A Multilevel Soft Quantize-and-Forward Scheme for Multiple Access Relay Systems

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***Abstract*—This paper proposes the novel technique of multilevel threshold based soft quantization (MLT-SQ) for a multiple access relay system (MARS). The scheme is suitable for systems using binary phase-shift keying (BPSK) and network coding at the relay. In the proposed MLT-SQ protocol, the relay evaluates the reliabilities, expressed as log-likelihood ratios (LLRs), of the received signals from the two sources. It then computes the LLRs of the network-coded packet and quantizes these using a set of optimized multilevel thresholds, forwarding the resulting "*quantized soft symbols*" to the destination. We provide the derivation for the bit error rate (BER) at the destination, based on which we optimize the multilevel thresholds to minimize the BER. Compared to competing schemes, the performance of our system is superior in terms of BER when the same amount of channel state information (CSI) is exploited.**

I. Introduction

Cooperative communication for wireless networks promises improved transmit diversity and spectral efficiency [1]. A judiciously designed signal forwarding technique at the relay and an accurate detection technique at the destination can greatly enhance the system performance. Two classical relaying protocols are in use; amplify-and-forward (AF) and decode-and-forward (DF). With the AF strategy, the relay transmits an amplified version of its received signal to the destination. AF does not perform any noise suppression; therefore, it suffers from severe noise propagation and power inefficiency issues. By use of a detector/decoder at the relay, the DF protocol is able to re-generate the transmitted signal so that the noise propagation can be avoided. But any decoding error in the regenerated signal can cause a performance degradation at the destination.

A recently proposed and promising relay protocol called *soft information relaying* (SIR) has gained significant attention [2]-[5]. In [2], the authors studied the implementation of SIR in conjunction with distributed turbo coding (DTC). A scheme for soft re-encoding and soft parity symbol forwarding was proposed in [3]. A soft forwarding technique based on symbol-wise mutual information (SMI) was investigated in [4] using physical layer network coding (PLNC) in the two-way relay channel. Recently, a soft decode-compress-forward scheme was proposed in [5]; this work featured a new model, referred to as the *soft scalar model*, to facilitate the LLR computation at the destination. A drawback of these unquantized soft forwarding schemes is that a heuristic and not quite accurate model must be adopted for the equivalent noise (comprising contributions from both channel and relay operations), in order to form LLRs at the destination. This is an inherent problem which originates due to the unquantized nature of the signal transmitted from the relay.

An LLR-threshold based soft quantize-and-forward protocol was presented in [6]-[9]. In these works, the relay evaluates the reliabilities of the received symbols (from each source) and selects the corresponding level to send to the destination. When the symbol LLR is sufficiently large (i.e., the symbol decision is deemed to be sufficiently reliable), the relay transmits the hard decision for the symbol; otherwise it is silent. Therefore, these works [6]-[8] may be regarded as a soft forwarding scheme based on three-level LLR quantization in simple relay networks; however, the extension to multilevel quantization and a multiple access relay is not trivial. Also, in [9], a mutual information based soft information forwarding scheme was proposed.

In this paper, we propose a new framework for designing a *multilevel* threshold based soft quantization (MLT-SQ) protocol using multiple thresholds which are optimized for minimum overall bit error rate (BER) at the destination in a multiple access relay system using network coding. To the best of the authors' knowledge, such a soft quantization scheme (i.e., based on multilevel optimized thresholds) has not previously been reported in the literature. Simulation results are presented which compare the performance of the proposed scheme to competing schemes. We compare with three-level soft forwarding, uncoded DF, and a 'Genie Aided' scheme as benchmarks; as another competing scheme, we also include the link-adaptive regeneration (LAR) protocol as proposed in [8]. The simulation results demonstrate that the proposed MLT-SQ scheme efficiently mitigates error propagation in a power efficient manner when compared to the benchmark schemes.

II. System Model

In this paper, we consider a four-terminal topology as shown in Fig. 1. It is assumed that there is a direct transmission

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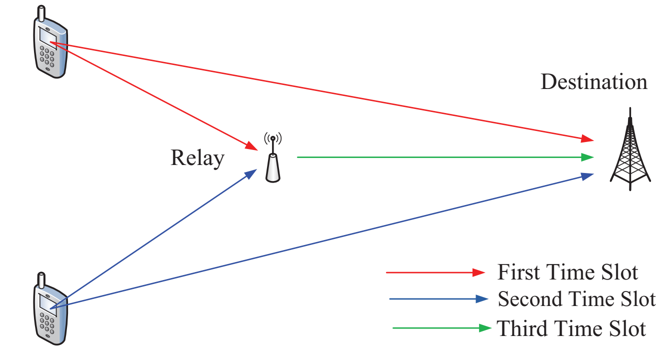

Figure 1. The proposed multiple access relay system in half-duplex mode.

from the sources to the destination. The sources[1] S_1 and S_2 transmit an uncoded frame, of length N, of BPSK modulated symbols in the first and second time slots respectively; these are received both by the relay and the destination. In the third time slot, the relay aids the destination by transmitting a network coded message based on the signals received in the first and second time slots. We assume that all nodes have only one antenna working in a half-duplex mode.

We denote[2] by h_{iR}, h_{iD}, and h_{RD} where $i \in \{S_1, S_2\}$, the channel coefficients between i and R, between i and D, and between R and D, respectively. The corresponding distances between nodes are denoted by d_{iR}, d_{iD}, and d_{RD} respectively. We assume that h_{iR}, h_{iD}, and h_{RD} are independent and identically Rayleigh distributed. The channel gains are related to the corresponding distances by the attenuation exponent γ , i.e., $\lambda_{iR} = 1/(d_{iR})^{\gamma}$, $\lambda_{iD} = 1/(d_{iD})^{\gamma}$, and $\lambda_{RD} = 1/(d_{RD})^{\gamma}$ respectively. We consider quasi-static fading channels, i.e., the channel coefficients are constant during one transmission phase, and change independently from one phase to another. In each time slot, the source bit $u_{i,j} \in \{0, 1\}$ is mapped to a BPSK symbol $x_{i,j} \in \{-1, 1\}$ via the mapping $0 \mapsto +1$, $1 \mapsto -1$. The received signals at the relay and destination corresponding to the source i in the jth time slot are given by

$$\begin{aligned} y_{iR,j} &= \sqrt{P_i} h_{iR} x_{i,j} + n_{iR,j}, \\ y_{iD,j} &= \sqrt{P_i} h_{iD} x_{i,j} + n_{iD,j}, \end{aligned} \tag{1}$$

where $n_{iR,j}$ and $n_{iD,j}$ are i.i.d. real Gaussian random variables, each having zero mean and the same variance $\sigma^2 = N_0/2$. Also, P_i is the transmit power constraint from node i (here we assume $P_1 = P_2 = 1$).

[1]Unless otherwise stated, in this work S, R, D stand for source, relay, and destination respectively. Throughout the paper all vectors are taken to be row vectors. Also, vectors are denoted by bold letters and the i^{th} element by an italic letter. We use regular letters to denote scalars (including random variables). For a random variable x, we use $\mathbb{E}[x]$ to denote the expected value of x. The soft information corresponding to symbol a is represented by $\tilde{a}$. The notation $\mathrm{sgn}(\cdot)$ indicates the sign of the variable in the bracket.

[2]Unless otherwise stated, in this work we assume $i \in \{S_1, S_2\}$ and $j \in \{1, 2, \ldots, N\}$.

As the network coding operation in the hard decision domain, i.e., XOR, between two bits is equivalent to the multiplication of the corresponding BPSK symbols, the network coded symbol $x_{R,j}$ can be obtained via $x_{R,j} = \hat{x}_{1,j}\hat{x}_{2,j}$, where $\hat{x}_{i,j}$ is the hard decision of $x_{i,j}$ at the relay. The relay may then transmit these network coded symbols to the destination in order to achieve diversity (BPSK modulation being assumed for retransmission), at a power of P_R. However, at low source-relay SNR, $x_{i,j}$ is often detected incorrectly, and forwarding hard decisions can result in erroneous symbols being propagated to the destination. Instead of simply checking the polarity of the relay received signal $y_{iR,j}$, in our proposed MLT-SQ scheme the relay performs a soft decision. More specifically, it extracts the appropriate soft information by computing each LLR $L_{i,j} = \ln\left(\frac{p(x_{i,j}=+1|y_{iR,j})}{p(x_{i,j}=-1|y_{iR,j})}\right) = \frac{2h_{iR}}{\sigma^2} y_{iR,j}$. The relay computes the LLR value $L_{R,j}$ for each network coded symbol $x_{R,j}$ after detection at the relay as follows:

$$L_{R,j} = 2\tanh^{-1}\left(\tanh\left(L_{S_1,j}/2\right)\tanh\left(L_{S_2,j}/2\right)\right) \tag{2}$$

As an attempt to achieve diversity, if the LLR value in (2) shows that the confidence regarding the decision is high, i.e., the absolute LLR value is larger than the lowest preset optimal threshold, then a corresponding soft quantized value $\tilde{x}_{R,j} = f(L_{R,j})$ is forwarded to the destination, where the function $f(.)$ will be elucidated in Section III.

III. Multilevel Threshold based Soft Quantization Scheme

In what follows, we will focus on the relay processing, i.e., MLT-SQ and relay-destination transmission, especially the soft-message preparation and soft network coding. The LLR $L_{R,j}$ of the jth soft network coded bit as given by (2) is quantized according to

$$\tilde{x}_{R,j} = \begin{cases} \mathrm{sgn}(L_{R,j}) & |L_{R,j}| \geq |L_1| \\ \frac{2}{3}\mathrm{sgn}(L_{R,j}) & |L_2| \leq |L_{R,j}| < |L_1| \\ \frac{1}{3}\mathrm{sgn}(L_{R,j}) & |L_3| \leq |L_{R,j}| < |L_2| \\ 0 & |L_{R,j}| < |L_3|. \end{cases} \tag{3}$$

Here we have 7 symmetrical quantization levels (3 positive, 3 negative, and the zero level), with a spacing of $1/3$ between adjacent levels.

The equation presented in (3) may be viewed as a soft symbol mapping which acts as a substitute for the $\tanh$ function as used in [2]-[5]. The principal advantage of the proposed soft symbol mapping is there is a finite number of possible transmit levels rather than an infinite number as would be produced by the $\tanh$ function mapping; this facilitates exact LLR computation at the destination. The signal transmitted by the relay to the destination is given by

$$y_{RD,j} = \sqrt{P_R}\beta h_{RD}\tilde{x}_{R,j} + n_{RD,j}, \tag{4}$$

where $n_{RD,j}$ is a Gaussian noise with zero mean and variance $\sigma^2 = N_0/2$, $j = 1, 2, \ldots, N$, P_R is the relay transmit power and it is set to be $P_R = 1$, here the factor β is chosen to satisfy the transmit power constraint at the relay, i.e., $\beta =$

$\sqrt{\frac{1}{\frac{1}{N}\sum_{j=1}^{N}|\tilde{x}_{R,j}^2|}}$. The formation of optimal thresholds[3] L_k will be explained in the next sections. The instantaneous channel state information (CSI) of h_{iD} and h_{RD} are assumed to be available at the relay when the relay performs its optimization of quantization levels.

A. Derivation of Optimal Multilevel Thresholds

In this section, we derive the optimal individual thresholds in order to minimize the BER at the destination. We remark that the approach proposed here is different from existing approaches reported in the literature which use only a single positive threshold (3 levels) [6]-[9]. In the analysis, as mentioned, we only consider three positive thresholds (7 levels) for ease of presentation; the analysis can easily be extended to any number of positive thresholds. We denote by $\varepsilon_{c,k}$ the event that the magnitude of bit LLR $|L_{R,j}|$ lies above the kth threshold and $\tilde{x}_{R,j}$ has the *correct* sign, i.e., $\text{sgn}(\tilde{x}_{R,j}) = \text{sgn}(x_{R,j})$ where we define $x_{R,j} = x_{1,j}x_{2,j}$. Similarly, we denote by $\varepsilon_{e,k}$ the event that the magnitude of bit LLR $|L_{R,j}|$ lies above the kth threshold and $\tilde{x}_{R,j}$ has the *incorrect* sign, i.e., $\text{sgn}(\tilde{x}_{R,j}) \neq \text{sgn}(x_{R,j})$. Finally, the event ε_s represents the event that $|L_{R,j}|$ is smaller than the smallest threshold L_3, i.e., the relay is silent. These events are illustrated as follows:

$$\begin{array}{lcll}
\varepsilon_{c,1} & : & |L_{R,j}| \geq |L_1|, & \text{sgn}(\tilde{x}_{R,j}) = \text{sgn}(x_{R,j}), \\
\varepsilon_{e,1} & : & |L_{R,j}| \geq |L_1|, & \text{sgn}(\tilde{x}_{R,j}) \neq \text{sgn}(x_{R,j}), \\
\varepsilon_{c,2} & : & |L_2| \leq |L_{R,j}| < |L_1|, & \text{sgn}(\tilde{x}_{R,j}) = \text{sgn}(x_{R,j}), \\
\varepsilon_{e,2} & : & |L_2| \leq |L_{R,j}| < |L_1|, & \text{sgn}(\tilde{x}_{R,j}) \neq \text{sgn}(x_{R,j}), \\
\varepsilon_{c,3} & : & |L_3| \leq |L_{R,j}| < |L_2| & \text{sgn}(\tilde{x}_{R,j}) = \text{sgn}(x_{R,j}), \\
\varepsilon_{e,3} & : & |L_3| \leq |L_{R,j}| < |L_2|, & \text{sgn}(\tilde{x}_{R,j}) \neq \text{sgn}(x_{R,j}), \\
\varepsilon_s & : & |L_{R,j}| < |L_3|. &
\end{array} \tag{5}$$

Consequently, the average BER at the destination $P_{error,i}$ for source i is expressed as

$$\begin{aligned} P_{error,i} &= P_{i,1}^{(c)} \Pr(\varepsilon_{c,1}) + P_{i,2}^{(c)} \Pr(\varepsilon_{c,2}) \\ &+ P_{i,3}^{(c)} \Pr(\varepsilon_{c,3}) + P_{i,1}^{(e)} \Pr(\varepsilon_{e,1}) + P_{i,2}^{(e)} \Pr(\varepsilon_{e,2}) \\ &+ P_{i,3}^{(e)} \Pr(\varepsilon_{e,3}) + P_i^{(s)} \Pr(\varepsilon_s), \end{aligned} \tag{6}$$

where $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$ respectively indicate the bit error rate at the destination: where the relay transmits the kth quantization level and this has the *correct* sign; where the relay transmits the kth quantization level and this has the *incorrect* sign; and where the relay stays silent. The average BER of two sources at the destination is denoted by $P_{error} = \frac{1}{2}(P_{error,1} + P_{error,2})$. Now, we concentrate on determining $\{\Pr(\varepsilon_{c,k})\}$ and $\{\Pr(\varepsilon_{e,k})\}$ for $k = 1, 2, 3$, and $\Pr(\varepsilon_s)$. We begin by investigating the PDF of $L_{R,j}$ at the relay node. For a given channel realization, the PDF of $L_{R,j}$, conditioned on the underlying BPSK symbol being equal to $+1$, can be approximated to a Gaussian random variable having PDF

$$P_{L_{R,j}}(L) = \frac{1}{\sqrt{2\pi\sigma_L^2}} \exp\left(-\frac{(L-\mu_L)^2}{2\sigma_L^2}\right), \tag{7}$$

[3]The subscript $k \in \{1, 2, 3\}$ indicates a positive threshold level.

where the relationship described in [10] holds, i.e., that the variance is twice the mean ($\sigma_L^2 = 2\mu_L$). Next, the probabilities we seek may be expressed in terms of the PDF $P_{L_{R,j}}(L)$ and the multilevel thresholds L_k:

$$\begin{aligned} \Pr(\varepsilon_{c,1}) &= \textstyle\int_{L_1}^{+\infty} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_{c,2}) &= \textstyle\int_{L_2}^{L_1} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_{c,3}) &= \textstyle\int_{L_3}^{L_2} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_{e,1}) &= \textstyle\int_{-\infty}^{-L_1} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_{e,2}) &= \textstyle\int_{-L_1}^{-L_2} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_{e,3}) &= \textstyle\int_{-L_2}^{-L_3} p_{L_{R,j}}(L)dL, \\ \Pr(\varepsilon_s) &= \textstyle\int_{-L_3}^{L_3} p_{L_{R,j}}(L)dL. \end{aligned} \tag{8}$$

Note also that

$$\Pr(\varepsilon_s) = 1 - \sum_{k=1}^{3} \left[\Pr(\varepsilon_{c,k}) + \Pr(\varepsilon_{e,k})\right] . \tag{9}$$

The optimum thresholds are provided in the following theorem.

Theorem 1. *The set of optimal thresholds $\{L_k^*\}$ which minimize the overall BER at the destination can be expressed as*

$$\begin{aligned} L_1^* &= \ln\left[\frac{P_1^{(e)} - P_2^{(e)}}{P_2^{(c)} - P_1^{(c)}}\right], \quad L_2^* = \ln\left[\frac{P_2^{(e)} - P_3^{(e)}}{P_3^{(c)} - P_2^{(c)}}\right], \\ L_3^* &= \ln\left[\frac{P_3^{(e)} - P^{(s)}}{P^{(s)} - P_3^{(c)}}\right], \end{aligned} \tag{10}$$

where $P_k^{(c)} = \frac{1}{2}\left(P_{1,k}^{(c)} + P_{2,k}^{(c)}\right)$, $P_k^{(e)} = \frac{1}{2}\left(P_{1,k}^{(e)} + P_{2,k}^{(e)}\right)$, and $P^{(s)} = \frac{1}{2}\left(P_1^{(s)} + P_2^{(s)}\right)$.

Proof: We take the partial derivative of the average BER $P_{error} = \frac{1}{2}(P_{error,1} + P_{error,2})$ in (6) with respect to L_1, L_2 L_3 and set the result in each case equal to zero, $\frac{\partial(P_{error})}{\partial L_k} = 0$. For the case of L_1, we obtain

$$\begin{aligned} &\left(P_2^{(c)} - P_1^{(c)}\right) \frac{\partial\left(\Pr\left(\varepsilon_{c,1}\right)\right)}{\partial L_1} + \left(P_2^{(c)} - P^{(s)}\right) \cdot \\ &\frac{\partial\left(\Pr\left(\varepsilon_{c,2}\right)\right)}{\partial L_1} + \left(P_3^{(c)} - P^{(s)}\right) \frac{\partial\left(\Pr\left(\varepsilon_{c,3}\right)\right)}{\partial L_1} = \\ &\left(P_1^{(e)} - P_2^{(e)}\right) \frac{\partial\left(\Pr\left(\varepsilon_{e,1}\right)\right)}{\partial L_1} + \left(P_2^{(e)} - P^{(s)}\right) \cdot \\ &\frac{\partial\left(\Pr\left(\varepsilon_{e,2}\right)\right)}{\partial L_1} + \left(P_3^{(e)} - P^{(s)}\right) \frac{\partial\left(\Pr\left(\varepsilon_{e,3}\right)\right)}{\partial L_1} \end{aligned} \tag{11}$$

where we have used (9); this simplifies to

$$\begin{aligned} &\left(P_2^{(c)} - P_1^{(c)}\right) \frac{\partial(\Pr(\varepsilon_{c,1}))}{\partial L_1} = \\ &\left(P_1^{(e)} - P_2^{(e)}\right) \frac{\partial(\Pr(\varepsilon_{e,1}))}{\partial L_1}, \end{aligned} \tag{12}$$

In line with the assumption on the PDF of $P_{L_{R,j}}(L)$ as given in (7), and the fact that $\sigma_L^2 = 2\mu_L$, the optimal threshold L_1^* follows as in (10). Similarly, we can derive the optimal thresholds for L_2^* and L_3^* as in (10). This completes the proof. ■

The probabilities $P_k^{(c)}$, $P_k^{(e)}$ and $P^{(s)}$, which appear in (10), are derived in the next section.

B. Destination LLR formation

It not trivial to compute the LLRs at the destination corresponding to the relay transmission. Using Bayes' theorem, and denoting the quantization levels by $\{s_k\}$, we obtain

$$L_{RD,j} = \ln\left(\frac{\sum_{s_k>0} p(y_{RD}^j|s_k)p_k}{\sum_{s_k<0} p(y_{RD}^j|s_k)p_k}\right), \quad (13)$$

where by symmetry, we can write $p_k = p(s_k) = p(-s_k)$ for any quantization level s_k, these values being estimated at the destination. Also, from Gaussianity of the channel noise we have $p(y_{RD}^j|\pm s_k) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(y_{RD}^j \mp s_k)^2}{2\sigma^2}\right)$.

The LLR values at the destination corresponding to the source transmissions are $L_{iD,j} = \frac{2h_{iD}}{\sigma^2} y_{iD,j}$. The extrinsic LLR for bit $x_{i,j}$, determined from the network coding operation between [4] $x_{\bar{i},j}$ and $\tilde{x}_{R,j}$, is represented by

$$L_{iD,j}^E = 2\tanh^{-1}\left(\tanh\left(L_{\hat{i}D,j}/2\right)\tanh\left(L_{RD,j}/2\right)\right). \quad (14)$$

The combined LLR at the destination, denoted by $L_{D,j}$, is computed as $L_{D,j} = L_{iD,j} + L_{iD,k,j}^E$.

C. Error Probability Analysis

As we can see from (10), the optimal thresholds L_k^* are based on $P_{i,k}^{(c)}$, $P_{i,k}^{(e)}$, and $P_i^{(s)}$, which in turn are based on the channel realizations h_{1D}, h_{2D} and h_{RD}. Now, we will determine these probabilities when perfect CSI (i.e., knowledge of h_{1D}, h_{2D} and h_{RD}) is available at the relay. The received LLRs at the destination, i.e., $L_{iD,j}$, $L_{RD,j}$, and $L_{iD,k,j}^E$, given $h_{i,D}$ and h_{RD}, are approximately Gaussian distributed with their variances being twice the absolute value of their means [10]. The mean of $L_{iD,j}$ can be approximated as $m_{L_{iD}} = \mathbb{E}(L_{iD,j}) \triangleq \frac{2h_{iD}^2 x_{i,j}}{\sigma^2}$ and the mean value of $L_{RD,j}$ conditioned on the transmission of level s_k by the relay can be approximated as $m_{L_{RD,k}} = \mathbb{E}(L_{RD,j}|s_k) \triangleq \frac{2h_{RD}^2 s_k^2 \mathrm{sgn}(x_{R,j})}{\sigma_{RD}^2}$. Then the mean value of the extrinsic LLR $L_{iD,k,j}^E$, denoted by $m_{L_{iD,k}^E}$, can be calculated by employing the function $\phi(z)$ where $z \in (0,\infty]$ introduced in [10] i.e.,

$$\phi(z) = \begin{cases} 1 - \frac{1}{\sqrt{4\pi z}}\int_{-\infty}^{\infty}\tanh(\frac{u}{2})\exp\left(-\frac{(u-z)^2}{4z}\right)du, & z>0,\\ 1 & z=0.\end{cases} \quad (15)$$

It is easy to check that $\phi(z)$ is a continuous and monotonically decreasing function on $z \in (0,\infty]$, with $\phi(0) = 1$ and $\phi(\infty) = 0$. Using (15), we obtain the following mean value for $L_{iD,k,j}^E$ as

$$\begin{aligned} m_{L_{iD,k}^E} &= x_{i,j}\mathrm{sgn}\left(\tilde{x}_{R,j}\right)\phi^{-1}\left(\phi\left(|m_{L_{iD,j}^E}|\right)\right.\\ &\left.+\phi\left(|m_{L_{RD,k}^E}|\right) - \phi\left(|m_{L_{iD}}|\right)\phi\left(|m_{L_{RD,k}}|\right)\right). \end{aligned} \quad (16)$$

As in [10], when $z > 0$, the function $\phi(.)$ is bounded by $\sqrt{\frac{\pi}{z}}\exp\left(-\frac{z}{4}\right)\left(1-\frac{3}{z}\right) < \phi(z) < \sqrt{\frac{\pi}{z}}\exp\left(-\frac{z}{4}\right)\left(1+\frac{1}{7z}\right)$. As derived in [10], when z is large enough, the upper and lower bounds converge to $\sqrt{\frac{\pi}{z}}\exp\left(-\frac{z}{4}\right)$. Therefore, given z_1 and z_2, when both of them are large enough, we have

$$\phi(z_1) + \phi(z_2) - \phi(z_1)\phi(z_2) \approx \max\left(\phi(z_1), \phi(z_2)\right). \quad (17)$$

In a similar manner, we can approximate the mean $m_{L_{iD,k}^E}$ in the high SNR regime as $m_{L_{iD,k}^E} \approx x_{i,j}\mathrm{sgn}(\tilde{x}_{R,j})\phi^{-1}(\max(\phi(m_{L_{iD,j}}), \phi(m_{L_{RD,k,j}})))$ and this can be approximated to $m_{L_{iD,k}^E} = \frac{x_{i,j}\mathrm{sgn}(\tilde{x}_{R,j})}{\sigma^2}\min(h_{iD}^2, h_{RD}^2(s_k)^2)$. The combined LLR $L_{D,j}$ can be considered as approximately Gaussian distributed with mean $m_{L_{D,k}} = m_{L_{iD}} + m_{L_{iD,k}^E}$ and variance $\sigma_{L_{D,k}}^2 = 2(|m_{L_{iD,k}}| + |m_{L_{iD,k}^E}|)$. The instantaneous error probabilities can be obtained by utilizing the $Q(.)$ function, $Q(z) = \int_z^{\infty}\exp\left(-\frac{u^2}{2}\right)du$. In the case of $\varepsilon_{c,k}$, we have $\tilde{x}_{R,j} = \tilde{x}_{ij}\tilde{x}_{ij}$. In this case, $m_{L_{iD}}$ and $m_{L_{iD,k}^E}$ have the same signs and thus $|m_{L_{iD}} + m_{L_{iD,k}^E}| = |m_{L_{iD}}| + |m_{L_{iD,k}^E}|$. Therefore we obtain the error probability of $x_{i,j}$ as

$$P_{i,k}^{(c)} = Q\left(\sqrt{\frac{|m_{L_{iD}}| + |m_{L_{iD,k}^E}|}{2}}\right). \quad (18)$$

In the event of incorrect information forwarding from the relay, i.e., when $\varepsilon_{e,k}$ occurs, we have $\mathrm{sgn}(\tilde{x}_{R,j}) = -x_{R,j}$. In this case, $m_{L_{iD}}$ and $m_{L_{iD,k}^E}$ have opposite signs and thus we have $|m_{L_{iD}} + m_{L_{iD,k}^E}| = ||m_{L_{iD}}| - |m_{L_{iD,k}^E}||$. The value of $P_{i,k}^{(e)}$ is then

$$\begin{cases} Q\left(\sqrt{\frac{(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)^2}{2(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)}}\right), & |m_{L_{iD}}| > |m_{L_{iD,k}^E}|,\\ 1-Q\left(\sqrt{\frac{(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)^2}{2(|m_{L_{iD}}|-|m_{L_{iD,k}^E}|)}}\right), & |m_{L_{iD}}| \le |m_{L_{iD,k}^E}|.\end{cases} \quad (19)$$

When the relay is silent, i.e., when ϵ_s occurs, we have $P_{i,s} = Q\left(\sqrt{|m_{\lambda_{x_iD}}|/2}\right)$. By use of (18),(19) and $P_{i,s}$ we can compute the optimum positive values of three-level thresholds.

IV. Simulation results and Discussion

In this section, simulation results are provided to demonstrate the effectiveness of the proposed system. In the simulations, we use uncoded BPSK signaling with a frame length of $N = 10,000$. All channels are assumed to exhibit quasi-static fading, i.e., the channel co-efficients h_{1D}, h_{2D}, h_{RD}, h_{1R} and h_{2R} are constant for each transmission phase and change independently from one phase to the next.

We consider the case of a symmetric MARC system, where the relay is located midway between the two sources and the destination, i.e., $d_{1R} = d_{2D} = d_{RD} = 0.5$. The distances between the sources and the destination are normalized to unity, i.e., $d_{1D} = d_{2D} = 1$. The attenuation exponent was chosen to be $\gamma = 2$.

Fig. 2 displays the BER for the proposed uncoded MLT-SQ scheme. In this work, we assume optimization of the three positive threshold levels, i.e., the values L_1^*, L_2^* and L_3^* in

[4] The subscript $\bar{i}$ refers to the opposite source when source i is under consideration.

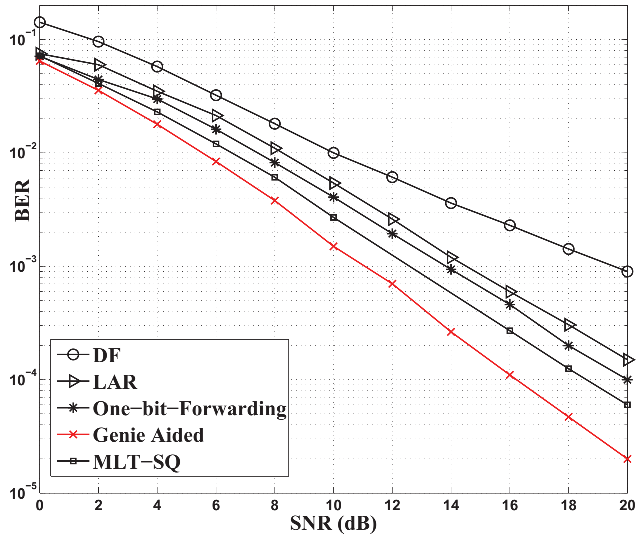

Figure 2. Error rate performance of the proposed MLT-SQ scheme, together with that of competing schemes, in a quasi-static fading environment.

Theorem 1 (and symmetric negative levels). As one of the baselines, we use the conventional DF protocol for comparison in Fig. 2. In this simulation, the relay decodes the received signals from S_1 and S_2. It makes hard decisions based on the *a priori* channel LLRs and then transmits network-coded symbols from the relay. As shows in the simulation, it has the worst error performance as compared to other protocols and it does not achieve the full diversity order (i.e., 2).

As a benchmark for the thresholding scheme, we here assume that, for any given frame, the relay transmission is error-free. This protocol, known as the 'Genie Aided' protocol, is rather unrealistic but gives us a useful benchmark for the proposed scheme. We have also presented the simulation result for LAR, a power scaling scheme where the relay located power scalar w is given by $w = \min\left(\frac{\gamma_{SR,min}}{\gamma_{RD}}, 1\right)$ where γ_{SR} and γ_{RD} are the channel SNR of $S - R$ and $R - D$ links respectively. In the present context, we have $w = \min\left(\frac{\min\left(h^2_{S_1R}, h^2_{S_2R}\right)}{h^2_{RD}}, 1\right)$.

In addition, we have also simulated one-bit forwarding (MLT-SQ with a single positive level). In this technique, we have three threshold levels. If the amplitude of the relay received reliability $|L_{R,j}|$ is higher than the threshold value L^*_T, then the relay transmits the symbol $\text{sgn}(L_{R,j})$; otherwise it stays silent (this forms three threshold levels). As we can see in the simulation, it has improved BER performance over uncoded DF and LAR.

As can be seen from the simulation results, the proposed MLT-SQ scheme has superior BER performance over the three-level forwarding case, which may be viewed as (conditional) one-bit forwarding. It is also shown in the simulation results that the proposed MLT-SQ scheme displays better performance over the uncoded DF, one-bit forwarding and LAR. The MLT-SQ scheme yields considerable gains in the regime of low source-relay SNR, since here erroneous detection at the relay results in the forwarding of incorrect symbols to the destination. Note that the proposed MLT-SQ system offers a clear advantage for channels which exhibit strong variations in SNR, since it is robust to such variations.

V. Conclusion

We have developed a novel optimized soft quantization scheme (MLT-SQ) based on cooperative network coding in a multiple access relay system. We present the BER analysis of the proposed MLT-SQ scheme and used this to optimize each level of the network coded multilevel thresholding scheme in order to minimize the BER at the destination. Instead of forwarding hard decisions, our proposed scheme forwards soft quantized values based on the magnitude of the network coded LLRs. We compared our proposed technique with other relevant competing schemes and displayed significant improvement of performance in terms of BER.

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