Soft Decode-and-Forward using LDPC coding in Half-Duplex Relay Channels

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Abstract—**This paper proposes a new soft decode-and-forward (SDF) protocol using LDPC codes in the half-duplex relay channels. In order for the encoding of the additional parity-check symbols at the relay to be linear and systematic, we introduce a structured rate-compatible (RC) LDPC code. We then develop the soft-decoding and soft-re-encoding algorithms for the proposed RC-LDPC code, which allows the relay to forward soft messages to the destination when the relay fails to decode the source's message. Furthermore, we propose a new method, which we refer to as *soft fading*, to compute the log-likelihood ratio (LLR) of the received signal at the destination for the SDF protocol. We show that our proposed method performs better when compared to a previous reported method in literature.**

I. INTRODUCTION

Cooperative communication [1], [2] for wireless networks promise improved transmit diversity and increased spectral efficiency. In the context the integration of cooperation and coding, the use of a decode-and-forward (DF) strategy instead of the simple amplify-and-forward (AF) strategy has proven to be a particularly useful in providing additional coding gain [2]. Distributed channel coding (DCC) schemes for the three-nodes relay channel represent one important subset in implementing cooperation with coding. These DCC schemes can be classified into two groups of capacity achieving codes: the turbo codes [3], [4] and the LDPC codes [5], [6], [7].

While the DCC schemes using a DF strategy always assume that the source-relay channel is error free, this cannot always be guaranteed in practice. In order to mitigate the effect of error propagation at the relay, the soft DF (SDF) protocol was proposed in [8], [9], [10] using distributed turbo codes. The work in [9] models the soft-errors introduced by the relay as Gaussian noise, which will then be added at the destination when computing the received log-likelihood ratio (LLR). However, the work in [10] numerically shows that the LLR distribution at the destination for the turbo code's used in [9] is not Gaussian. In another class of work [11], distributed turbo product codes (DTPC) with soft information relaying (SIR) over cooperative networks are proposed. In this work we investigate the use of SIR with LDPC codes.

The main challenge of performing SIR using LDPC codes is the soft-re-encoding at the relay. It is a well known result that the capacity-achieving LDPC codes have irregular degree distributions (i.e. the code has random structure), resulting to the issue of having a low complexity systematic encoder.

In a relay channel, the encoder becomes more complicated because the rate-compatible (RC) LDPC codes [6] form nested sequence of code bits where the parity bits of higher rate codes for the source-relay channel are embedded in those of lower rate codes for the source-destination.

This paper represents the first attempt to implement SIR within LDPC codes. We propose a structured RC-LDPC code with linear and systematic encoder for the additional parity bits at the relay. Based on this systematic code's structure, a soft-re-encoding of the additional parity bits at the relay is developed. Then, we propose a new method to compute the LLR of the received signal at the destination under the SDF protocol, where the soft-errors introduced at the relay are described via fading coefficients. We refer to our new method as *soft fading*. Finally, we show that the proposed *soft fading* method performs better when compared to a previous reported method in literature.
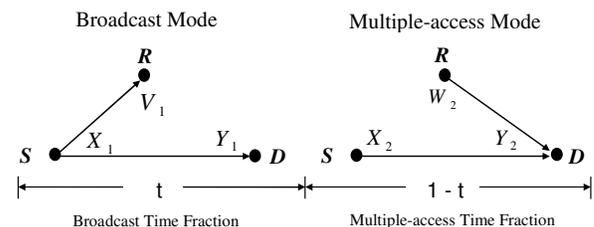
Figure 1. Half-duplex relay channels.

II. SYSTEM MODEL AND LDPC CODING SCHEME

A. Half-duplex relay model

The half-duplex relay channel is illustrated in Fig. 1, where one source (S) sends independent messages to the destination (D), and in doing so it is aided by one half-duplex relay (R). Given a time window L, the total of $N = N_1 + N_2$ symbols are received at D. In a fraction of time tL (namely, broadcast (BC) mode), S broadcasts N_1 symbols that can be received by both R and D. In the remaining fraction of time $t'L = (1 - t)L$ (namely, multiple-access (MAC) mode), both S and R transmit N_2 symbols to D simultaneously. We denote the source transmitted signal, relay received signal, relay transmitted signal and destination received signal by using X, V, W and Y (see Fig. 1), respectively. In order

to distinguish between the transmission modes, we denote the BC mode using subscript 1 and the MAC mode using subscript 2. With the above notations, the half-duplex relay channel is defined by

$$V_1 = h_{SR}X_1 + N_{R1} \tag{1}$$
$$Y_1 = h_{SD}X_1 + N_{D1} \tag{2}$$
$$Y_2 = h_{SD}X_2 + h_{RD}W_2 + N_{D2} \tag{3}$$

where, h_{ij} is the channel realization between node i and node j, while N_{D1} and N_{D2} are the noise realizations at the destination in BC and MAC modes, respectively, and N_{R1} is the noise realization at the relay in BC mode. All N_{R1}, N_{D1} and N_{D2} noises are Gaussian with zero mean and unit variance ($\sigma^2 = 1$), i.e. $\mathcal{N}(0,1)$. In this work, the source and the relay transmit with equal power P and BPSK modulation is employed. The nodes S, R and D are assumed to lie on a straight line[1], where we normalized the distance between S and D to unity, and d denotes the position of R relative to S with $0 \leq d \leq 1$. With this setup, the channel gains are $h_{SD} = 1$, $h_{SR} = \frac{1}{\sqrt{d^\alpha}}$ and $h_{RD} = \frac{1}{\sqrt{(1-d)^\alpha}}$, where α is the path-loss exponent. In this paper, we choose $\alpha = 2$ for our plots. Thus, perfect global channel knowledge is assumed at all received nodes.

B. LDPC coding for half-duplex relay systems

From the achievable rate point of view, the relay coding strategy can achieve two fundamental extremes. In one extreme, R and S simultaneously send completely independent information, which interfere with each other; in the other extreme, R and S simultaneously send identical information (i.e. $X_2 = W_2$), which forms a coherent transmission and corresponds to the relay coding strategy in this paper. Based on the latter strategy, S transmits a binary LDPC code $\underline{w}_1$ with length N_1 and dimension K in the BC mode, i.e. with rate $R_1 = K/N_1$. The code $\underline{w}_1$ is defined by a $(N_1 - K) \times N_1$ parity check matrix H_1. Assuming perfect decoding at R[2], both S and R transmit another codeword segment $\underline{w}_2$ with length N_2 that only contains additional parity-check bits in the MAC mode. Thus, D will receive the full codeword $\underline{w} = [\underline{w}_1 \underline{w}_2]$ with rate $R_c = K/N$. This full codeword is defined by a $(N - K) \times N$ parity check matrix H as illustrated in Fig. 2(a), which includes H_1. Note that the sub-matrix H_2 is a lower triangular parity-check matrix. As a consequence of this, $\underline{w}_2$ can be linearly encoded at S and R based on the LDPC code $\underline{w}_1$ using *back-substitution* [12]. The rate compatibility and linear encoder of $\underline{w}_2$ using H_2 are the properties of the full codeword $\underline{w} = [\underline{w}_1 \underline{w}_2]$. We refer the full LDPC code $\underline{w} = [\underline{w}_1 \underline{w}_2]$ as the rate-compatible LDPC codes. Fig. 2(b) summarizes the transmission model using the proposed RC-LDPC codes in the half-duplex relay channels, which can be

written as follows

$$\underline{V}_1 = \sqrt{P} \cdot h_{SR} \cdot \underline{w}_1 + \underline{N}_{R1} \tag{4}$$
$$\underline{Y}_1 = \sqrt{P} \cdot h_{SD} \cdot \underline{w}_1 + \underline{N}_{D1} \tag{5}$$
$$\underline{Y}_2 = \sqrt{P} \cdot (h_{SD} + h_{RD}) \cdot \underline{w}_2 + \underline{N}_{D2} \tag{6}$$

where the codeword vectors transmitted in the BC and MAC modes can be expressed as $\underline{w}_1 = [w_{1,1}, w_{1,2}, \ldots, w_{1,N_1}]^T$ and $\underline{w}_2 = [w_{2,1}, w_{2,2}, \ldots, w_{2,N_2}]^T$, respectively. Similarly, the received vectors can be expressed as $\underline{V}_1 = [v_{1,1}, v_{1,2}, \ldots, v_{1,N_1}]^T$, $\underline{Y}_1 = [y_{1,1}, y_{1,2}, \ldots, y_{1,N_1}]^T$ and $\underline{Y}_2 = [y_{2,1}, y_{2,2}, \ldots, y_{2,N_2}]^T$.

(a)

(b)
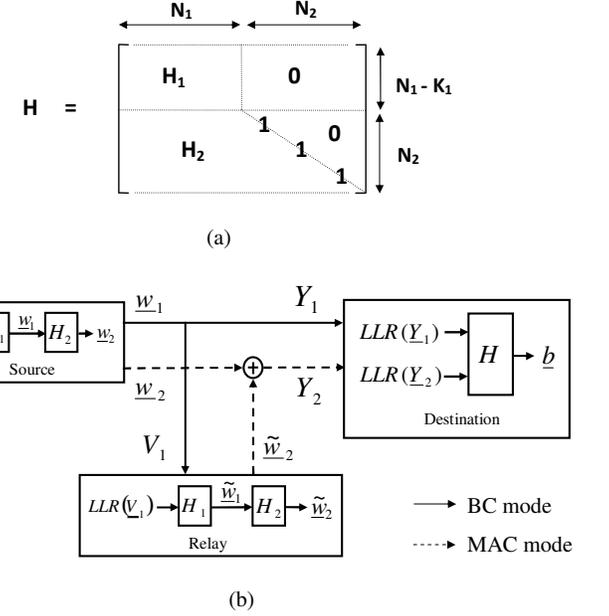
Figure 2. (a) Rate-compatible structure of the parity-check matrix for half-duplex relay codes. (b) Transmission model with rate-compatible LDPC codes and iterative decoding. The relay only decodes $\underline{w}_1$ using H_1, while the destination decodes $\underline{w} = [\underline{w}_1 \underline{w}_2]$ using H.

III. SOFT DECODE AND FORWARD

In this section, we consider the case where the relay fails to decode LDPC code $\underline{w}_1$, i.e. $\widetilde{\underline{w}}_1 \neq \underline{w}_1$ (refer to Fig. 2(b)), and we explain how the SIR can be performed using the RC-LDPC code structure introduced in the previous section.

A. Calculation of soft code bits at the relay

The first step required for SIR is the calculation of the LLR $L(\underline{w}_1/\underline{V}_1)$ of $\underline{w}_1$ from the received sequence $\underline{V}_1$ at the relay. Since $\underline{w}_1$ is a valid LDPC codeword, this computation can be easily performed using the Belief Propagation (BP) decoder of H_1. We denote $L^p(\underline{w}_1/\underline{V}_1)$ as the LLR of $\underline{w}_1$ after p iteration of the BP decoder. From the computed LLR values, the soft bits of $\underline{w}_1$ is given by

$$\widehat{\underline{w}}_1 = tanh(L^p(\underline{w}_1/\underline{V}_1)/2). \tag{7}$$

The second step required for SIR is the encoding of the parity bits $\underline{w}_2$ from the reliability of soft bits $\widehat{\underline{w}}_1$. The parity

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[1]Even though we assume all the nodes lie on a straight line, the extension to any topology of relay channels is straightforward.

[2]Code $\underline{w}_1$ is designed with a rate that achieves the capacity of the S-R channel.

bits $\underline{w}_2$ can be encoded linearly using *back-substitution* since H_2 has a lower triangular structure. The value of the first soft bit $\widehat{w}_{2,1}$ in vector $\widehat{\underline{w}}_2$ can be calculated using

$$\widehat{w}_{2,1} = \prod_{k \in B_1} \widehat{w}_{1,k} \qquad (8)$$

while the remaining soft bits in vector $\widehat{\underline{w}}_2$ are given by

$$\widehat{w}_{2,j} = \prod_{k \in B_1} \widehat{w}_{1,k} \prod_{k \in B_2, k \neq j} \widehat{w}_{2,k} \qquad \text{for } j = 2, \ldots, N_2 \qquad (9)$$

where B_1 (B_2) is the set of message nodes incident to check node j that carries the soft information from vector $\widehat{\underline{w}}_1$ ($\widehat{\underline{w}}_2$), i.e. the message nodes that involve in the parity check equation j. After all soft bits in $\widehat{\underline{w}}_2$ have been computed, the signal transmitted from the relay can be written as

$$\widetilde{\underline{w}}_2 = \sqrt{P} \cdot \beta \cdot \widehat{\underline{w}}_2 \qquad (10)$$

where the power normalization per transmission β can be calculated from the transmitted power constraint at the relay, given by

$$\beta = \sqrt{\frac{1}{\frac{1}{N_2} \sum_{i=1}^{N_2} |\widehat{w}_{2,i}|^2}}. \qquad (11)$$

Remark 1: For the case where relay can successfully decode $\underline{w}_1$ using the BP decoder of H_1, the signs of $L(\underline{w}_1/\underline{V}_1)$ satisfy all parity check equations spanned by H_1. When this happens, the soft bit value of $\widehat{w}_{1,i}$ is either 1 or -1, i.e. the hard decision for a 0 or 1 binary bit $\breve{w}_{1,i}$. As a consequence, the encoding of a binary bit $\breve{w}_{2,i}$ using *back-substitution* can be written as [12]

$$\breve{w}_{2,i} = \left[\sum_{j=1}^{N_1} \oplus (H_2(i,j) \cdot \breve{w}_{1,j}) \right] \oplus \left[\sum_{j=1}^{i-1} \oplus (H_2(i,j+N_1) \cdot \breve{w}_{2,j}) \right]$$

where the $\Sigma \oplus$ and $\oplus$ symbols denote the summation and addition operators, respectively, in binary field. The power normalization for the hard decision encoding is $\beta = 1$ because $\widehat{w}_{2,i} \in \{-1, +1\}$. Note that this is the standard DF scheme.

B. Calculation of LLR at the destination

In order to perform the BP iterative decoding using the H matrix, D has to compute the input LLRs for the codeword vectors $\underline{w}_1$ and $\underline{w}_2$ based on the received signals $\underline{Y}_1$ and $\underline{Y}_2$, respectively. Here, we start by discussing the LLR computation for the DF scheme, i.e. for the case where R can correctly decode $\underline{w}_1$. Under the DF scheme, the received LLRs $L(y_{1,i}/w_{1,i})$ and $L(y_{2,i}/w_{2,i})$ at D for each bit $w_{1,i}$ and $w_{2,i}$, respectively, can be computed using

$$L(y_{1,i}/w_{1,i}) = 2 \cdot \frac{\sqrt{P} \cdot h_{SD}}{\sigma^2} \cdot y_{1,i} \qquad (12)$$

$$L(y_{2,i}/w_{2,i}) = 2 \cdot \frac{\sqrt{P} \cdot (h_{SD} + h_{RD})}{\sigma^2} \cdot y_{2,i}. \qquad (13)$$

Note that $L(y_{2,i}/w_{2,i})$ is obtained because S and R coherently transmit the same value of the parity check bit $w_{2,i}$.

For the SDF scheme where R fails to correctly decode $\underline{w}_1$, R will transmit the soft bits $\beta\widehat{\underline{w}}_2$ instead of the hard

decision bit of $\underline{w}_2$. Under SIR, the received LLR $L(y_{1,i}/w_{1,i})$ for $w_{1,i}$ remains the same as in (13), since there are no changes in the transmission of LDPC code $\underline{w}_1$ in the BC mode. The only difference is the computation of $L(y_{2,i}/w_{2,i})$ because S and R are transmitting two different vectors $\underline{w}_2$ and $\beta\widehat{\underline{w}}_2$, respectively, during the MAC mode. Hence, the accurate and precise LLR computation of $L(y_{2,i}/w_{2,i})$ requires the knowledge of the error probability $Pr(\beta\widehat{w}_{2,i} \neq w_{2,i})$.

In [9], the knowledge of $Pr(\beta\widehat{w}_{2,i} \neq w_{2,i})$ was modeled as $\widehat{w}_{2,i} = w_{2,i}(1 - \overline{n}_s)$, where $\overline{n}_s$ is the equivalent noise introduced at R. This noise can then be added when computing the received LLR at D. In this paper, we refer $\overline{n}_s$ as *soft noise*. The *soft noise* $\overline{n}_s$ is assumed to be Gaussian with the mean value of $\mu_{\overline{n}_s} = E\{1 - w_{2,i} \cdot \widehat{w}_{2,i}\}$ and the variance value of $\sigma^2_{\overline{n}_s} = E\{(1 - w_{2,i} \cdot \widehat{w}_{2,i} - \mu_{\overline{n}_s})^2\}$ [9]. Now, we will briefly explain how the statistics[3] of $\overline{n}_s$ can be incorporated in the computation of $L(y_{2,i}/w_{2,i})$ when soft bits $\widehat{\underline{w}}_2$ are transmitted by R. Using the *soft noise* model, the signal transmitted from R in (10) can be written as

$$\widetilde{w}_{2,i} = \sqrt{P} \cdot \beta_{sn} \cdot w_{2,i}(1 - \overline{n}_s) \qquad (14)$$

and the normalized factor β_{sn} can be computed by using[4]

$$\beta_{sn} = \sqrt{\frac{1}{(1 - \mu_{\overline{n}_s})^2 + \sigma^2_{\overline{n}_s}}}. \qquad (15)$$

At D, the received signal corresponding to the transmissions from S and R in the MAC mode is given by

$$y_{2,i} = \sqrt{P} \cdot \overline{h}_E \cdot w_{2,i} + \overline{n}_E \qquad (16)$$

where $\overline{h}_E = h_{SD} + h_{RD}\beta_{sn}(1 - \mu_{\overline{n}_s})$ and $\overline{n}_E$ is the equivalent Gaussian noise at D, with zero mean and variance $\sigma^2_E = \sigma^2 + |h_{RD}\beta_{sn}|^2\sigma^2_{\overline{n}_s}$. The approximated LLR $L_{sn}(y_{2,i}/w_{2,i})$ can then be computed using

$$L_{sn}(y_{2,i}/w_{2,i}) = 2\frac{\sqrt{P} \cdot (\overline{h}_E)}{\sigma^2_E} y_{2,i}. \qquad (17)$$

Here, we propose a new method called the *soft fading* to incorporate the knowledge of $Pr(\beta\widehat{w}_{2,i} \neq w_{2,i})$ when computing the received LLR at D. Our proposed method represents the relationship between the soft bit $\widehat{w}_{2,i}$ and the correct bit $w_{2,i}$ in the following manner

$$\widehat{w}_{2,i} = \overline{h}_s \cdot w_{2,i} \qquad (18)$$

where $\overline{h}_s$ can be seen as the fading coefficient, i.e. the soft bit $\widehat{w}_{2,i}$ is the received signal of $w_{2,i}$ through a channel with multiplicative fading of $\overline{h}_s$. Here, we call $\overline{h}_s$ the *soft fading*

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[3] All statistical values of the soft noise (i.e. $\mu_{\overline{n}_s}$ and $\sigma^2_{\overline{n}_s}$) are computed offline numerically. These statistical values are stored at D for the computation of received LLR during real time transmission. Hence, R does not need to provide any extra information to D.

[4] β_{sn} is the average normalized factor calculated on the assumption that $\overline{n}_s$ is Gaussian with mean $\mu_{\overline{n}_s}$ and variance $\sigma^2_{\overline{n}_s}$. Note that β_{sn} is not equal to β in (11), where β is the power normalization factor for every codeword $\underline{w}_2$ during real time transmission.

variable. Using this method, the received signal at D in MAC mode can be written as

$$
\begin{aligned}
y_{2,i} &= \sqrt{P} \cdot (h_{SD} \cdot w_{2,i} + h_{RD} \cdot \beta_{sf} \cdot \widehat{w}_{2,i}) + N_{D2} \\
&= \sqrt{P} \cdot (h_{SD} + h_{RD} \cdot \beta_{sf} \cdot \overline{h}_s) \cdot w_{2,i} + N_{D2} \\
&= \overline{h}_F \cdot w_{2,i} + N_{D2}
\end{aligned}
\tag{19}
$$

where the normalized factor β_{sf} can be computed by using[5]

$$
\beta_{sf} = \sqrt{\frac{1}{E(\overline{h}_s^2)}} \qquad \text{for} \quad E(\overline{h}_s^2) > 0.
\tag{20}
$$

If we know the probability density function (pdf) of $\overline{h}_F$, the received LLR of $L(y_{2,i}/w_{2,i})$ can be written as [13]

$$
\log \frac{\int_{-\infty}^{+\infty} f_{Y_2|W_2,\overline{H}_F}(y_{2,i}|w_{2,i} = +1, \overline{h}_F) f_{\overline{H}_F}(\overline{h}_F) d\overline{h}_F}{\int_{-\infty}^{+\infty} f_{Y_2|W_2,\overline{H}_F}(y_{2,i}|w_{2,i} = -1, \overline{h}_F) f_{\overline{H}_F}(\overline{h}_F) d\overline{h}_F}
\tag{21}
$$

where

$$
f_{Y_2|W_2,\overline{H}_F}(y_{2,i}|w_{2,i}, \overline{h}_F) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y_{2,i} - \overline{h}_F \cdot w_{2,i})^2}{2\sigma^2}\right]
$$

and $f_{\overline{H}_F}(\overline{h}_F)$ is the pdf of $\overline{h}_F$. Note that the only random variable in $\overline{h}_F$ is the *soft fading variable* $\overline{h}_s$. So, $f_{\overline{H}_F}(\overline{h}_F)$ solely depends on the pdf of *soft fading variable* $\overline{h}_s$ denoted by $f_{\overline{H}_s}(\overline{h}_s)$ in which its closed form expression is unknown. Even though the full knowledge of $f_{\overline{H}_s}(\overline{h}_s)$ is unknown, there are still some important properties about $f_{\overline{H}_s}(\overline{h}_s)$ that can be obtained by studying the characterization of the soft bits $\widehat{w}_{2,i}$. Below are the two properties of $f_{\overline{H}_s}(\overline{h}_s)$:

- The *soft fading variable* $\overline{h}_s$ is a continuous random variable with value between 1 and -1, i.e. $-1 \leq \overline{h}_s \leq 1$.
- $0 \leq E(\overline{h}_s^2) \leq 1$: The value of $E(\overline{h}_s^2) = 0$ is achieved when R does not transmit any signal to D in the MAC mode, i.e. R remains silent. The value of $E(\overline{h}_s^2) = 1$ is achieved only if $\widehat{w}_{2,i} = w_{2,i}$, which in turn can only occur for the case of DF scheme.

Since the range of $\overline{h}_s$ is small (i.e. $-1 \leq \overline{h}_s \leq 1$), we can approximate the continuous random variable $\overline{h}_s$ using a discrete random variable $\overline{h}_s^*$ without losing much information. This approximation can be done by quantizing the continuous value of $\overline{h}_s$ into $K + 1$ discrete values. Let $S = \{-1, -1+\Delta, -1+2\Delta, \ldots, -1+(K-1)\Delta, +1\}$ be the set of the sample space for $\overline{h}_s^*$, where Δ is the quantization size[6]. The probability mass function (pmf) of the discrete random variable $\overline{h}_s^*$ can be written as

$$
f_{\overline{H}_s}(\overline{h}_s^*) = Pr_{\overline{H}_s}(\overline{h}_s^* = i) \qquad \text{for } i \in S.
\tag{22}
$$

Now, the challenge is to obtain the pmf of $\overline{h}_s^*$ given in (22). Here, we obtain the pmf of $\overline{h}_s^*$ by generating the

histogram of $\overline{h}_s^*$ numerically, which can be performed offline before the real time transmission take place[7]. Once the pmf (histogram) is generated, the normalized factor β_{sf} in (20) can be computed. Finally, the LLR $L(y_{2,i}/w_{2,i})$ in (21) can be easily approximated and computed using

$$
\log \frac{\Sigma_{i\in S} f_{Y_2|W_2,\overline{H}_F}(y_{2,i}|w_{2,i} = +1, \overline{h}_F^*) Pr_{\overline{H}_s}(\overline{h}_s = i)}{\Sigma_{i\in S} f_{Y_2|W_2,\overline{H}_F}(y_{2,i}|w_{2,i} = -1, \overline{h}_F^*) Pr_{\overline{H}_s}(\overline{h}_s = i)}
\tag{23}
$$

where $\overline{h}_F^* = \sqrt{P} \cdot (h_{SD} + h_{RD} \cdot \beta_{sf} \cdot i)$.
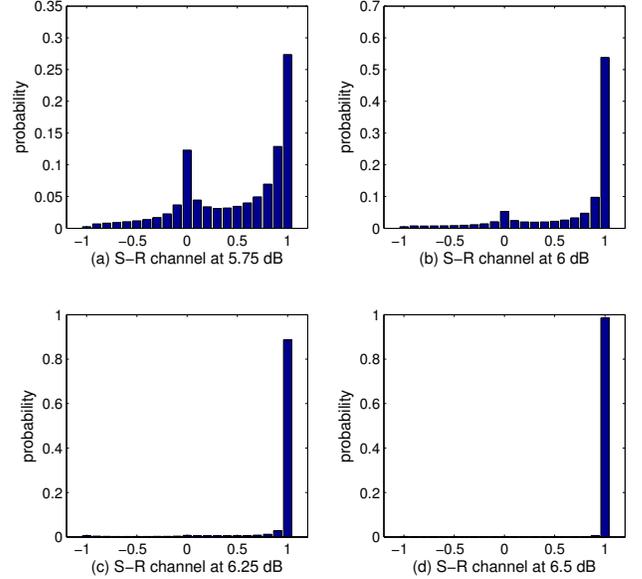
Figure 3. The numerical pmf for the *soft fading variable* using the proposed RC-LDPC code that operates at $R_c = 0.45$ and $R_1 = 0.9$ for different SNR of S-R channel. The code's block length is 20000 bits and 1000 samples of block errors are collected at the relay to generate the pmf.

IV. NUMERICAL RESULTS

We evaluate the performance of the proposed SIR using the RC-LDPC codes for the following half-duplex relay channel; $d = 0.29$; $t = 0.5$; $\underline{w}_1$ and $\underline{w} = [\underline{w}_1 \underline{w}_2]$ have the rate $R_1 = 0.9$ and $R_c = 0.45$, respectively. Using the results in [14], the capacity bound for the above half-duplex relay channel can be achieved with the transmission power of $P = -4.9$ dB. The transmission power P is equal to the SNR of the S-D channel because the distance between S and D is normalized to unity, and the noise at D is Gaussian with zero mean and unit variance.

Before the real time transmission takes place, all statistics of the *soft noise* and *soft fading* methods are computed numerically. Fig. 3 illustrates the numerical pmf (histograms) of *soft fading variables* $\overline{h}_s^*$ for different SNR of S-R channel using the proposed RC-LDPC code designed at the rate of

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[5]β_{sf} is an averaged normalized factor computed based on the distribution of *soft fading variable* $\overline{h}_s$. β_{sf} is not equal to β in (11), where β is the power normalization factor for every codeword $\underline{w}_2$ during real time transmission.

[6]Note that the smaller the quantization size Δ, the more accurate the discrete distribution to the original continuous distribution.

[7]This offline computation is similar to the case of *soft noise* modeling. The numerical pmf (histogram) can then be stored at D for the LLR computation during real time transmission. Hence, the transmission complexity of the *soft fading* model is similar to the *soft noise* model.

$R_1 = 0.9$ and $R_c = 0.45$. Note that the pmf is purely dependent on the quality of S-R channel. While the quality of S-R channel improves, the pmf of *soft fading variables* $\overline{h}_s^*$ becomes closer to the case when S-R channel is error free (i.e. $E(h_s^2) = 1$).
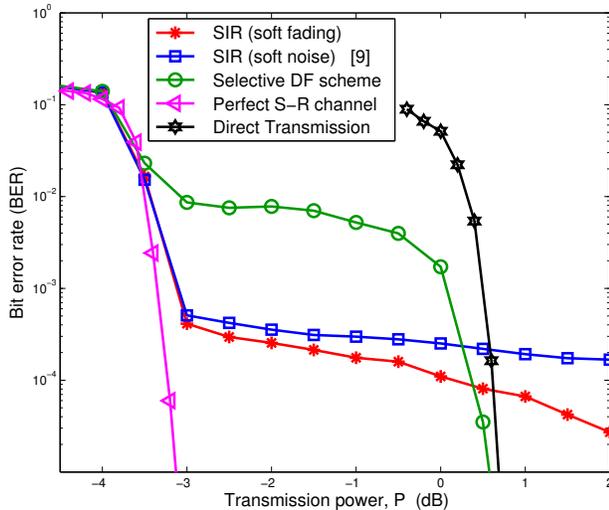
Figure 4. The BER performances for different cases using the proposed RC-LDPC code, which operates with rate $R_c = 0.45$ at the destination.

Fig. 4 depicts the simulation results at D using the proposed RC-LDPC code. First, there are two distinct waterfall regions, which correspond to the BER performances for two ideal cases. The first waterfall region at low SNR corresponds to the code's performance under the assumption that the S-R channel is error free. The second waterfall region at higher SNR corresponds to the code's performance when there is no cooperation from R in MAC mode, i.e. R is always silent in MAC mode. The latter case can also be seen as the code's performance for direct link transmission from S to D. There is an improvement of about 4 dB between these two ideal cases, which shows the benefit of using relay.

Next, we evaluate the code's performance at D when there are transmission errors at R. In the simulations, we fixed the SNR of the S-R channel at 6.25 dB (the Shannon limit is at 5.75 dB) to demonstrate the impact of transmission errors at the relay and how SIR can be applied to mitigate these errors. We start by evaluating the state-of-the-art scheme called Selective DF, where R will only transmit if it can decode $\underline{w}_1$ in BC mode. Otherwise, R will just be silent. For Selective DF scheme, the error in S-R channel causes significant error floor performance at D for the SNR region between the two ideal cases. The BER curve then decreases sharply once the SNR is equal or greater than the capacity of the direct transmission since D can decode the full codeword $\underline{w} = [\underline{w}_1 \underline{w}_2]$ by only using the received signal from S. The high error floor performance at the low SNR region for the Selective DF scheme can be significantly improved when SIR is applied at R. Unfortunately, performing SIR at the high

SNR when the S-R channel is in error does not greatly reduce the BER curves as in the case of the Selective DF scheme. However, the high SNR region is not the region of interest for SIR because the purpose of utilizing relay in the first place is to help the source's transmission when the quality of the S-D channel is poor. That is R is not needed if the channel quality between S and D is good in the first place. Finally, Fig. 4 demonstrates the performance of SIR using two different LLR computations at D. We can see that our proposed *soft fading* method achieves better BER performance when compared to the *soft noise* method introduced in [9].

V. CONCLUSION

This paper proposed a new SDF protocol using structured RC-LDPC codes in the half-duplex relay channels. We developed the soft-decoding and soft-re-encoding algorithms for the proposed RC-LDPC codes. We proposed a new method, which we refer as *soft fading*, to compute the LLR of the received signal at the destination. We demonstrated that the *soft fading* method achieves better BER performance as compared to a previous reported method in literature.

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