the dashed boxes denote the signals' reception processes and the solid ones denote the signals' transmission processes. Since N relays are arranged in the round-robin way, we define $\mathbf{x}_s \triangleq [x_{s_1,1}, x_{s_2,1}, x_{s_1,2}, x_{s_2,2}, \cdots, x_{s_1,N}, x_{s_2,N}]^T$ as a system frame which is composed of symbols from two sources. All symbols are equally probable from the QAM constellation set Q with zero means and variances $2P$ where P is the average total network transmission power over a time slot. The symbol vector in the system frame for s_k $(k = 1, 2)$ is denoted as $\mathbf{x}_{s_k} = [x_{s_k,1}, x_{s_k,2}, \cdots, x_{s_k,N}]^T$. The symbol vector received by relays is denoted as $\mathbf{y}_r = [y_{r_1}, y_{r_2}, \cdots, y_{r_N}]^T$, in which the i-th element $(i = 1, \cdots, N)$ is the signal received by r_i. The symbol vector received by d_k is denoted as $\mathbf{y}_{d_k} = [y_{d_k,1}, y_{d_k,2}, \cdots, y_{d_k,2N-1}, y_{d_k,2N}]^T$.

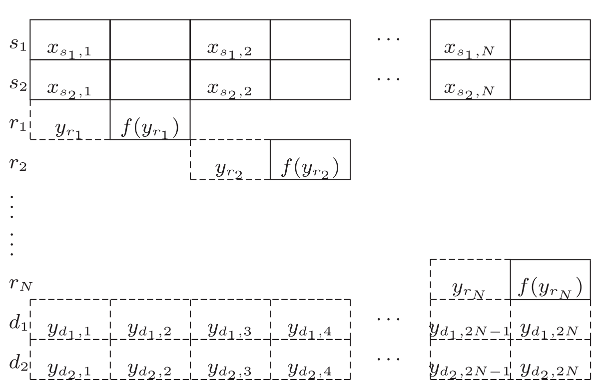

Fig. 2 Frame structure and relaying scheme in NBK protocol.

The channel model is represented in Fig. 3. All nodes are constrained by half-duplexing and each relay is isolated from the other relays. All channel coefficients $\hbar_1$, $\hbar_2$, $g_{1,i}$, $g_{2,i}$, $h_{i,1}$, and $h_{i,2}$, for $i = 1, \ldots, N$ are assumed to be flat fading with i.i.d. zero-mean, circularly symmetric complex Gaussian distribution and quasi-static in at least one frame period. So their magnitudes are Rayleigh distributed. $\hbar_k$, $g_{k,i}$, $h_{i,k}$ denote the channel coefficients between the k-th source and the k-th destination, the k-th source and the i-th relay, the i-th relay and the k-th destination with variances $\sigma_\hbar^2$, σ_g^2, σ_h^2 respectively. We assume that $\sigma_\hbar^2 = 1, \sigma_g^2 = \eta_g \sigma_\hbar^2, \sigma_h^2 = \eta_h \sigma_\hbar^2$. v_{r_i} and v_{d_k} denote the noises observed by r_i and d_k respectively, which are all i.i.d. Gaussian distribution with zero means and variances σ^2. Then we define the system SNR as $\rho \triangleq \frac{P}{\sigma^2}$.

The total transmission power consumed by sources and relays during a system frame period is

$$\mathcal{E}\left\{\mathbf{k}_1^T|\mathbf{x}_{s_1}|^2 + \mathbf{k}_2^T|\mathbf{x}_{s_2}|^2 + \mathbf{t}^T|\mathbf{x}_r|^2\right\} = 2NP, \tag{1}$$

where $|\mathbf{x}_{s_1}|^2 = [|x_{s_1,1}|^2, \cdots, |x_{s_1,N}|^2]^T$ (so as to $|\mathbf{x}_{s_2}|^2$ and $|\mathbf{x}_r|^2$), $\mathbf{k}_k = [\kappa_{k,1}, \cdots, \kappa_{k,N}]^T$ is the power allocation vector of s_k, in which the i-th element is the power allocation factor for the i-th symbol of $\mathbf{x}_{s_k}$, $\mathbf{t} = [\tau_1, \cdots, \tau_N]^T$ is the power allocation vector of relays, in which the i-th element is the power allocation factor of r_i, and $\mathbf{x}_r = [x_{r_1}, \cdots, x_{r_N}]^T$ denotes the transmission vector of relays, in which the i-th element is the transmitted symbol of r_i. Note that the transmitted signal x_{r_i} is the superposition of $x_{r_i,1}$ and $x_{r_i,2}$ which are the decoded symbols of $x_{s_1,i}$ and $x_{s_2,i}$ respectively.

In RNC protocols, symbols from two sources are jointly decoded by relays, i.e., for the i-th relay r_i, the received signal y_{r_i} is

$$y_{r_i} = g_{1,i}x_{s_1,i} + g_{2,i}x_{s_2,i} + v_{r_i}. \tag{2}$$

Then joint maximum likelihood (ML) decoding is performed at r_i, i.e.,

$$(x_{r_i,1}, x_{r_i,2}) = \underset{(x_{s_1,i}, x_{s_2,i}) \in Q}{\arg\min} \left\{|y_{r_i} - (g_{1,i}x_{s_1,i} + g_{2,i}x_{s_2,i})|^2\right\}. \tag{3}$$

By definition, the difference between RCNC and RGNC is the way by which $x_{r_i,1}$ and $x_{r_i,2}$ are superposed. In RCNC

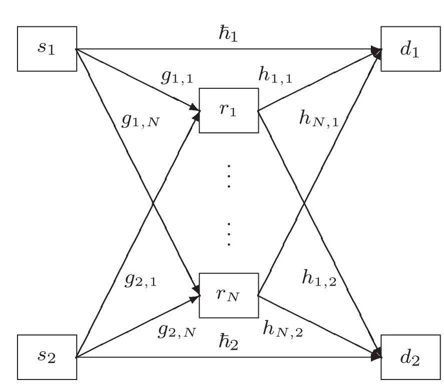

Fig. 3 Cooperative multicast channels with N relays.

protocol, $x_{r_i} = \frac{1}{\sqrt{2}}(x_{r_i,1}+x_{r_i,2})$ while in RGNC protocol, both decoded symbols are firstly demapped into two bit streams. Then the two streams are combined into a new one by Galois field operation. A new symbol x_{r_i} to be transmitted by r_i is thus produced by remapping the new bit stream. We denoted such process as $x_{r_i} = x_{r_i,1} \oplus x_{r_i,2}$ and rewrite (2) in matrix form as

$$\mathbf{y}_r = \mathbf{X}_r\mathbf{g} + \mathbf{v}_r, \tag{4}$$

where $\mathbf{X}_r$ is the $N \times 2N$ symbol matrix and $\mathbf{g}$ is the $2N \times 1$ channel vector, i.e.,

$$\mathbf{X}_r = \begin{pmatrix} x_{s_1,1} & x_{s_2,1} & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & x_{s_1,N} & x_{s_2,N} \end{pmatrix},$$
$$\mathbf{g} = [g_{1,1}, g_{2,1}, g_{1,2}, g_{2,2}, \cdots, g_{1,N}, g_{2,N}]^T, \tag{5}$$

and $\mathbf{v}_r = [v_{r_1}, \cdots, v_{r_N}]^T$ is the $N \times 1$ noise vector with $\mathbf{v}_r \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N)$. Then we focus on the signals received by d_k, i.e.,

$$\begin{aligned} y_{d_k,1} &= \hbar_k x_{s_k,1} + v_{d_k,1} \\ y_{d_k,2} &= h_{1,k}x_{r,1} + v_{d_k,2} \\ &\vdots \\ y_{d_k,2N-1} &= \hbar_k x_{s_k,N} + v_{d_k,2N-1} \\ y_{d_k,2N} &= h_{N,k}x_{r,N} + v_{d_k,2N}. \end{aligned} \tag{6}$$

We rewrite (6) in matrix form as

$$\mathbf{y}_{d_k} = \mathbf{X}_{d_k}\mathbf{h} + \mathbf{v}_{d_k}, \tag{7}$$

where $\mathbf{X}_{d_k}$ is the $2N \times (N+1)$ symbol matrix and $\mathbf{h}$ is the $(N+1) \times 1$ channel vector, i.e.,

$$\mathbf{X}_{d_k} = \begin{pmatrix} x_{s_k,1} & 0 & \cdots & 0 \\ 0 & x_{r_1} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ x_{s_k,N} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & x_{r_N} \end{pmatrix},$$
$$\mathbf{h} = [\hbar_k, h_{1,k}, h_{2,k}, \cdots, h_{N,k}]^T, \tag{8}$$

and $\mathbf{v}_{d_k} = [v_{d_k,1}, v_{d_k,2}, \cdots, v_{d_k,2N-1}, v_{d_k,2N}]^T$ is the $2N\times1$ noise vector with $\mathbf{v}_{d_k} \sim \mathcal{N}(0, \sigma^2\mathbf{I}_{2N})$. ML decoding is applied to d_k after every $2N$ time slots, i.e.,

$$\hat{\mathbf{x}}_s = \arg\min_{\mathbf{x}_s} \left\{ \sum_{i=1}^{N} |y_{d_k,2i-1} - \hbar_k x_{s_k,i}|^2 + \sum_{i=1}^{N} |y_{d_k,2i} - h_{i,k}x_{r_i}|^2 \right\}. \tag{9}$$

3. System Frame Error Probability

We measure the performance of the RCNC protocol and RGNC protocol in terms of the system frame error probability (SFEP). We define that a system frame $\mathbf{x}_s$ is successfully transmitted if and only if both destinations can successfully decode $\mathbf{x}_s$. So the SFEP of the multicast network can be calculated as

$$P_{sys} = P_r + (1 - P_r)(P_{d_1} + P_{d_2}), \tag{10}$$

where P_r is the FEP of the relays and P_{d_k} is the FEP of d_k on the condition that relays have successfully decoded the system frame. We denote PEP as the average pairwise error probability and R as the bit quantity per-channel use. Since we have in total 2^{2R} codewords, there is FEP $= 2^{2RN}$PEP. So to deduce P_r and P_{d_k}, we first focus on the corresponding PEP, i.e., $P_{PE,r}$ and P_{PE,d_k} respectively.

3.1 PEP of System Frame in Relays

For the $P_{PE,r}$, $2N$ symbols in the system frame $\mathbf{x}_s$ are decoded by N relays in the fashion that every two symbols in the frame are decoded by a different relay. Since all symbols in the system frame $\mathbf{x}_s$ are i.i.d., we conclude the following theorem.

Theorem 1: Let $u_{s_k,i} = \sqrt{\kappa_{k,i}/P}(x_{s_k,i} - \hat{x}_{s_k,i})$ is the normalized decoding error value of the symbol $x_{s_k,i}$. Suppose that $\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2) \neq 0$. Then when ρ is large enough, the average PEP jointly decoded by the N relays is

$$P_{PE,r} = \frac{(2N-1)!!2^{(2N-1)}\eta_g^{-N}\rho^{-N}}{N!\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2)}, \tag{11}$$

where $(2N+1)!! = 1 \cdot 3 \cdot 5 \cdot (2N+1)$.

Proof: According to [12], $P_{PE,r}$ can be expressed as

$$P_{PE,r} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{E}_{\mathbf{g}}\left\{\exp\left(-\rho\frac{\mathbf{g}^H\mathbf{U}_r^H\mathbf{I}_N^{-1}\mathbf{U}_r\mathbf{g}}{8\sin^2\theta}\right)\right\} \mathrm{d}\theta, \tag{12}$$

where $\mathbf{U}_r$ is the normalized decoding error matrix of $\mathbf{X}_r$, which can be expressed as

$$\mathbf{U}_r = \begin{pmatrix} u_{s_1,1} & u_{s_2,1} & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & u_{s_1,N} & u_{s_2,N} \end{pmatrix}. \tag{13}$$

Note that for a random column vector $\mathbf{z} \sim \mathcal{N}(0, \mathbf{\Sigma_z})$ and a Hermitian matrix $\mathbf{H}$, there is $\mathcal{E}[\exp(-\mathbf{z}^H\mathbf{H}\mathbf{z})] = 1/\det(\mathbf{I} + \mathbf{\Sigma_z}\mathbf{H})$. Then we take expectation with respect to $\mathbf{g}$ and rewrite (12) as

$$\begin{aligned} P_{PE,r} &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \det\left(\mathbf{I}_{2N} + \frac{\eta_g\rho}{8\sin^2\theta}\mathbf{U}_r^H\mathbf{U}_r\right)^{-1} \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\theta}{\prod_{i=1}^{N}\left(1 + \frac{\eta_g\rho|u_{s_1,i}|^2}{8\sin^2\theta} + \frac{\eta_g\rho|u_{s_2,i}|^2}{8\sin^2\theta}\right)}. \end{aligned} \tag{14}$$

When ρ is large enough,

$$P_{PE,r} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(8\sin^2\theta)^N \eta_g^{-N}\rho^{-N}}{\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2)}\,\mathrm{d}\theta. \tag{15}$$

On the other hand,

$$\int_0^{\frac{\pi}{2}} \sin^{2N}\theta\,\mathrm{d}\theta = \frac{1\cdot 3\cdot 5\cdots(2N-1)}{2\cdot 4\cdot 6\cdots 2N}\frac{\pi}{2}. \tag{16}$$

Plug (16) into (15), and then we get

$$P_{PE,r} = \frac{(2N-1)!!2^{(2N-1)}\eta_g^{-N}\rho^{-N}}{N!\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2)}. \tag{17}$$

Then we complete the proof. ■

3.2 PEP of System Frame in Destinations

Due to the symmetry of the statistical channel model, the two destinations have the same PEP. Without loss of generality, we focus on the PEP of d_k, i.e., P_{PE,d_k} and deduce P_{PE,d_k} on the condition that relays have successfully decoded the system frame. Then we conclude the following theorem.

Theorem 2: Suppose that $\sum_{i=1}^{N}|u_{s_1,i}|^2\prod_{i=1}^{N}|u_{r_i}|^2 \neq 0$. Then when relays can transmit the proper symbols to destinations and ρ is large enough, the average PEP of d_1 for the RCNC protocol is

$$P_{PE,d_k} = \frac{(2N+1)!!2^{(2N+1)}\eta_h^{-N}\rho^{-(N+1)}}{(N+1)!\sum_{i=1}^{N}|u_{s_k,i}|^2\prod_{i=1}^{N}|u_{r_i}|^2}, \tag{18}$$

where $u_{s_k,i} = \sqrt{\kappa_{k,i}/P}(x_{s_k,i} - \hat{x}_{s_k,i})$ is the normalized decoding error value of $x_{s_k,i}$ and $u_{r_i} = \sqrt{\tau_i/P}(x_{r_i} - \hat{x}_{r_i})$ is the normalized decoding error value of x_{r_i}.

Proof: P_{PE,d_k} can be expressed as

$$P_{PE,d_k} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{E}_{\mathbf{h}}\left\{\exp\left(-\rho\frac{\mathbf{h}^H\mathbf{U}_{d_k}^H\mathbf{I}_{2N}{}^{-1}\mathbf{U}_{d_k}\mathbf{h}}{8\sin^2\theta}\right)\right\}\mathrm{d}\theta, \tag{19}$$

where $\mathbf{U}_{d_k}$ is the normalized decoding error matrix of $\mathbf{X}_{d_k}$, which can be expressed as

$$\mathbf{U}_{d_k} = \begin{pmatrix} u_{s_k,1} & 0 & \cdots & 0 \\ 0 & u_{r_1} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ u_{s_k,N} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & u_{r_N} \end{pmatrix}. \tag{20}$$

Take expectation with respect to $\mathbf{h}$ in (19) and then

$$P_{PE,d_k} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \det\left(\mathbf{I}_{N+1} + \frac{\rho}{8\sin^2\theta}\boldsymbol{\Sigma}_{\mathbf{h}}\mathbf{U}_{d_k}^H\mathbf{U}_{d_k}\right)^{-1}\mathrm{d}\theta, \tag{21}$$

where $\boldsymbol{\Sigma}_{\mathbf{h}} = \mathrm{diag}(1, \eta_h, \cdots, \eta_h)$. The matrix in (21) is

$$\mathbf{I}_{N+1} + \frac{\rho}{8\sin^2\theta}\boldsymbol{\Sigma}_{\mathbf{h}}\mathbf{U}_{d_k}^H\mathbf{U}_{d_k} = \mathrm{diag}\Big(1 + \lambda\sum_{i=1}^{N}|u_{s_k,i}|^2, 1 + \eta_h\lambda|u_{r_1}|^2, \cdots, 1 + \eta_h\lambda|u_{r_N}|^2\Big). \tag{22}$$

where $\lambda = \frac{\rho}{8\sin^2\theta}$. Then (21) can be written as

$$P_{PE,d_k} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\theta}{\left(1 + \lambda\sum_{i=1}^{N}|u_{s_k,i}|^2\right)\prod_{i=1}^{N}(1 + \eta_h\lambda|u_{r_i}|^2)}. \tag{23}$$

When ρ is large enough, we get the P_{PE,d_k} as

$$\begin{aligned} P_{PE,d_k} &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(8\sin^2\theta)^{N+1}\eta_h^{-N}\rho^{-(N+1)}}{\sum_{i=1}^{N}|u_{s_k,i}|^2\prod_{i=1}^{N}|u_{r_i}|^2}\,\mathrm{d}\theta \\ &= \frac{(2N+1)!!2^{2N+1}\eta_h^{-N}\rho^{-(N+1)}}{(N+1)!\sum_{i=1}^{N}|u_{s_k,i}|^2\prod_{i=1}^{N}|u_{r_i}|^2}. \end{aligned} \tag{24}$$

Then we complete the proof. ■

Based on the two theorems, we turn to the SFEP. When ρ is large enough, (10) can be approximated as

$$\begin{aligned} P_{sys} \approx& P_r + P_{d_1} + P_{d_2} \\ =& 2^{2RN}P_{PE,r} + 2\cdot 2^{2RN}P_{PE,d_k} \\ =& 2^{2RN}\frac{(2N-1)!!2^{(2N-1)}\eta_g^{-N}\rho^{-N}}{N!\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2)} + \\ & 2\cdot 2^{2RN}\frac{(2N+1)!!2^{(2N+1)}\eta_h^{-N}\rho^{-(N+1)}}{(N+1)!\sum_{i=1}^{N}|u_{s_k,i}|^2\prod_{i=1}^{N}|u_{r_i}|^2}. \end{aligned} \tag{25}$$

However, the expression of P_{sys} does not mean that the N-relay multicast system can acquire the N-order diversity. For example, consider the denominator of P_r in (25), if one factor in $\prod_{i=1}^{N}(|u_{s_1,i}|^2 + |u_{s_2,i}|^2)$ is equal to 0 (In fact, approaches to $\frac{1}{\rho}$ since we omit $\frac{1}{\rho}$ in the deduction of the two theorems), the system diversity will decrease 1. In the sequel, we turn to the power allocation and precoding to increase both the coding gain and diversity gain.

4. Performance Improvements

According to the SFEP of the two RNC protocols, we go on with the detailed performance analysis. We first optimize the power allocation of RCNC and RGNC respectively in terms of SFEP. Then we propose the precoder design to achieve higher diversity gain.

4.1 Power Allocation

We then have a detailed discussion on Eq. (25). When ρ is large enough, we only consider the mostly happened cases, which cause the system only to achieve the 1-order diversity gain. We first consider P_r in (25). The mostly happened cases are that $N-1$ factors of the $\prod_{i=1}^{N}(|u_{s_1,i}|^2+|u_{s_2,i}|^2)$ are equal to 0. Since there are N such cases, when ρ is large enough,

$$P_r = 2^{2R}\sum_{i=1}^{N}\frac{2\eta_g^{-1}\rho^{-1}}{|u_{s_1,i}|^2+|u_{s_2,i}|^2}+O(\rho^{-2})+\cdots+O(\rho^{-N})$$
$$\approx 2^{2R}\sum_{i=1}^{N}\frac{2\eta_g^{-1}\rho^{-1}}{|u_{s_1,i}|^2+|u_{s_2,i}|^2}. \tag{26}$$

Then we focus on P_{d_k}. In RCNC protocol, if relays successfully decode the $\mathbf{x}_s$, then we have $x_{r_i,1}=x_{s_1,i}$ and $x_{r_i,2}=x_{s_2,i}$. So P_{d_k} of RCNC can be written as

$$P_{d_k}^{RCNC} = 2^{2RN}\frac{(2N+1)!!2^{(2N+1)}\eta_h^{-N}\rho^{-(N+1)}}{(N+1)!\sum_{i=1}^{N}|u_{s_k,i}|^2\prod_{i=1}^{N}|u_{r_i,1}+u_{r_i,2}|^2}, \tag{27}$$

where $u_{r_i,k}=\sqrt{\tau_i/2P}(x_{s_k,i}-\hat{x}_{s_k,i})$. Obviously, when joint decoding is performed at d_k, the wrongly decoding event at $x_{s_{\bar{k}},i}$ occurs with the most probability, where $\bar{k}$ is the complementary element of k in set $\{1,2\}$. So if ρ is large enough,

$$P_{d_k}^{RCNC} \approx 2^{R}\sum_{i=1}^{N}\frac{2\eta_h^{-1}\rho^{-1}}{|u_{r_i,\bar{k}}|^2}. \tag{28}$$

On the other hand, in RGNC protocol, since $x_{r_i}\in Q$ is isolated from $x_{s_k,i}$, when ρ is large enough, we only consider the mostly happened cases that only 1-order diversity gain can be achieved by the system. Then the FEP of d_k in RGNC can be approximated as

$$P_{d_k}^{RGNC} \approx 2^{R}\sum_{i=1}^{N}\frac{2\rho^{-1}}{|u_{s_k,i}|^2}+2^{R}\sum_{i=1}^{N}\frac{2\eta_h^{-1}\rho^{-1}}{|u_{r_i}|^2}. \tag{29}$$

Then we focus on the power allocation schemes of the two protocols. Due to the symmetry of the channel model, it is obvious that power allocated for each symbol of the two sources should be equal, i.e., $\mathbf{k}_1=\mathbf{k}_2=[\kappa,\cdots,\kappa]^T$. Meanwhile, power allocated for each relay should be equal, i.e., $\mathbf{t}=[\tau,\cdots,\tau]^T$. So the optimal power allocation schemes is to study the proper relation between κ and τ. Then we have the following theorem.

Theorem 3: When ρ is large enough, the statistical channel state information based optimal power allocation is to choose the power allocation factors as

$$\tau = \frac{\sqrt{\eta_g}}{\sqrt{\eta_g}+\sqrt{2^{R-2}\eta_h}} \quad \text{for RCNC},$$
$$\tau = \frac{\sqrt{\eta_g}}{\sqrt{\eta_g}+\sqrt{2^{R-1}\eta_h+2}} \quad \text{for RGNC}. \tag{30}$$

κ can be worked out according to the power constraint $2\kappa+\tau=1$.

Proof: When ρ is large enough, we rewrite (10) as $P_{sys}=P_r+P_{d_1}+P_{d_2}$. Since $\mathcal{E}(|x_{s_k,i}-\hat{x}_{s_k,i}|^2)=4P$ and in RGNC, $\mathcal{E}(|x_{r_i}-\hat{x}_{r_i}|^2)=4P$, the expectations of the decoding error value $\mathcal{E}(|u_{s_k}|^2)=4\kappa$, $\mathcal{E}(|u_{r_i,k}|^2)=2\tau$ and $\mathcal{E}(|u_{r_i}|^2)=4\tau$. Then in RCNC protocol,

$$\mathcal{E}(P_{sys}^{RCNC}) = 2^{2R}N\frac{\eta_g^{-1}\rho^{-1}}{4\kappa}+2\cdot 2^{R}N\frac{\eta_h^{-1}\rho^{-1}}{\tau}$$
$$= 2^{R+1}N\rho^{-1}\left(\frac{2^{R-3}\eta_g^{-1}}{\kappa}+\frac{\eta_h^{-1}}{\tau}\right). \tag{31}$$

The optimal power allocation in RCNC is to

$$\text{minimum} \quad \frac{2^{R-3}\eta_g^{-1}}{\kappa}+\frac{\eta_h^{-1}}{\tau},$$
$$\text{subject to} \quad 2\kappa+\tau=1. \tag{32}$$

So we get

$$\tau = \frac{\sqrt{\eta_g}}{\sqrt{\eta_g}+\sqrt{2^{R-2}\eta_h}} \quad \text{for RCNC}. \tag{33}$$

In RGNC protocol,

$$\mathcal{E}(P_{sys}^{RGNC}) = 2^{2R}N\frac{\eta_g^{-1}\rho^{-1}}{4\kappa}+2^{R}N\frac{\rho^{-1}}{\kappa}+2^{R}N\frac{\eta_h^{-1}\rho^{-1}}{\tau}$$
$$= 2^{R}N\rho^{-1}\left(\frac{2^{R-2}\eta_g^{-1}}{\kappa}+\frac{1}{\kappa}+\frac{\eta_h^{-1}}{\tau}\right) \tag{34}$$

The optimal power allocation in RGNC is to

$$\text{minimum} \quad \frac{2^{R-2}\eta_g^{-1}}{\kappa}+\frac{1}{\kappa}+\frac{\eta_h^{-1}}{\tau},$$
$$\text{subject to} \quad 2\kappa+\tau=1. \tag{35}$$

So we get

$$\tau = \frac{\sqrt{\eta_g}}{\sqrt{\eta_g}+\sqrt{2^{R-1}\eta_h+2}} \quad \text{for RGNC}. \tag{36}$$

Then we complete the proof. ■

In the numerical results, we select $\eta_g=\eta_h=1$ and consider the two-relay scenario. Figure 4 and Fig. 5 show the power allocation schemes under different values of 2κ. From Theorem 3, the optimal power allocation scheme is related with the transmission rate R. So Fig. 4 considers the scenario with transmission rate $R=2$, i.e., 2 bit per-channel use (BPCU) and Fig. 5 considers another scenario, where transmission rate is $R=4$, i.e., 4 BPCU. When $R=2$, we can get the optimal power allocation schemes as $2\kappa=\frac{1}{2}$ for RCNC and $2\kappa=\frac{2}{3}$ for RGNC according to (30). Meanwhile, when $R=4$, we follow the same way and get the optimal power allocation schemes as $2\kappa=\frac{2}{3}$ for RCNC and $2\kappa=$

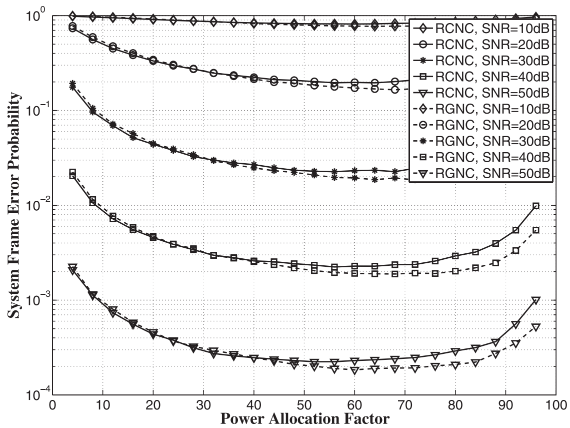

Fig. 4 2 BPCU based system frame error probability under different power allocation schemes for the two protocols with the channel gains $\eta_g = \eta_h = 1$. We select the number of relays as $N = 2$. Abscissa is the percentile value of 2κ.

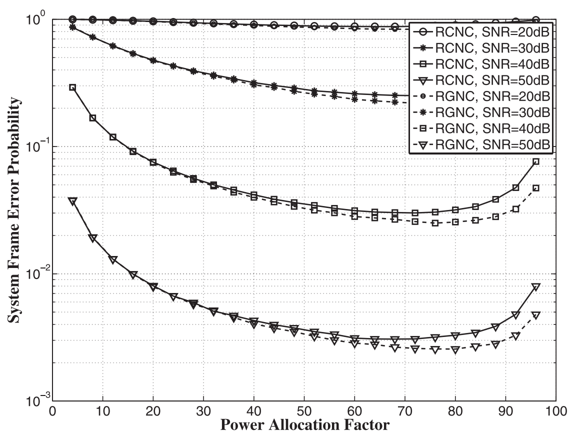

Fig. 5 4 BPCU based system frame error probability under different power allocation schemes for the two protocols with the channel gains $\eta_g = \eta_h = 1$. We select the number of relays $N = 2$. Abscissa is the percentile value of 2κ.

$\frac{\sqrt{10}}{\sqrt{10}+1} \approx 0.76$ for RGNC. Figure 4 and Fig. 5 validate the predictions of Theorem 3.

4.2 Precoder Design

From the analysis of the power allocation, it is obvious that the system only reach the 1-order diversity, which is due to that the transmission scheme can not ensure the denominators of P_{sys} in (25) to be nonzero. So the optimal power allocation only improves the coding gain while remains the poor diversity gain.

We then turn to the precoder design to achieve higher diversity gain. We follow the similar way of precoder design as that in [13] where precoder is designed for MIMO systems. In MIMO systems, one kind of space-time precoders is designed as the normalized Vandermonde matrix to achieve full diversity gain [13], i.e.,

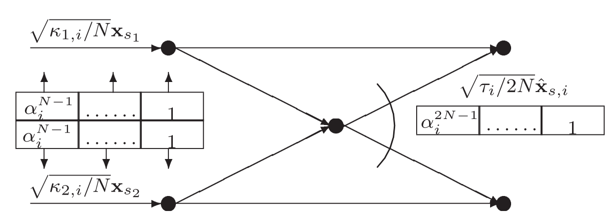

Fig. 6 The joint source-relay precoder design for RCNC in $2-N-2$ multicast system, where we consider the i-th relay. $\hat{\mathbf{x}}_{s,i}$ denotes the system frame decoded by the i-th relay.

$$\mathbf{\Theta} = \frac{1}{\sqrt{L}} \begin{pmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{L-1} \\ 1 & \alpha_2 & \cdots & \alpha_2^{L-1} \\ \vdots & \vdots & & \vdots \\ 1 & \alpha_L & \cdots & \alpha_L^{L-1} \end{pmatrix}_{L\times L} \tag{37}$$

where $\{\alpha_i\}_{i=1}^{L}$ have unit modulus. And L is an integer. If $L = 2^m$, where m is an arbitrary positive integer, then $\alpha_i = e^{j\pi(4i-1)/2L}$; else if $L = 3 \times 2^m$, then $\alpha_i = e^{j\pi(6i-1)/3L}$.

We denote $\mathbf{\Theta}_s$ and $\mathbf{\Theta}_r$ as the precoders for the sources and relays respectively. We select $\mathbf{\Theta}_s$ as the precoder in (37), where we make $L = N$. For $\mathbf{\Theta}_r$, we first choose a precoder $\mathbf{\Theta}_{2N}$ as that in (37), where we make $L = 2N$. Then $\mathbf{\Theta}_r$ is obtained by selecting arbitrary N rows from $\mathbf{\Theta}_{2N}$, i.e.,

$$\mathbf{\Theta}_s = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{N-1} \\ 1 & \alpha_2 & \cdots & \alpha_2^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & \alpha_N & \cdots & \alpha_N^{N-1} \end{pmatrix}_{N\times N}$$

$$\mathbf{\Theta}_r = \frac{1}{\sqrt{2N}} \begin{pmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{2N-1} \\ \vdots & \vdots & & \vdots \\ 1 & \alpha_i & \cdots & \alpha_i^{2N-1} \\ \vdots & \vdots & & \vdots \\ 1 & \alpha_N & \cdots & \alpha_N^{2N-1} \end{pmatrix}_{N\times 2N} \tag{38}$$

Figure 6 shows the proposed joint source-relay precoder scheme in RCNC protocol, which need each relay participate to decode the whole system frame $\mathbf{x}_s$, not one symbol of the frame. Then the transmission scheduling strategy should be rearranged during a frame period. Figure 7 shows the improved scheduling strategy which composes of two stages. In the first stage, each relay receives all the signals from the two sources, decodes the system frame, and applies the precoders to the decoded frame. Only those relays which successfully decode the system frame participate to the second stage, where relays transmit the processed symbols in turn. The following theorem illuminates that the system after precoding can achieve N-order diversity gain.

Theorem 4: After applying precoders to the new scheduling strategy, RCNC protocol can achieve the N-order diversity gain.

Proof: In the new transmission scheme with precoding, since all relays should decode the system frame, we

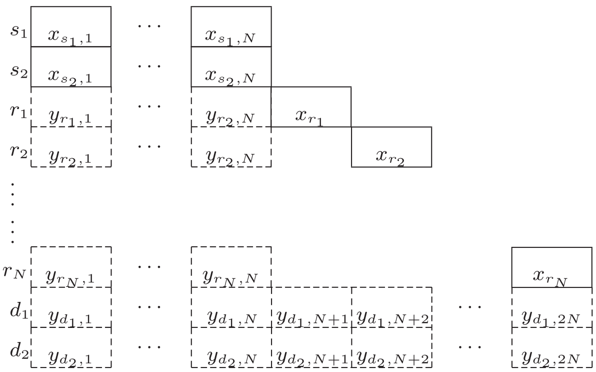

Fig. 7 Improved transmitting order for proposed precoding.

first focus on the situation that n of the total N relays can successfully decode the system frame. In this situation, the PEP of d_k will be

$$P_{PE,d_k}^{RCNC} = \frac{(2n+1)!!2^{(2n+1)}\eta_h^{-n}\rho^{-(n+1)}}{(n+1)!\sum_{i=1}^{n}|\mu_{s_k,i}|^2\prod_{i=1}^{n}|\mu_{s,i}|^2}, \tag{39}$$

where

$$|\mu_{s_k,i}|^2 = \frac{1}{N}\left|\sum_{j=0}^{N-1}\alpha_i^j u_{s_k,j}\right|^2,$$

$$|\mu_{s,i}|^2 = \frac{1}{2N}\left|\sum_{j=0}^{N-1}\alpha_i^{2j}u_{r_j,1} + \sum_{j=0}^{N-1}\alpha_i^{2j+1}u_{r_j,2}\right|^2. \tag{40}$$

When ρ is large enough, we only consider the wrongly decoding event of $\mathbf{x}_{s_{\bar{k}}}$ at d_k. Then the FEP of d_k is

$$\begin{aligned} P_{d_k}^{RCNC} &= 2^{RN}\frac{(2n-1)!!2^{(2n-1)}\eta_h^{-n}\rho^{-n}}{n!\prod_{i=1}^{n}\frac{1}{N}\left|\sum_{j=0}^{N-1}\alpha_i^{2j+1}u_{r_j,\bar{k}}\right|^2} + O(\rho^{-(n+1)}) \\ &\approx 2^{RN}\frac{(2n-1)!!2^{(2n-1)}\eta_h^{-n}\rho^{-n}}{n!\prod_{i=1}^{n}\frac{1}{N}\left|\sum_{j=0}^{N-1}\alpha_i^{2j+1}u_{r_j,\bar{k}}\right|^2}. \end{aligned} \tag{41}$$

According to the principle of precoders design, the denominator of (41) will be equal to zero if and only if $\mathbf{x}_{s_{\bar{k}}}$ can be successfully decoded by d_k, which means that the system achieves the n-order diversity on the condition that n of N relays successfully decode the system frame. Then we turn to the event that n relays successfully decode $\mathcal{D}(n)$, which means that n relays decode the system frame. So the probability of event $\mathcal{D}(n)$ is

$$P_{\mathcal{D}(n)} = \binom{N}{n}(1-P(r))^n P(r)^{N-n} \tag{42}$$

where $P(r) = 2^{2RN}2\eta_g^{-1}\rho^{-1}/\sum_{i=1}^{N}(|\mu_{s_1,i}|^2+|\mu_{s_2,i}|^2)$ is the frame error probability of each relay. When ρ is large enough, the SFEP after applying precoder is

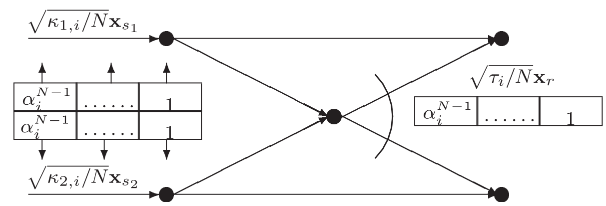

Fig. 8 The joint source-relay precoder design for RGNC in $2-N-2$ multicast system, where we consider the i-th relay. $\mathbf{x}_r = [x_{r_1}, \cdots, x_{r_N}]^T$, where $x_{r_i} \in Q$.

$$\begin{aligned} P_{sys}^{RCNC} &= \sum_{n=0}^{N} P_{\mathcal{D}(n)} 2P_{d_k}^{RCNC} \\ &\approx \sum_{n=0}^{N} 2\binom{N}{n} P(r)^{N-n} P_{d_k}^{RCNC} \end{aligned} \tag{43}$$

It is obvious that $P_{sys}^{RCNC} \sim \rho^{-N}$. So the RCNC protocol can achieve N-order diversity gain after precoding. Then we complete the proof. ■

In RGNC protocol, we follow the same scheduling strategy of RCNC, i.e., all relays participate to decode the system frame $\mathbf{x}_s$. Only the relays set $\mathcal{D}(n)$ which can successfully decode the system frame combines the $\mathbf{x}_s$ in Galois field and form the frame $\mathbf{x}_r$. $\mathbf{\Theta}_s$ is then used as the precoder matrixes for both the two sources and the relays. Figure 8 shows the precoders for the sources and relays. Note that $\mathbf{x}_r = [x_{r_1}, \cdots, x_{r_N}]^T$ is the frame to be transmitted by $\mathcal{D}(n)$. However, this precoding scheme can not achieve the full diversity gain since the signals transmitted by relays are isolated from the signals transmitted by sources, which cause the symbols in $s \to d$ link can not be protected by relays. So the system only achieve the 1-order diversity gain. And the precoder in RGNC only improve the performance by providing more coding gain.

In numerical results we choose the transmission rate $R = 4$, i.e., 4 BPCU and the relay number $N = 2$. We firstly study the effect of precoders on the $r \to d$ link. So we suppose that each $s \to r$ channel quality is so perfect that no decoding errors happen at relays. We denote such scenario as $\eta_g = \infty$, $\eta_h = 1$. See Fig. 9, in RCNC, the slope of the SFEP curve after precoding means that the prcoder of each relay makes the $r \to d$ link achieve full diversity gain, i.e., 2-order diversity gain. However, even after precoding in RGNC, the system only achieves 1-order diversity gain as analyzed in the priori paragraph, i.e., the signals transmitted by relays are isolated from the signals transmitted by sources. Precoding in RGNC only brings more coding gain. Then we consider the common scenario $\eta_g = \eta_h = 1$. From the slope of the SFEP curves in Fig. 10, we can see that when there are no precoders, the both protocols only achieve the 1-order diversity gain even with the optimal power allocation. In RCNC, precoders in sources and relays help the system to achieve the full diversity gain, i.e., 2-order diversity gain as Theorem 4 has predicted. While after precoding in RGNC, the whole system only achieve 1-order diversity gain due to the bottleneck of the $r \to d$ link, which can also

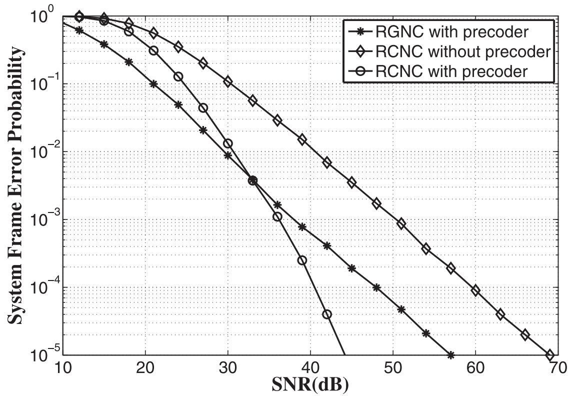

Fig. 9 4 BPCU based system frame error probability of RCNC and RGNC, where the channel gains $\eta_g = \infty$, $\eta_h = 1$. We consider the 2-relay scenario with and without precoders.

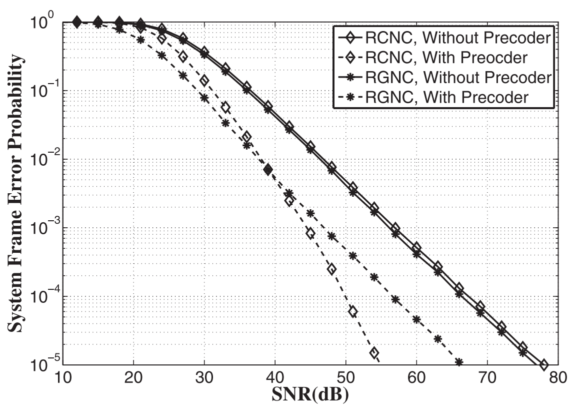

Fig. 10 4 BPCU based system frame error probability of RCNC and RGNC, where the channel gains $\eta_g = \eta_h = 1$. We consider the 2-relay scenario with and without precoders.

be seen from Fig. 10.

5. Conclusion

We propose two protocols for the $2-N-2$ multicast systems in complex field and Galois field to achieve higher system throughput by consuming the less transmission time slots. Meanwhile, we define and deduce the system frame error probability as the measurement to evaluate the two protocols. According to the expressions of SFEP, we conclude that a proper power allocation scheme can improve the system performance. However, power allocation can not enhance the system diversity gain. Precoder is then designed to achieve higher system performance. We also design the corresponding improved scheduling strategy as well. Simulations show the proposed precoder can distinctly improve the system performance.

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